Vectors

Representations Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

When to Use this Video

- In Phys 101 as part of recitation or during Module 3: Cartesian Coordinates and Vectors, Lecture 3.
- In Math 201 as part of recitation or during Unit 1: Basic Linear Algebra, Lecture 1:Vectors, the basics.
- Prior knowledge: Students should be able to add, decompose, and scale vectors represented as arrows and algebraically.

Key Information

Duration: 15:35
Narrator: Prof. Daniel Hastings
Materials Needed:
• Paper
• Pencil

Learning Objectives

By the end of this video, students will be able to:

- Understand the properties of vectors by using displacement as an example.
- Determine whether a physical quantity can be represented using vectors.

Motivation

- Through defining and exploring the vector as a representation of displacement, rather than just an arrow, students are prompted to use this key example in order to recall the mathematical properties of vectors.
- Students have a difficult time using vectors mathematically. They recall the definition, but forget what it means to scale or add vectors. This video emphasizes the linear properties of the vector.
- Experts understand that the vector is a powerful representational tool. This video revolves around two activities that ask students to determine what physical quantities have magnitude and direction, and then determine whether or not they have the necessary linearity conditions to be represented by vectors.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Compile a list of physical quantities that have magnitude and direction, and determine whether or not these quantities can be represented with vectors.
- Determine how to add vectors by using intuition about displacements.
- Determine if forces and rotations have vector properties.

Video Highlights

The table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:01	MIT lobby 7 gift search	Segment motivates the need for direction as well as magnitude as a quantity of interest.
1:43	Chapter 1: motivating examples	Examples motivate the definition of the vector, illustrate its importance, and encourage students to think about where they have seen vectors in their daily lives.
2:00	Activity: Make a list of physical quantities that have both magnitude and direction	
2:29	Wind tunnel demonstration	Segment motivates the use of vectors in engineering, providing scaffolding for vector fields with a smoke visualization over an airplane model in the Wright Brothers Wind Tunnel at MIT.
3:34	Chapter 2: review of vector properties	Definition and properties of vectors
4:40	Chapter 3: displacement as canonical vector example	Using displacement to motivate vector properties
5:22	Activity: understand vector addition through displacements	
6:37	Chapter 4: representing physical quantities with vectors	This section determines whether forces and rotations have the necessary properties to be adequately represented by vectors. Force presents a true example, proved via experiment, while rotations provide a non-example.
7:20	Force balance demo	Provides demo to show that forces do add like vectors. Two opportunities for student involvement provided.
12:17	Rotations counter-example	Animations that show why rotations cannot be represented using vectors. Student activity provided.

Video Summary

This video consists of four chapters: an introduction to the concept of the vector, a brief review of vector properties, an exploration of vector properties using physical intuition based on displacements, and finally, a discussion of how to recognize when a physical quantity can be represented by a vector, which works through the example of force and the non-example of rotation.

Phys 101 Materials

When appropriate, the guide is accompanied by additional materials to aid in the delivery of some of the activities and discussions.

Pre-Video Materials

In addition to these materials, there are two PowerPoint® presentations that are available to use in your own lecture. One presents projections, decomposition, and vector algebra in different notations. These are in the file "Vectors_Projection_Algebra.ppt". A presentation that may aid your lecture on the dot product and cross product can be found in the supplemental materials labeled "Vector_Products.ppt". The concept check questions that appear below can be found in "Vectors_Concept_Checks.pdf".

1. The following problem is adapted from problem 5.2 on page 10 of Hoffmann (1975) (see reference list.)

Determine the total distance travelled from the initial position, and in which direction for the following journeys.

(a) Travel 3m south, followed by travelling another 7m south.

(b) Travel 5 light years upwards, followed by 2 light years downwards.

(c) Travel 4 km east; then 3 km north.

2. Have students create a list of physical quantities that can be described by a magnitude and direction. After students have made their own list, have them break into small groups of 2 or 3 students, compare lists, and compile quantities. (This activity appears as the first pause in the video. So if you plan to use the video in class, you may wish to wait to use this activity during the video.)

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3. Ask students to determine the initial velocity of a ball thrown by someone on a bicycle.

