CHAPTER 1 INTRODUCTION TO CALCULUS

Section 1.1 Velocity and Distance (page 6)

1 $v = 30, 0, -30; v = -10, 20$ **3** $v(t) = \begin{cases} 2 \text{ for } 0 < t < 10 \\ 1 \text{ for } 10 < t < 20 \\ -3 \text{ for } 20 < t < 30 \end{cases}$ $v(t) = \begin{cases} 0 & \text{for } 0 < t < T \\ \frac{1}{T} & \text{for } T < t < 2T \\ 0 & \text{for } 2T < t < 3T \end{cases}$

5 $25; 22; t + 10$ **7** $6; -30$ **9** $v(t) = \begin{cases} 20 \text{ for } & t < .2 \\ 0 \text{ for } & t > .2 \end{cases}$ $f(t) = \begin{cases} 20t & \text{for } t \le .2 \\ 4 & \text{for } t \ge .2 \end{cases}$ **11** $10\%; 12\frac{1}{2}\%$

13 $f(t) = 0, 30(t-1), 30; f(t) = -30t, -60, 30(t-6)$ **15** Average 8, 20 **17** $40t - 80$ for $1 \le t \le 2.5$

21 $0 \le t \le 3, -40 \le f \le 20; 0 \le t \le 3T, 0 \le f \le 60T$ **23** $3 - 7t$ **25** $6t - 2$ **27** $3t + 7$

29 Slope -2; $1 \le f \le 9$ **31** $v(t) = \begin{cases} 8 & \text{for } 0 < t < T \\ -2 & \text{for } T < t < 5T \end{cases}$ $f(t) = \begin{cases} 8t & \text{for } 0 \le t \le T \\ 10T - 2t & \text{for } T \le t \le 5T \end{cases}$

33 $\frac{9}{5}C + 32$; slope $\frac{9}{5}$ **35** $f(w) = \frac{w}{1000}$; slope = conversion factor **37** $1 \le t \le 5, 0 \le f \le 2$

39 $0 \le t \le 5, 0 \le f \le 4$ **41** $0 \le t \le 5, 1 \le t \le 32$ **43** $\frac{1}{2}t + 4; \frac{1}{2}t + \frac{7}{2}; 2t + 12; 2t + 3$

45 Domains $-1 \le t \le 1$: ranges $0 \le 2t + 2 \le 4$, $-3 \le t - 2 \le -1$, $-2 \le -f(t) \le 0$, $0 \le f(-t) \le 2$

47 $\frac{3}{2}V; \frac{3}{2}V$ **49** input * input $\to A$; input $+A \to$ output | input * input $\to A$; input $+A \to B$ | $B * B \to C$; $B + C \to$ output | input $+1 \to A$; $A * A \to B$; $A + B \to$ output

51 $3t + 5, 3t + 1, 6t - 2, 6t - 1, -3t - 1, 9t - 4$; slopes 3,3,6,6,–3,9

53 The graph goes up and down *twice*. $f(f(t)) = \begin{cases} 2(2t) & 0 \le t \le 1.5 & 12 - 2(12 - 2t) & 3 \le t \le 4.5 \\ 12 - 4t & 1.5 \le t \le 3 & 2(12 - 2t) & 4.5 \le t \le 6 \end{cases}$

Section 1.2 Calculus Without Limits (page 14)

1 $2 + 5 + 3 = 10; f = 1, 3, 8, 11; 10$ **3** $f = 3, 4, 6, 7, 7, 6$; max f at $v = 0$ or at break from $v = 1$ to -1

5 $1.1, -2, 5; f(6) = 6.6, -11, 4; f(7) = 7.7, -13, 9$ **7** $f(t) = 2t$ for $t \le 5$, $10 + 3(t - 5)$ for $t \ge 5$; $f(10) = 25$

9 7, 28, $8t + 4$; multiply slopes **11** $f(8) = 8.8, -15, 14; \frac{\Delta f}{\Delta t} = 1.1, -2, 5$

13 $f(x) = 3052.50 + .28(x - 20{,}350)$; then 11,158.50 is $f(49{,}300)$ **15** $19\frac{1}{4}\%$

17 Credit subtracts 1,000, deduction only subtracts 15% of 1000 **19** All $v_j = 2; v_j = (-1)^{j-1}; v_j = (\frac{1}{2})^j$

