19 Discrete-Time Sampling

Recommended Problems

P19.1

Consider Figures P19.1-1 and P19.1-2, and determine $X(\Omega)$, $P(\Omega)$, $x_p[n]$, and $X_p(\Omega)$.

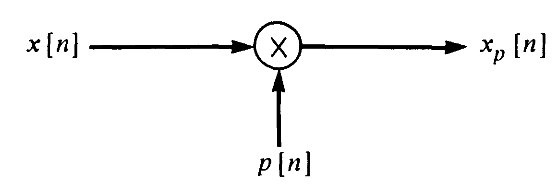

Figure P19.1-1

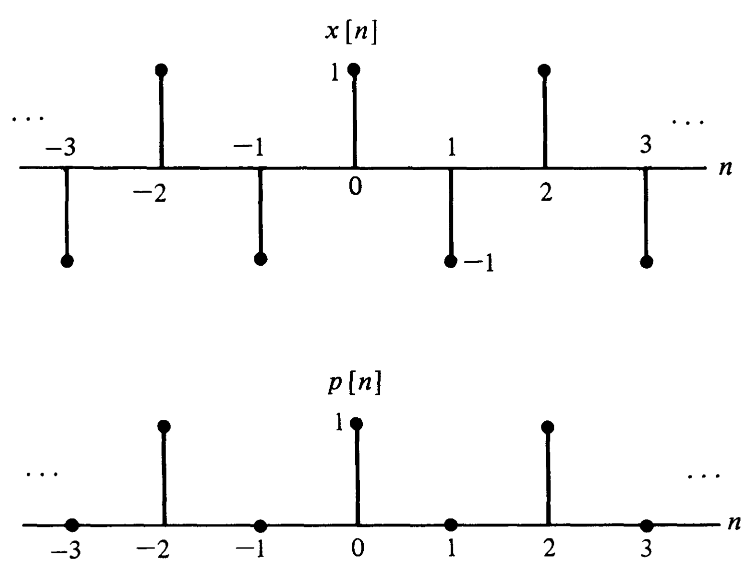

Figure P19.1-2

P19.2

$x[n]$ has a transform $X(\Omega)$. Determine in terms of $X(\Omega)$ the transforms of the signals in parts (a) and (b).

(a) $x_s[n] = \begin{cases} x[n/2], & n \text{ even}, \\ 0, & n \text{ odd} \end{cases}$

(b) $x_d[n] = x[2n]$, i.e., $x_d[n]$ is $x[n]$ decimated.

(c) If $X(\Omega)$ is as given in Figure P19.2, sketch $X_s(\Omega)$ and $X_d(\Omega)$ for parts (a) and (b).

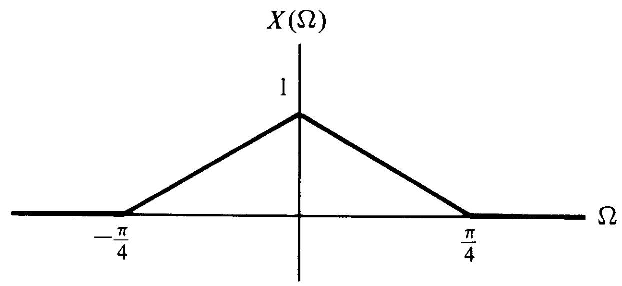

Figure P19.2

P19.3

Consider the system in Figure P19.3-1.

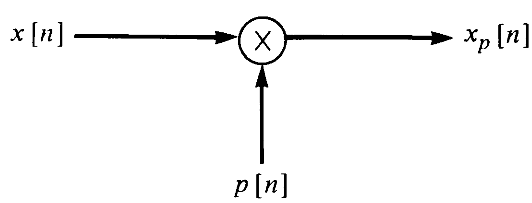

Figure P19.3-1

(a) If $p[n]$ is given by Figure P19.3-2 sketch $P(\Omega)$ for $N = 1, 2$, and L, an arbitrary integer.

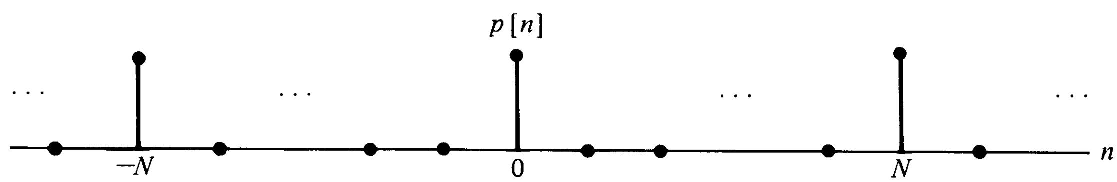

Figure P19.3-2

(b) For each of the discrete-time spectra in Figures P19.3-3 and P19.3-4, determine the maximum sampling period N such that $x[n]$ is reconstructible from its samples $x_p[n]$ using an ideal lowpass filter.