Break students into small groups of 4 to 5 students. Have students determine the average initial velocity that they can throw a ball. Also have them determine the average velocity at which they ride a bicycle.

Have them take measurements outside of class as homework, or during recitation. They should design both the approach and the experiment. If done in a large lecture style class, they can work in small groups and determine reasonable estimations.

The problem is for students to discuss what the initial velocity of a ball thrown while riding a bicycle. This is an interesting problem because it will bring up issues of relative velocity, which is worth discussing, and could provide scaffolding for discussions of relativity later. The point of this activity is to recognize that velocity adds like a vector. It is important for students to consider what happens when the ball is thrown in a variety of directions. Have students first discuss in small groups, and then share with the class. If this is done in a recitation setting, you could have students actually make the measurements to determine that velocity adds like a vector.

Post-Video Materials

1. At the end of the video, students are asked to determine if a list of quantities can be represented using vectors or not. This activity requires that students think about each quantity, determine if it can be described using magnitude and direction, and explore their intuition for these quantities to determine if the quantity scales and adds like a vector. This is a very difficult activity.

Because students in Physics 101 may not be familiar with torque yet, you may want to focus on acceleration, velocity, time, and temperature. Break students up into a few small groups, and give each group one scalar and one vector quantity to ponder.

Many students should already have a sense that time is a scalar, and that velocity and acceleration are vectors. The idea is to get students to think deeply about why this is true, and how they would design an experiment to prove this.

2. Why are 1-dimensional vectors equivalent to real numbers?

Have students break up into small groups and discuss why this is true. If students had difficulty determining why temperature or time were best represented by scalars in the previous problem, this discussion should help.

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3. Consider the following. A mechanical object moves according to some force with velocity $v(t)$. Now take this object onto a train that moves at constant velocity w.

(a) What is the velocity that describes the same object while on the train?

(b) What is the acceleration of the object initially? And while on the train?

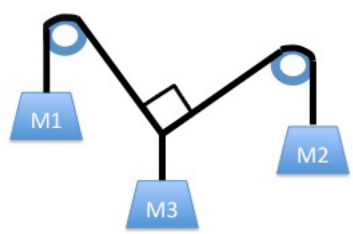
(c) Explain what this says about the force required in both situations.

Note: Something very strange happens when the train is moving so fast it is near the speed of light. Einstein showed that, in this case, velocity does not quite add like a vector anymore. However, for the kinds of velocities we might see around us, we rarely need to worry about this, unless we are particle physicists!

4. Vector Concept Check (Appendix A1)

The following problem was adapted from example figure 5.1 on page 9 of Hoffmann, B. (1975). *About Vectors*. New York: Dover Publications Inc.

Consider the following system of masses and pulleys, which is at rest. The angle between the strings attached to mass 1 and mass 2 is 90 degrees. Which best describes the relationship between the masses?



(a) M1 < M2 < M3

(b) M1 = M2 = M3

(c) M3 < M1 < M2

(d) M3 $= \sqrt{\text{M1}^2 + \text{M2}^2}$

(e) System not possible

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Math 201 Materials

When appropriate, the guide is accompanied by additional materials to aid in the delivery of some of the activities and discussions.

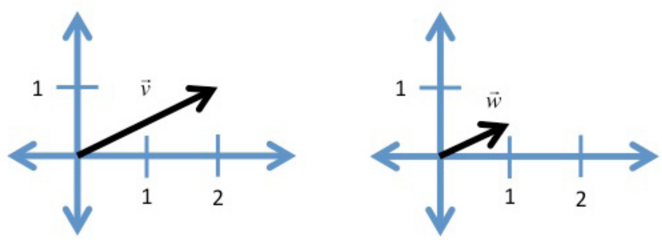
Pre-Video Materials

In addition to these materials, there are two PowerPoint® presentations that are available to use in your own lecture. One presents projections, decomposition, and vector algebra in different notations. These are in the file "Vectors_Projection_Algebra.ppt". A collection of slides that may aid in your presentation of the dot product and cross product can be found in the supplemental materials labeled "Products.ppt".

These materials are to reinforce ideas briefly discussed in the vectors video.

