MITOCW | torque

Here are two people. They're both standing, but they're standing in two completely different ways. Which person would it be easier to push over? If you wanted to push this person [left] over, where would you apply a force? What about this [right] person? The answers to all these questions can be explained using the concept of torque. This video is part of the Representations video series. Information can be represented in words, through mathematical symbols, graphically, or in 3-D models. Representations are used to develop a deeper and more flexible understanding of objects, systems, and processes. Hi. I'm Sanjay Sarma. Professor of mechanical engineering at MIT, In this video, we'll be talking torque and balance. In order to understand these core concepts, you'll need a working knowledge of vectors and their uses. Specifically, you must be familiar with force, displacement, and torque. We will also assume that you know how to compute a cross product, and how to use the Right-Hand Rule, and that you have done problems involving the center of mass of an object. Our objective is to improve your ability to draw torque diagrams, and give you some practice with setting them up. By the end you should also understand what is needed for human beings to balance. We'll start with an activity. Everyone stand up and spread out across the room. You'll need a partner for this activity. When I say "go," your goal is to carefully push your partner over. Use the smallest amount of force you can. You will switch partners halfway through, so be gentle. When you push, consider where you should push, and in what direction. Try many different approaches. Here are some questions that may help you think about this in a scientific manner. When it comes to your push, where will you push? What direction will you push? How hard will you push? Consider your partner as well: how is your partner standing? What is the floor like under your partner's feet? Can your partner balance well? Are you ready? Go! Switch partners! Your teacher will now lead you in a short discussion about this activity. Pause the video here. Let's continue our investigation of torque and balance in the human body. Take a look at this next video clip, in which MIT researchers Colin Fredericks and Jennifer French demonstrate the effectiveness of properly applied torque. [Colin speaking] This is called "Sai son" stance. This is a basic stance in martial arts. Known for it's stability along a particular direction. This is my colleague, mathematician Jennifer French. Jen knows the stance is strong along certain directionsâ€"such as the one she is pulling in now. [laughing] Oops. It can also withstand a great deal of force in the opposite direction. I'm not going to be knocked over if she is pushing or pulling along that particular line. However, Jen knows the stance's weak point. If she pushes along a different line, it'll knock me right down to the ground. This isn't just martial arts. This is science. Torque controls my ability to balance and we're going to show you how today. [Sanjay speaking] To analyze the situation, let's look at what physical properties are important here. What forces do you think are involved? Pause the video to discuss. Next, draw a simple diagram that you can use to find the net torque on this man. Pause the video while you do this. The simplest way to represent this man is with a rectangle. You should remove any other details, and draw our forces so that we can tell exactly where they are applied to his body. Finally, if you were to draw a diagram showing someone resisting a push, how would you do it? Which of these three is most appropriate? Pause the video to discuss. You may be wondering why we can use a two-dimensional diagram to discuss a three-dimensional situation. The reason is that all of our forces are applied in the same plane, simplifying the problem. While our torque vectors point in and out of the screen on this diagram, we can represent that fairly easily. In a more complex situation, we may need to draw something more fully three-dimensional, as our torques might point in other directions. This next video clip will walk you through a partial analysis of this situation. You will need a way to take notes and draw diagrams while you watch. [Jen speaking] I couldn't push Colin over by applying force in the stable direction. Yet when I applies the same force in a different direction, I could push him over. How did I know this? Torque. Here you see two 2-dimensional views representing Colin. The width of the base represents the distance between his feet. The wide square is the view of Colin from the side and stable states. The narrow rectangle is the view of Colin from straight on. Lets draw in the forces acting on Colin as I push him in the stable directions. Start by drawing the center of mass. Which we assume is at the center of the square. The force of gravity pulls straight down with magnitude three halfs. There is also the force of my push of length one. Which we place in the location where the force is applied. This push shifts Colin's weight entirely to his back foot. So the normal force is applied there with a magnitude