In each case, specify the associated cutoff frequencies for the lowpass filter.

(i)

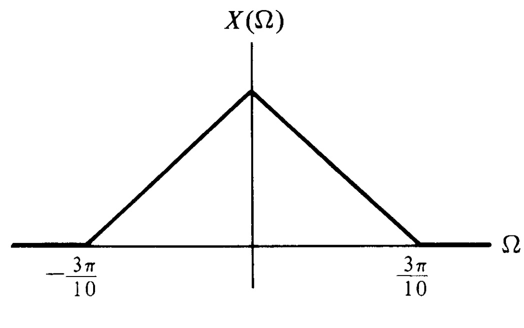

Figure P19.3-3

(ii)

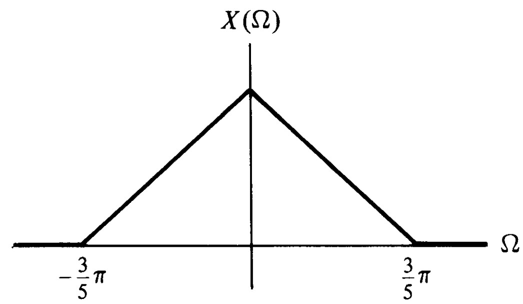

Figure P19.3-4

P19.4

Suppose the signal $x(t)$ is processed as shown in Figure P19.4-1.

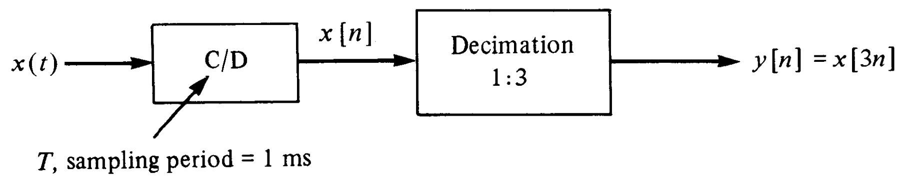

Figure P19.4-1

(a) The system in Figure P19.4-1 can be replaced by the one in Figure P19.4-2. Find T_1.

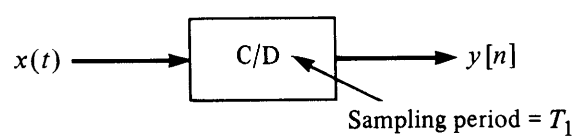

Figure P19.4-2

(b) Let $X(\omega)$ be given as in Figure P19.4-3. Find $X(\Omega)$ and $Y(\Omega)$.

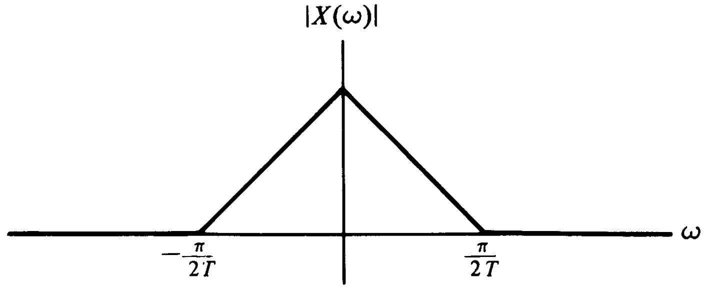

Figure P19.4-3

P19.5

As discussed in Section 8.7 and illustrated in Figure 8.40 of the text as well as in Figure P19.5-1 below, the procedure for interpolation or upsampling by an integer factor N can be thought of as the cascade of two operations. The first system, system A, corresponds to inserting $(N - 1)$ zero sequence values between each sequence value of $x[n]$, so that

$$x_p[n] = \begin{cases} x_d\left[\dfrac{n}{N}\right], & n = 0, \pm N, \pm 2N, \ldots, \\ 0, & \text{otherwise} \end{cases}$$

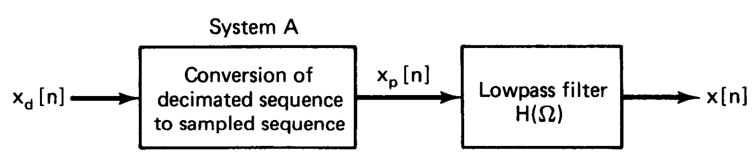

Figure P19.5-1

For exact bandlimited interpolation, $H(\Omega)$ is an ideal lowpass filter.

(a) Determine whether system A is linear.

(b) Determine whether system A is time-invariant.

(c) For $X_d(\Omega)$ as sketched in Figure P19.5-2, with $N = 3$, sketch $X_p(\Omega)$.