21 L's have area 1,3,5,7 **23** $f_j = j$; sum $j^2 + j$; sum $\frac{j^2}{2} + \frac{j}{2}$ **25** $(101^2 - 99^2)/2 = \frac{400}{2}$ **27** $v_j = 2j$ **29** $f_{31} = 5$

31 $a_j = -f_j$ **33** $0; 1; .1$ **35** $v = 2, 6, 18, 54; 2 \cdot 3^{j-1}$ **37** $\frac{\Delta f}{\Delta t} = 1, .7177, .6956, .6934 \to \ln 2 = .6931$ in Chapter 6

39 $v_j = -(\frac{1}{2})^j$ **41** $v_j = 2(-1)^j$, sum is $f_j - 1$ **45** $v = 1000, t = 10/V$

47 M, N **51** $\sqrt{9} < 2 \cdot 9 < 9^2 < 2^9; (\frac{1}{9})^2 < 2(\frac{1}{9}) < \sqrt{1/9} < 2^{1/9}$

Section 1.3 The Velocity at an Instant (page 21)

1 $6, 6, \frac{13}{2}a, -12, 0, 13$ **3** 4, 3.1, $3 + h$, 2.9 **5** Velocity at $t = 1$ is 3 **7** Area $f = t + t^2$, slope of f is $1 + 2t$

9 F; F; F; T **11** $2; 2t$ **13** $12 + 10t^2; 2 + 10t^2$ **15** Time 2, height 1, stays above $\frac{3}{4}$ from $t = \frac{1}{2}$ to $\frac{3}{2}$

17 $f(6) = 18$ **21** $v(t) = -2t$ then $2t$ **23** Average to $t = 5$ is 2; $v(5) = 7$ **25** $4v(4t)$ **27** $v_{\text{ave}} = t, v(t) = 2t$

Section 1.4 Circular Motion (page 28)

1 $10\pi, (0, -1), (-1, 0)$ **3** $(4\cos t, 4\sin t)$; 4 and $4t$; $4\cos t$ and $-4\sin t$

5 $3t$; $(\cos 3t, \sin 3t)$; $-3\sin 3t$ and $3\cos 3t$ **7** $x = \cos t; \sqrt{2}/2; -\sqrt{2}/2$ **9** $2\pi/3; 1; 2\pi$

11 Clockwise starting at (1,0) **13** Speed $\frac{2}{\pi}$ **15** Area 2 **17** Area 0

19 4 from speed, 4 from angle **21** $\frac{1}{4}$ from radius times 4 from angle gives 1 in velocity
23 Slope $\frac{1}{2}$; average $(1-\frac{\sqrt{3}}{2})/(\pi/6) = \frac{3(2-\sqrt{3})}{\pi} = .256$ **25** Clockwise with radius 1 from (1,0), speed 3
27 Clockwise with radius 5 from (0,5), speed 10 **29** Counterclockwise with radius 1 from (cos 1, sin 1), speed 1
31 Left and right from (1,0) to (−1,0), $v = -\sin t$ **33** Up and down between 2 and −2; start $2\sin\theta$, $v = 2\cos(t+\theta)$
35 Up and down from $(0,-2)$ to $(0,2)$; $v = \sin\frac{1}{2}t$ **37** $x = \cos\frac{2\pi t}{360}, y = \sin\frac{2\pi t}{360}$, speed $\frac{2\pi}{360}, v_{\text{up}} = \cos\frac{2\pi t}{360}$

Section 1.5 A Review of Trigonometry (page 33)

1 Connect corner to midpoint of opposite side, producing 30° angle **3** π **7** $\frac{\theta}{2\pi} \to$ area $\frac{1}{2}r^2\theta$
9 $d = 1$, distance around hexagon < distance around circle **11** T; T; F; F
13 $\cos(2t+t) = \cos 2t\cos t - \sin 2t\sin t = 4\cos^3 t - 3\cos t$
15 $\frac{1}{2}\cos(s-t) + \frac{1}{2}\cos(s+t); \frac{1}{2}\cos(s-t) - \frac{1}{2}\cos(s+t)$ **17** $\cos\theta = \sec\theta = \pm 1$ at $\theta = n\pi$
19 Use $\cos(\frac{\pi}{2} - s - t) = \cos(\frac{\pi}{2} - s)\cos t + \sin(\frac{\pi}{2} - s)\sin t$ **23** $\theta = \frac{3\pi}{2}+$ multiple of 2π
25 $\theta = \frac{\pi}{4}+$ multiple of π **27** No θ **29** $\phi = \frac{\pi}{4}$ **31** $|OP| = a, |OQ| = b$