equal and opposite that of force p. The force of friction is also at this foot. Equal and opposite the magnitude and direction of the push force. Recall that torque is r cross F. That is, torque occurs when a force is applied some displacement distance from a reference point. We can choose any reference point we like, and the net torque we compute will be the same. For this problem I will choose the reference point to be the pivot point. Why don't you try this problem but choose the center of mass as the reference point. Lets get back to the problem. The push force and the gravitation force are applied some distance from the pivot point at the foot. So they will create torque. However, the friction force and normal force are applied at the pivot point. So R is zero and we can ignore that when we compute the net torque. Lets compute the torque due to the push force. The vector r is the displacement from the pivot point to where the force is applied. We decompose this vector into its x and y components. We use a coordinate system with origin at the pivot point to determine the r vector to be 4i + 3j. The magnitude of the push force is negative one i. The magnitude of torque can be found as the magnitude of r times the magnitude of F times sin theta. Where theta is the angle between the two vectors. Thus the magnitude is the area of the parallelogram formed by r and F. Since the area of this parallelogram is also the area of the rectangle formed by F and r sin theta or the component of r that is perpendicular to F. We can see this magnitude visually as the area of this rectangle. We will show the magnitudes in this way because it is very easy to see the relative magnitudes of the various torques. So... For the push force the area is 3, which mean the magnitude of the torque is 3. The direction of the torque vector is along the axis of rotation caused by this force. We find the direction using the right hand wall. Point our fingers along r and curl along F. In this case we find the direction to be out of the board or positive k. So this tells us that the torque due to the push force is equal to 3k. We can also find the torque vector by taking the determinate of the following three by three matrix. The first row is i, j, and k. The second row are the x, y, and z components of the r vector. And the third row are the x, y , and z components of the force vector. Because r and F lie in the same plane, the z components are zero. In our example, the force is pointing in the negative i direction and r we found to be 4 i plus 3 j. Computing it this way, we also find that the torque is 3k. Now let's find the torque due to gravity. The vector r is found by connecting the pivot point to the center of mass. We leave to you to pause the video and to determine the components of r using this coordinate system. We find the magnitude of the torque by taking the component of r that is perpendicular to F pointing in the x direction, and the magnitude is the area of the rectangle between these two vectors. Again, we find the direction using the right hand rule. We find the torque to be pointing into the board at the pivot point. Since the x component of the r vector is 2, and the force due to gravity is 3 halves, pointing in the negative j direction. The magnitude of the torque due to gravity is 3 and it points in the negative k direction. So the torque is negative 3 k at the pivot point. Adding the push torque and the gravity torque together we find that we get zero. This means that there is no rotation about the pivot point. Now let's look in the unstable direction. We have all the same forces as before- force of gravity at the center of mass, the push force, a normal force at the back foot, as well as a friction force. The magnitudes of these forces are all the same as in the other diagram. Let's compute the torque due to the push force. The vector r connects the pivot point to the place where the force is applied. Using our coordinate axes with the origin placed at the pivot point, we find this vector to be i plus 3 j. We find the magnitude of the torque vector is the area of the rectangle formed by force vector and the component of r, perpendicular to F. Thus the area is one times three, or three. In order to find the direction, we use the right hand rule. Point our fingers along r, curl in the direction of F, and our thumb points out of the board, so the direction of our torque is k. Since the magnitude is 3, that tells us that torque due to the push is 3 k at the pivot point. Finally, let's find the torque due to gravity. Start by drawing the r vector from the pivot point to the center of mass. Now let's decompose this vector into it's x and y components. We leave this as an exercise to you, using the coordinate system shown here. The magnitude of the torque due to gravity at the pivot point is found by the area of the rectangle form by the force vector and the x component of the r vector. The direction is found using the right hand rule, pointing our fingers along r and curling around F. In other words, into the board or negative k. So the torque due to gravity is one-half times three-halves or three-fourths pointing in the negative k