(d) For $N = 3$, $X_d(\Omega)$ as in Figure P19.5-2, and $H(\Omega)$ appropriately chosen for exact bandlimited interpolation, sketch $X(\Omega)$.

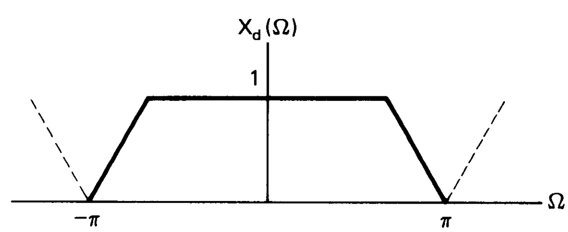

Figure P19.5-2

P19.6

Consider the sampling systems in Figure P19.6-1.

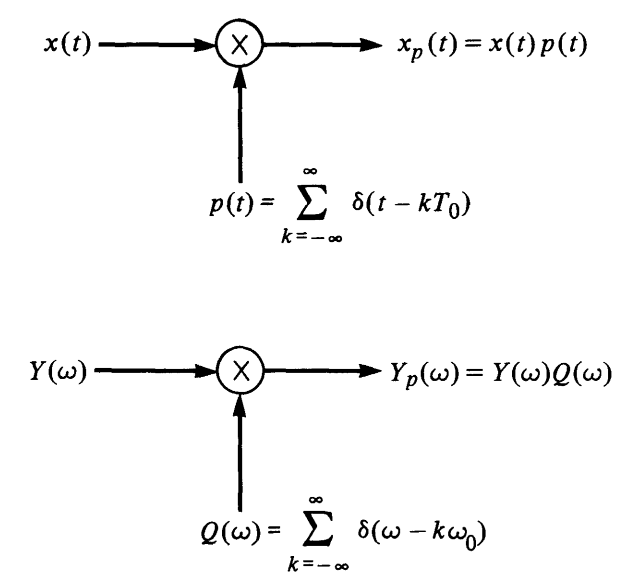

Figure P19.6-1

Let $x(t)$ and $X(\omega)$ be given as in Figure P19.6-2.

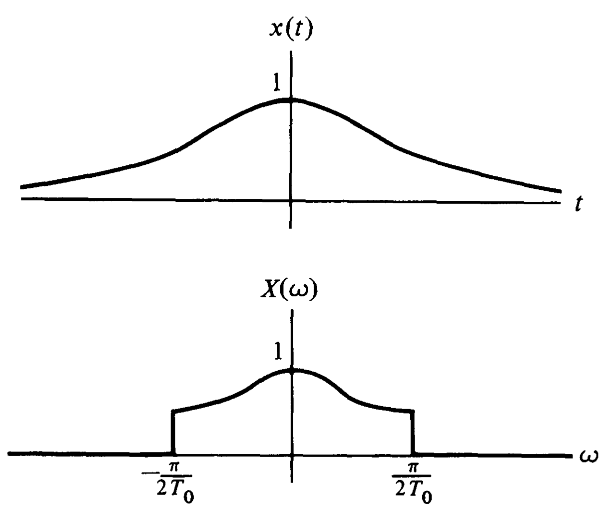

Figure P19.6-2

Let $y(t)$ and $Y(\omega)$ be given as in Figure P19.6-3.

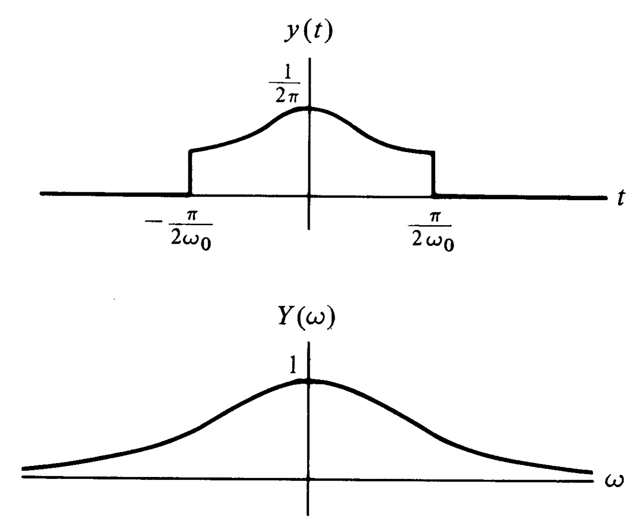

Figure P19.6-3

(a) Draw $x_p(t)$ and $Y_p(\omega)$.

(b) Find $X_p(\omega)$ and $y_p(t)$.

(c) Is $y_p(t)$ periodic? Does $Y_p(\omega)$ reflect this property?