CHAPTER 2 DERIVATIVES

Section 2.1 The Derivative of a Function (page 49)

1 (b) and (c) **3** $12+3h; 13+3h; 3; 3$ **5** $f(x)+1$ **7** -6 **9** $2x + \Delta x + 1; 2x+1$
11 $\frac{4}{t+\Delta t} - \frac{4}{t} = \frac{-4}{t(t+\Delta t)} \to \frac{-4}{t^2}$ **13** 7; 9; corner **15** $A = 1, \quad B = -1$ **17** F; F; T; F
19 $b = B; m$ and $M; m$ or undefined **21** Average $x_2 + x_1 \to 2x_1$
25 $\frac{1}{2}$; no limit (one-sided limits $1, -1$); 1; 1 if $t \neq 0, -1$ if $t = 0$ **27** $f'(3); f(4) - f(3)$
29 $2x^4(4x^3) = 8x^7$ **31** $\frac{du}{dx} = \frac{1}{2u} = \frac{1}{2\sqrt{x}}$ **33** $\frac{\Delta f}{\Delta x} = -\frac{1}{2}; f'(2)$ doesn't exist **35** $2f\frac{df}{dx} = 4u^3\frac{du}{dx}$

Section 2.2 Powers and Polynomials (page 56)

1 $6x^5; 30x^4; f'''''' = 720 = 6!$ **3** $2x+7$ **5** $1+2x+3x^2+4x^3$ **7** $nx^{n-1} - nx^{-n-1}$
9 $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$ **11** $-\frac{1}{x}, (-\frac{1}{x}) + 5$ **13** $x^{-2/3}; x^{-4/3}; -\frac{1}{9}x^{-4/3}$
15 $3x^2 - 1 = 0$ at $x = \frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$ **17** 8 ft/sec; − 8 ft/sec; 0 **19** Decreases for $-1 < x < \frac{1}{3}$
21 $\frac{(x+h)-x}{h(\sqrt{x+h}-\sqrt{x})} \to \frac{1}{2\sqrt{x}}$ **23** 1 5 10 10 5 1 adds to $(1+1)^5 (x = h = 1)$
25 $3x^2$; $2h$ is difference of x's **27** $\frac{\Delta f}{\Delta x} = 2x + \Delta x + 3x^2 + 3x\Delta x + (\Delta x)^2 \to 2x + 3x^2 =$ sum of separate derivatives
29 $7x^6; 7(x+1)^6$ **31** $\frac{1}{24}x^4$ plus any cubic **33** $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + C$ **35** $\frac{1}{24}x^4, \frac{1}{120}x^5$
37 F; F; F; T; T **39** $\frac{y}{x} = .12$ so $\frac{\Delta y}{\Delta x} = \frac{1}{2}(.12)$; six cents **41** $\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x}(\frac{c}{x+\Delta x} - \frac{c}{x}), \frac{dy}{dx} = -\frac{c}{x^2}$
43 $E = \frac{2x}{2x+3}$ **45** t to $\sqrt[3]{2}t$ **47** $\frac{1}{10}x^{10}; \frac{1}{n+1}x^{n+1}$; divide by $n+1 = 0$
49 $.7913, -3.7913, 1.618, -.618; 0, 1.266, -2.766$

Section 2.3 The Slope and the Tangent Line (page 63)

1 $\frac{-12}{x^2}; y - 6 = -3(x-2); y - 6 = \frac{1}{3}(x-2); y - 6 = -\frac{3}{2}(x-2)$ **3** $y + 1 = 3(x-1); y = 3x - 4$
5 $y = x; (3,3)$ **7** $y - a^2 = (c+a)(x-a); y - a^2 = 2a(x-a)$ **9** $y = \frac{1}{5}x^2 + 2; y - 7 = -\frac{1}{2}(x-5)$
11 $y = 1; x = \frac{\pi}{2}$ **13** $y - \frac{1}{a} = -\frac{1}{a^2}(x-a); y = \frac{2}{a}, x = 2a; 2$ **15** $c = 4$, tangent at $x = 2$