direction. The net torque about the pivot point is the sum of these two. In other words, the torque push plus the torque due to gravity is the net torque around the pivot point, which is two and a quarter the positive k direction. That means there is a total net rotation about the pivot point. Causing Colin to fall over. We leave it as an exercise for you to use the determinate in order to compute the torque vectors. If you computed the net torque using the center of mass as the reference point, you should notice that the net torques that you found are equal to the net torques that we computed here. [Sanjay speaking] Now that you've seen torque and balancing in detail, let's consider a more complex problem. This one is a bit tricky. I'll show you how it works. I'm going to take a chair and place it next to the wall. I'll put my toes up to the wall, and step back, toe-to-heel, twice. I step sideways until I'm over the chair. Then I bend forward until my head touches the wall, pick up the chair, and stand up... er... or not. This is something that most women can do, but men cannot. Maybe you think it's a trick? Here's a video of some of your professors trying to lift [Jen speaking]Okay, walk up to the wall. Yes. Then heel toe heel toe. That's it. One more time. And now translate over but don't come in or out. Okay. Now bend over to the the wall. Bend over. Put your head on the wall. Mmm-hmm. Pick the chair up. Now stand up. Hah! [laughing] That's fair enough. Don't hurt yourself. No, no. There was actually no way. [laughing] Toes against the wall. One. Two. Okay, translate over. You might be able to do it because his feet are proportionately small. Okay, down. Pick it up. [giggle] Pick up the chair first. Please pick up of the chair please. Now stand up. [laughing] Okay. One over. Bend over. Ohhh, okay. Alright. Okay, then do it one more time with the other foot. Okay, now can translate over a little bit? Just move over but don't go in or out. Well, no. You want a line there. Actually... be in a little. There you go. No no no- you're fine! Just even up your feet. Okay. Bend over. Put your hands on the wall, just so you don't hit your head. Okay... now pick up the chair. Now stand up. No no. Don't do it again. You have to pick up the chair. Clean pick up. Now stand up. Alright! [clapping] [Sanjay speaking] Now it's your turn to try. Get together with your partner again and bring a chair over to the wall. Why don't you pause the video and give it a try? Now that you've attempted the chair lift, let's return to our seats and discuss what happened. If you wanted to figure out why this happens, what information would you need? What assumptions would you have to make? Discuss the matter with your partner. Try to draw a diagram of the situation. Pause the video here while you work this out. Many of you should be wondering about the center of mass for this situation. This diagram shows typical locations for the center of mass in men (on the left) and women (on the right). Not only are men typically taller, but their center of mass is usually higher in their bodies. This is not always true, but it is fairly typical. Using this information, work with your partner to try to explain what is happening. Why can women lift the chair when men cannot? You do not need to obtain a numerical solution. Instead, use reasoned arguments, diagrams, and well-supported assumptions to prove your answer. Only use calculations if you cannot support your answer in any other way. Pause the video and give it a try. Are you ready to see the solution? Let's take a look. For this section we will measure torques around the center of mass. This will simplify our work so that we don't have to worry about the person's mass and the pull of gravity. This diagram shows the man lifting the chair. You can see that the center of mass is outside his body, and has been moved farther forward and down by the chair. This diagram shows the same for the woman. Her center of mass is also outside her body, but is much closer to her legs. If we draw a line for the man to indicate where his toes are, you see that the center of mass is beyond the edge of his foot! No matter how hard he tries, he simply cannot apply force in the right place to lift that chair. On our diagram, we can see that the torque will always point in the direction out of the screen. It will rotate the man counterclockwise, pushing his head harder into the wall. The harder he pushes, the worse it will be. The woman, however, has her center of mass above her feet. She can stand up because her feet are able to apply force in the proper location and direction. If we draw a torque diagram, we can see that the direction of torque applied by her feet will be into the screen. By applying more force, she can rotate her upper body clockwise and stand up, whereas the man cannot. Today we hope that you have improved your ability to draw torque diagrams, and to analyze torque problems that occur in the real world. Torque is an important quantity that comes into play in countless situations around us, from machinery to buildings to the simple act of walking. I hope you enjoyed this look at one of its fascinating applications.