Optional Problems

P19.7

Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to *sampling* $x[n]$ with sampling period 2 and $x_d[n]$ corresponds to *decimating* $x[n]$ by a factor of 2, so that

$$x_p[n] = \begin{cases} x[n], & n = 0, \pm 2, \pm 4, \ldots, \\ 0, & n = \pm 1, \pm 3, \ldots, \end{cases}$$

and

$$x_d[n] = x[2n]$$

(a) If $x[n]$ is as illustrated in Figure P19.7-1, sketch the sequences $x_p[n]$ and $x_d[n]$.

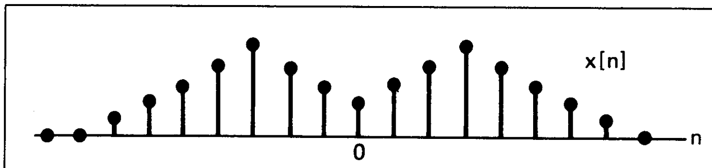

Figure P19.7-1

(b) If $X(\Omega)$ is as shown in Figure P19.7-2, sketch $X_p(\Omega)$ and $X_d(\Omega)$.

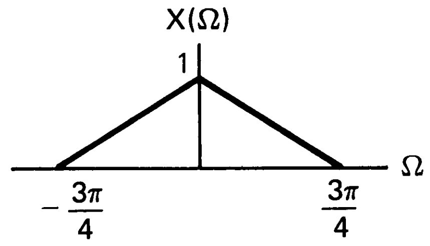

Figure P19.7-2

P19.8

Consider the system in Figure P19.8-1, where $X(\Omega)$ is as shown in Figure P19.8-2.

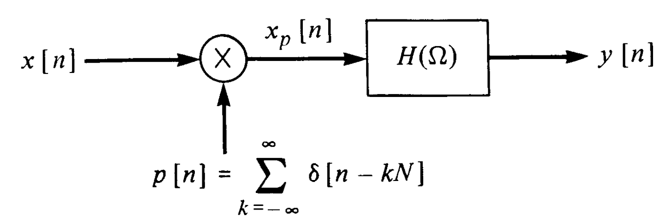

Figure P19.8-1

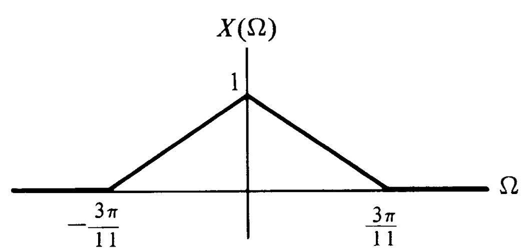

Figure P19.8-2

There is a range of values for N such that, with an appropriate choice for $H(\Omega)$, $y[n]$ will equal $x[n]$. For each allowable positive integer value of N,

(a) Draw $X_p(\Omega)$.

(b) Find an appropriate $H(\Omega)$ such that $y[n] = x[n]$.

P19.9

Consider the system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \frac{x[3n] + x[3n + 1] + x[3n + 2]}{3}$$

(a) For the sequence $x[n]$ in Figure P19.9, sketch $y[n]$.

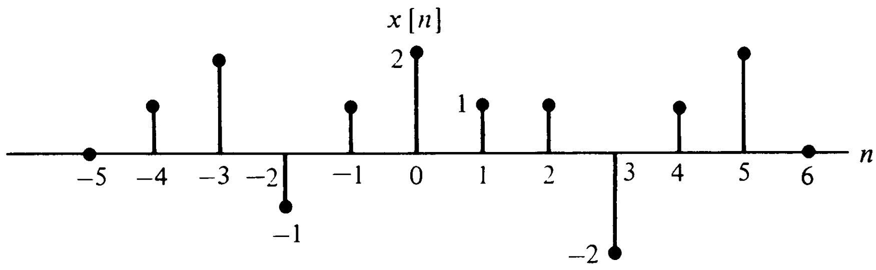

Figure P19.9

(b) Express the system as a combination of filtering and decimation.

P19.10

Consider the system in Figure P19.10, where

$$x_0[n] = \begin{cases} x\left[\dfrac{n}{N}\right], & n = kN, \\ 0, & n \neq kN,\ k \text{ an integer} \end{cases}$$

Find a constraint on $h[n]$ such that $y[kN] = x[k]$, for all k.

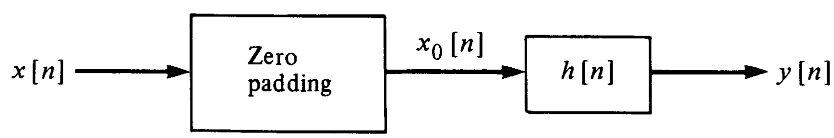

Figure P19.10