17 $(-3,19)$ and $(\frac{1}{3},\frac{13}{27})$ **19** $c=4, y=3-x$ tangent at $x=1$
21 $(1+h)^3; 3h+3h^2+h^3; 3+3h+h^2; 3$ **23** Tangents parallel, *same* normal
25 $y=2ax-a^2, Q=(0,-a^2)$; distance $a^2+\frac{1}{4}$; angle of incidence = angle of reflection
27 $x=2p$; focus has $y=\frac{x^2}{4p}=p$ **29** $y-\frac{1}{\sqrt{2}}=x+\frac{1}{\sqrt{2}}; x=-\frac{2}{\sqrt{2}}=-\sqrt{2}$
31 $y-a^2=-\frac{1}{2a}(x-a); y=a^2+\frac{1}{2}; a=\frac{\sqrt{3}}{2}$ **33** $(\frac{1}{x^2})(1000)=10$ at $x=10$ hours **35** $a=2$
37 $1.01004512; 1+10(.001)=1.01$ **39** $(2+\Delta x)^3-(8+6\Delta x)=6(\Delta x)^2+(\Delta x)^3$ **41** $x_1=\frac{5}{4}; x_2=\frac{41}{40}$
43 $T=8$ sec; $f(T)=96$ meters **45** $a>\frac{4}{5}$ meters/sec^2

Section 2.4 The Derivative of the Sine and Cosine (page 70)

1 (a) and (b) **3** $0;1;5;\frac{1}{5}$ **5** $\sin(x+2\pi); (\sin h)/h\to 1; 2\pi$ **7** $\cos^2\theta\approx 1-\theta^2+\frac{1}{4}\theta^4; \frac{1}{4}\theta^4$ is small
9 $\sin\frac{1}{2}\theta\approx\frac{1}{2}\theta$ **11** $\frac{3}{2};4$ **13** $PS=\sin h$; area $OPR=\frac{1}{2}\sin h<$ curved area $\frac{1}{2}h$
15 $\cos x=1-\frac{x^2}{2\cdot 1}+\frac{x^4}{4\cdot 3\cdot 2\cdot 1}-\cdots$ **17** $\frac{1}{2h}(\cos(x+h)-\cos(x-h))=\frac{1}{h}(-\sin x\sin h)\to-\sin x$
19 $y'=\cos x-\sin x=0$ at $x=\frac{\pi}{4}+n\pi$ **21** $(\tan h)/h=\sin h/h\cos h<\frac{1}{\cos h}\to 1$
23 Slope $\frac{1}{2}\cos\frac{1}{2}x=\frac{1}{2},0,-\frac{1}{2},\frac{1}{2}$; no **25** $y=2\cos x+\sin x; y''=-y$ **27** $y=-\frac{1}{3}\cos 3x; y=\frac{1}{3}\sin 3x$
29 In degrees $(\sin h)/h\to 2\pi/360=.01745$ **31** $2\sin x\cos x+2\cos x(-\sin x)=0$

Section 2.5 The Product and Quotient and Power Rules (page 77)

1 $2x$ **3** $\frac{-1}{(1+x)^2}-\frac{\cos x}{(1+\sin x)^2}$ **5** $(x-2)(x-3)+(x-1)(x-3)+(x-1)(x-2)$
7 $-x^2\sin x+4x\cos x+2\sin x$ **9** $2x-1-\frac{1}{\sin^2 x}$ **11** $2\sqrt{x}\sin x\cos x+\frac{1}{2}x^{-1/2}\sin^2 x+\frac{1}{2}(\sin x)^{-1/2}\cos x$
13 $4x^3\cos x-x^4\sin x+\cos^4 x-4x\cos^3 x\ \sin x$ **15** $\frac{1}{2}x^2\cos x+2x\sin x$ **17** 0 **19** $-\frac{8}{3}(x-5)^{-5/3}+\frac{8}{3}(5-x)^{-5/3}(=0?)$
21 $3(\sin x\cos x)^2(\cos^2 x-\sin^2 x)+2\cos 2x$ **23** $u'vwz+v'uwz+w'uvz+z'uvw$ **25** $-\csc^2 x-\sec^2 x$
27 $V=\frac{t\cos t}{1+t}, V'=\frac{\cos t-t\sin t-t^2\sin t}{(1+t)^2}$ $A=2(\frac{t}{t+1}+t\cos t+\frac{\cos t}{t+1})$ $A'=2(\cos t-t\sin t+\frac{1-\cos t}{(t+1)^2}-\frac{\sin t}{t+1})$
29 $10t$ for $t<10$, $\frac{50}{\sqrt{t-10}}$ for $t>10$ **31** $\frac{2t^3+3t^2}{(1+t)^2}; \frac{2t^3+6t^2+6t}{(1+t)^3}$
33 $u''v+2u'v'+uv''; u'''v+3u''v'+3u'v''+v'''$ **35** $\frac{1}{2}\sin^2 t; \frac{1}{2}\tan^2 t; \frac{2}{3}[(1+t)^{3/2}-1]$
39 T; F; F; T; F **41** degree $2n-1$/ degree $2n$ **43** $v(t)=\cos t-t\sin t(t\le\frac{\pi}{2}); v(t)=-\frac{\pi}{2}(t\ge\frac{\pi}{2})$
45 $y=\frac{2hx^3}{L^3}+\frac{3hx^2}{L^2}$ has $\frac{dy}{dx}=0$ at $x=0$ (no crash) and at $x=-L$ (no dive). Then $\frac{dy}{dx}=\frac{6Vh}{L}(\frac{x^2}{L^2}+\frac{x}{L})$ and $\frac{d^2y}{dx^2}=\frac{6V^2h}{L^2}(\frac{2x}{L}+1)$.

