1 Introduction

Recommended Problems

P1.1

Evaluate each of the following expressions for the complex number $z = \frac{1}{2}e^{j\pi/4}$.

(a) $Re\{z\}$

(b) $Im\{z\}$

(c) $|z|$

(d) $\sphericalangle z$

(e) z^* (* denotes complex conjugation)

(f) $z + z^*$

P1.2

Let z be an arbitrary complex number.

(a) Show that

$$Re\{z\} = \frac{z + z^*}{2}$$

(b) Show that

$$jIm\{z\} = \frac{z - z^*}{2}$$

P1.3

Using Euler's formula, $e^{j\theta} = \cos\theta + j\sin\theta$, derive the following relations:

(a) $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

(b) $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

P1.4

(a) Let $z = re^{j\theta}$. Express in polar form (i.e., determine the magnitude and angle for) the following functions of z:

(i) z^*

(ii) z^2

(iii) jz

(iv) zz^*

(v) $\dfrac{z}{z^*}$

(vi) $\dfrac{1}{z}$

(b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for $r = \frac{2}{3}$, $\theta = \pi/6$.

P1.5

Show that

$$(1 - e^{j\alpha}) = 2 \sin\left(\frac{\alpha}{2}\right) e^{j[(\alpha-\pi)/2]}$$

P1.6

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t + 2)$

(c) $x(2t + 2)$

(d) $x(1 - 3t)$

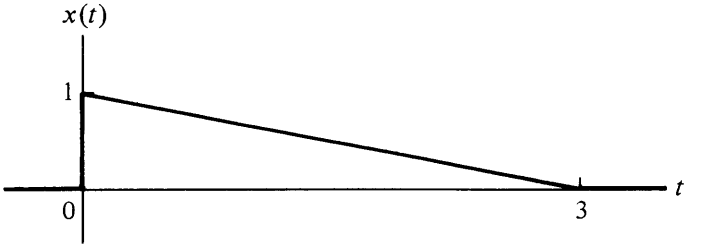

Figure P1.6

P1.7

Evaluate the following definite integrals:

(a) $\int_0^a e^{-2t}\, dt$

(b) $\int_2^\infty e^{-3t}\, dt$

Resource: Signals and Systems
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