1. Vector Concept Check (Appendix A2)

Which of the following descriptions of the vector **w** are true given the arrow descriptions for the vectors **v** and **w** pictured below?



(a) $2w=v$

(b) $w=i + 1/2\ k$

(c) $w=<1,1/2>$

(d) All of the above

(e) None of the above

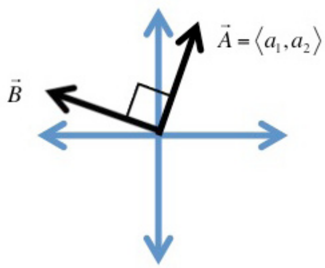
2. The vector $v=<1,2,-1>$ and the plane $x+2y-3z=5$ are

(a) Parallel

(b) Perpendicular

(c) Neither

3. Vector Concept Check (Appendix A3)



Which of the following represents the vector B?

(a) $< a_2 , a_1 >$

(b) $< -a_2 , a_1 >$

(c) $< a_2 , -a_1 >$

(d) $< -a_1 , a_2 >$

(e) None of the above

4. Have students create a list of physical quantities that can be described by a magnitude and direction. After students have made their own list, have them break into small groups of 2 or 3 students, compare lists, and compile quantities. (This activity appears as the first pause in the video. So if you plan to use the video in class, you may wish to wait to use this activity during the video.)

Post-Video Materials

1. At the end of the video, students are asked to determine if a list of quantities can be represented using vectors or not. This activity requires that students think about each quantity, determine if it can be described using magnitude and direction, and explore their intuition for these quantities to determine if the quantity scales and adds like a vector. This is a very difficult activity.

Many students should already have a sense that time is a scalar, and that velocity and acceleration are vectors. The idea is to really get students involved in thinking about why this is true. You can have students set up and do experiments, or you can have them work in small groups to describe the experiment they would use in order to test their hypothesis.

2. Why are 1-dimensional vectors equivalent to real numbers?

Have students break up into small groups and discuss why this is true. If students had difficulty determining why temperature or time were best represented by scalars in the previous problem, this discussion should help.

3. Vector Concept Check (Appendix A4)

This question checks a student's understanding of vector subtraction.

Additional Resources

Going Further

The following topics all rely on the basic understanding of vectors discussed in this video.

When exploring *dot products*, try to connect the projection of a vector onto another vector to some real world quantity in order to motivate dot products as a fundamental. You could start with the example shown of decomposing a displacement vector into two components pointing along particular directions as a motivating example.

When introducing the *cross-product*, have student think about what properties a product of two vectors that is also a vector should have. Can it be commutative? Why or why not? Should it be distributive? What about associative? If you start here, you can actually determine the direction of the cross product. Have students try to make up functions that will give them a well defined cross-product. Determine the positive attributes of the magnitude being the area of the parallelogram formed by the two vectors in the cross product.

Have students try to interpret cross-product properties in terms of torque, or torque properties in terms of cross-products. Explain how the intuition for one or the other helps in understanding.

Matrices are generalized vectors. Have students explore which vector properties are also true for matrices, and which aren't. Have students see if they can connect matrix multiplication to any vector products they already understand. These sorts of explorations can help students to understand matrix algebra, and the linearity properties involved.

An idea mentioned in the video: *matrices can represent rotations*. Have students explore 2 by 2 matrices that describe rotations, and see if they can determine the key properties, and why it is that they describe rotations. Not for the faint of heart, extend this to 3 dimensions.

Once matrices are understood, students can begin to understand changes of coordinates. Of particular difficulty for students is the *representation of vectors in spherical coordinates*. It is recommended that you spend considerable time looking at the matrix that converts between coordinate systems to understand how to go between Cartesian and spherical vector notation, and that a spherical vector is not simply the vector form of the spherical coordinate of the associated point.

Vector fields are the other natural place that vector knowledge is key. Remind students that the vector doesn't include the point of origin. Have students determine an appropriate "vector field algebra" for vectors that include the additional information of a point of origin.

A *vector field activity* one can do in class is to explore physical quantities that are vectors, and have students determine whether or not they are actually vector fields. Are there certain physical situations where a vector quantity becomes part of a vector field? How do you tell if a vector quantity in nature is part of a vector field or not? How does that change what you can do with the vector and how you represent it?