Section 2.6 Limits (page 84)

1 $\frac{1}{4}, L=0$, after $N=10$; $\frac{25}{24},\infty$, no N; $\frac{1}{4},0$, after 5; 1.1111, $\frac{10}{9}$, all n; $\sqrt{2},1$, after 38; $\sqrt{20}-4,\frac{1}{2}$, all n; $\frac{625}{256}, e=2.718\cdots$, after $N=12$. **3** (c) and (d)
5 Outside any interval around zero there are only a finite number of a's **7** $\frac{5}{2}$ **9** $\frac{f(h)-f(0)}{h}$ **11** 1
13 1 **15** sin 1 **17** No limit **19** $\frac{1}{2}$ **21** Zero if $f(x)$ is continuous at a **23** 2
25 .001, .0001, .005, .1 **27** $|f(x)-L|; \frac{4x}{1+x}$ **29** $0; X=100$ **33** $4;\infty;7;7$ **35** 3; no limit; 0; 1
37 $\frac{1}{1-r}$ if $|r|<1$; no limit if $|r|\ge 1$ **39** .0001; after $N=7$ (or 8?) **41** $\frac{1}{2}$
43 $9; 8\frac{1}{2}; a_n-8=\frac{1}{2}(a_{n-1}-8)\to 0$
45 $a_n-L\le b_n-L\le c_n-L$ so $|b_n-L|<\epsilon$ if $|a_n-L|<\epsilon$ and $|c_n-L|<\epsilon$

Section 2.7 Continuous Functions (page 89)

1 $c = \sin 1$; no c **3** Any $c; c = 0$ **5** $c = 0$ or 1; no c **7** $c = 1$; no c **9** no c; no c
11 $c = \frac{1}{64}; c = \frac{1}{64}$ **13** $c = -1; c = -1$ **15** $c = 1; c = 1$ **17** $c = -1; c = -1$
19 $c = 2, 1, 0, -1, \cdots$; same c **21** $f(x) = 0$ except at $x = 1$ **23** $\sqrt{x-1}$ **25** $-\frac{2x}{|x|}$ **27** $\frac{5}{x-1}$
29 One; two; two **31** No; yes; no **33** $xf(x), (f(x))^2, x, f(x), 2(f(x) - x), f(x) + 2x$ **35** F; F; F; T
37 Step; $f(x) = \sin \frac{1}{x}$ with $f(0) = 0$ **39** Yes; no; no; yes ($f_4(0) = 1$)
41 $g(\frac{1}{2}) = f(1) - f(\frac{1}{2}) = f(0) - f(\frac{1}{2}) = -g(0)$; zero is an intermediate value between $g(0)$ and $g(\frac{1}{2})$
43 $f(x) - x$ is ≥ 0 at $x = 0$ and ≤ 0 at $x = 1$

CHAPTER 3 APPLICATIONS OF THE DERIVATIVE

Section 3.1 Linear Approximation (page 95)

