In this lecture, we look in some depth into various properties of Poisson processes.

These properties would be quite hard to study if one were to proceed just analytically by manipulating formulas.

However, by using memorylessness and our intuitive understanding of what the Poisson process really is, they become quite simple.

And the mathematical manipulations can be avoided almost entirely.

We will start by arguing that the sum of independent Poisson random variables is Poisson.

But we will then establish the much stronger statement that if we merge two independent Poisson processes, we, again, obtain a Poisson process.

We will see that we can exploit this fact to solve some problems that would be quite difficult otherwise.

Once more, intuitive reasoning is the key.

Finally, we will spend some time discussing a phenomenon that goes under the name of random incidence.

The Poisson process has been running.

You show up at a certain time.

You look at the size of the inter-arrival interval during which you show up.

It turns out that this inter-arrival interval that you get to observe is not a typical one.

It tends to be larger than the typical inter-arrival interval.

We will understand what exactly is going on, build some intuition, and realize that this is a general phenomenon that also shows up in many other contexts.