References

Video references from MIT OCW:

- Lewin, Walter. *8.01 Physics I: Classical Mechanics, Fall 1999.* (Massachusetts Institute of Technology: MIT OpenCourseWare), http://ocw.mit.edu (Accessed 21 Dec, 2011). License: Creative Commons BY-NC-SA
  -Lecture 03 introduces vector algebra mostly 2-dimensionally
- Auroux, Denis. 1*8.02 Multivariable Calculus, Fall 2007.* (Massachusetts Institute of Technology: MIT OpenCourseWare), http://ocw.mit.edu (Accessed 21 Dec, 2011). License: Creative Commons BY-NC-SA
  -Lecture 01 introduces 3-dimensional vectors mathematically

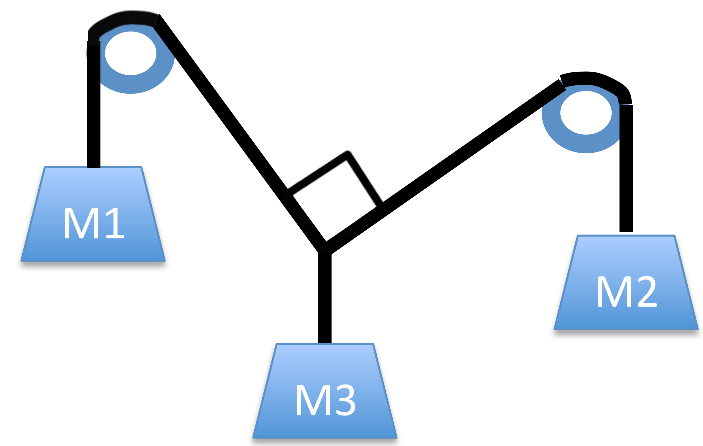
Here is a nice book that has material and problems about vectors that you can use or recommend to your students.

- Hoffmann, B. (1975). *About Vectors.* New York: Dover Publications Inc.

Here are a list of journal articles about vector knowledge and student understanding that you may find helpful, and which were consulted in the creation of the video and surrounding materials.

- Aguirre, J.M. (1988). Student Preconceptions about Vector Kinematics. *The Physics Teacher*, *26*(4), 212–216.
- Britton, S. and Henderson, J.(2009). Linear algebra revisited: an attempt to understand students' conceptual difficulties. *Int. J. of Math. Ed. in Sci. and Tech.*, *40*(7), 963–974.
- Dray, T. and Monague, C.A. (1999). The vector calculus gap. *PRIMUS*, *9*(1), 21–28.
- Knight, R.D. (1995). The Vector Knowledge of Beginning Physics Students. *The Physics Teacher, 33*(2), 74–77.
- Nguygen, N. and Meltzer, D.E. (2003). Initial understanding of vector concepts among students in introductory physics courses. *Am. J. Phys.*, *71*(6), 630–638.
- Shaffer, P.S. and McDermott, L.C. (2005). A research-based approach to improving student understanding of the vector nature of kinematical concepts. *Am. J. Phys.*, *73*(10), 921–931.
- Stewart, S. and Thomas, M.O.J. (2009). A framework for mathematical thinking: the case of linear algebra. *Int. J. of Math. Ed. in Sci. and Tech.*, *40*(7), 951–961.

Consider the following system of masses and pulleys, which is at rest.



(a) M1 < M2 < M3

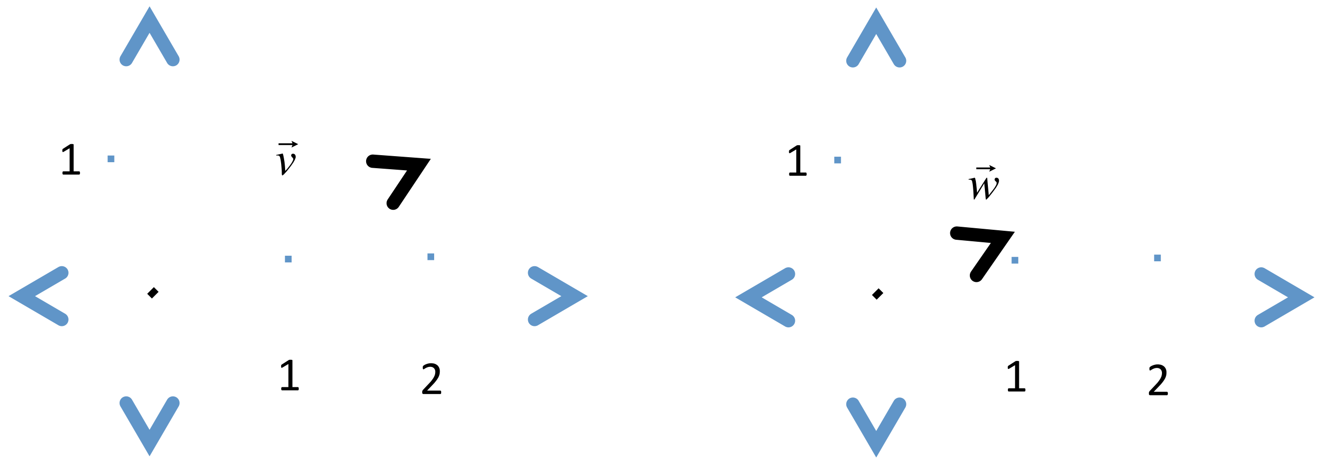
(b) M3 = M1 = M2

(c) M3 < M1 < M2

(d) $M3 = \sqrt{M1^2 + M2^2}$

(e) System not possible

Given the two vectors below, which of the following are true?



(a) $\vec{w} = \frac{1}{2}\vec{v}$

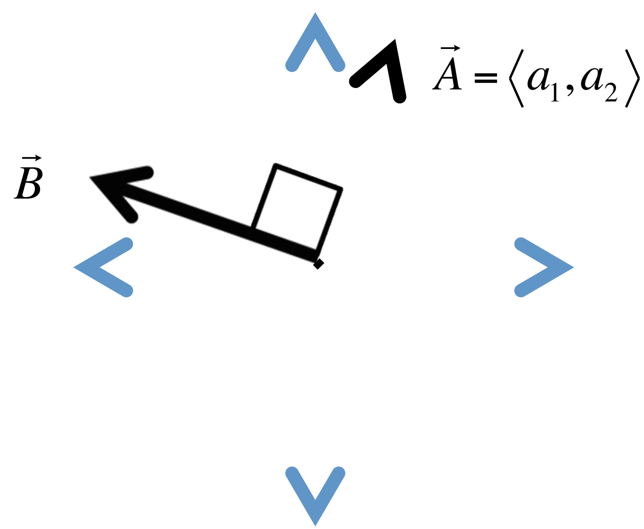
(b) $\vec{w} = \hat{i} + \frac{1}{2}\hat{j}$

(c) $\vec{w} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$

(d) All (a), (b), and (c)

(e) None of the above

Given the vector $\vec{A}$, which expression best describes vector $\vec{B}$?



(a) $\vec{B} = \langle a_2, a_1 \rangle$

(b) $\vec{B} = \langle -a_2, a_1 \rangle$

(c) $\vec{B} = \langle a_2, -a_1 \rangle$

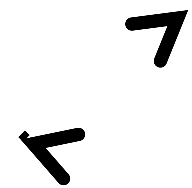
(d) $\vec{B} = \langle -a_1, a_2 \rangle$

(e) Both (b) and (c)

Given the vectors $\vec{a}$ and $\vec{b}$, which best describes the vector $\vec{b} - \vec{a}$?

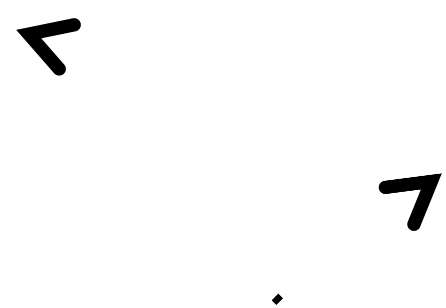
(a)

(b)



(c)

(d)



(e) Both (b) and (d)