1 $Y = x$ **3** $Y = 1 + 2(x - \frac{\pi}{4})$ **5** $Y = 2\pi(x - 2\pi)$ **7** $2^6 + 6 \cdot 2^5 \cdot .001$ **9** 1
11 $1 - 1(-.02) = 1.02$ **13** Error .000301 vs. $\frac{1}{2}$ (.0001)6 **15** $.0001 - \frac{1}{3}10^{-8}$ vs. $\frac{1}{2}(.0001)(2)$
17 Error .59 vs. $\frac{1}{2}(.01)(90)$ **19** $\frac{d}{dx}\sqrt{1-x} = \frac{-1}{2\sqrt{1-x}} = -\frac{1}{2}$ at $x = 0$
21 $\frac{d}{du}\sqrt{c^2+u} = \frac{1}{2\sqrt{c^2+u}} = \frac{1}{2c}$ at $u = 0, c + \frac{u}{2c} = c + \frac{x^2}{2c}$ **23** $dV = 3(10)^2(.1)$
25 $A = 4\pi r^2, dA = 8\pi r\, dr$ **27** $V = \pi r^2 h, dV = 2\pi rh\, dr$ (plus $\pi r^2\, dh$) **29** $1 + \frac{1}{2}x$ **31** 32nd root

Section 3.2 Maximum and Minimum Problems (page 103)

1 $x = -2$: abs min **3** $x = -1$: rel max, $x = 0$: abs min, $x = 4$: abs max
5 $x = -1$: abs max, $x = 0, 1$: abs min, $x = \frac{1}{2}$: rel max **7** $x = -3$: abs min, $x = 0$: rel max, $x = 1$: rel min
9 $x = 1, 9$: abs min, $x = 5$: abs max **11** $x = \frac{1}{3}$: rel max, $x = 1$: rel min, $x = 0$: stationary (not min or max)
13 $x = 0, 1, 2, \cdots$: abs min, $x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$: abs max **15** $|x| \leq 1$: all min, $x = -3$ abs max, $x = 2$ rel max
17 $x = 0$: rel min, $x = \frac{1}{3}$: abs max, $x = 4$: abs min
19 $x = 0$: abs min, $x = \pi$: stationary (not min or max), $x = 2\pi$: abs max
21 $\theta = 0$: rel min, $\tan\theta = -\frac{4}{3}$ ($\sin\theta = \frac{4}{5}$ and $\cos\theta = -\frac{3}{5}$ abs max, $\sin\theta = -\frac{4}{5}$ and $\cos\theta = \frac{3}{5}$ abs min), $\theta = 2\pi$: rel max
23 $h = \frac{1}{3}$(62″ or 158 cm); cube **25** $\frac{v}{av^2+b}$; $2\sqrt{ab}$ gallons/mile, $\frac{1}{2\sqrt{ab}}$ miles/gallon at $v = \sqrt{\frac{b}{a}}$
27 (b) $\theta = \frac{3\pi}{8} = 67.5°$ **29** $x = \frac{a}{\sqrt{3}}$; compare Example 7; $\frac{a}{b} = \sqrt{3}$
31 $R(x) - C(x)$; $\frac{R(x)-C(x)}{x}$; $\frac{dR}{dx} - \frac{dC}{dx}$; profit **33** $x = \frac{d-a}{2(b-e)}$; zero **35** $x = 2$
37 $V = x(6 - \frac{3x}{2})(12 - 2x); x \approx 1.6$ **39** $A = \pi r^2 + x^2, x = \frac{1}{4}(4 - 2\pi r); r_{\min} = \frac{2}{2+\pi}$
41 max area 2500 vs $\frac{10000}{\pi} = 3185$ **43** $x = 2, y = 3$ **45** $P(x) = 12 - x$; thin rectangle up y axis
47 $h = \frac{H}{3}, r = \frac{2R}{3}, V = \frac{4\pi R^2 H}{27} = \frac{4}{9}$ of cone volume
49 $r = \frac{HR}{2(H-R)}$; best cylinder has *no* height, area $2\pi R^2$ from top and bottom (?)
51 $r = 2, h = 4$ **53** 25 and 0 **55** 8 and $-\infty$
57 $\sqrt{r^2 + x^2} + \sqrt{q^2 + (s-x)^2}$; $\frac{df}{dx} = \frac{x}{\sqrt{r^2+x^2}} - \frac{s-x}{\sqrt{q^2+(s-x)^2}} = 0$ when $\sin a = \sin c$
59 $y = x^2 = \frac{3}{2}$ **61** $(1, -1), (\frac{13}{5}, -\frac{1}{5})$ **63** $m = 1$ gives nearest line **65** $m = \frac{1}{3}$ **67** equal; $x = \frac{1}{2}$
69 $\frac{1}{x}x^2$ **71** True (use sign change of f'')
73 Radius R, swim $2R\cos\theta$, run $2R\theta$, time $\frac{2R\cos\theta}{v} + \frac{2R\theta}{10v}$; *max* when $\sin\theta = \frac{1}{10}$, *min* all run

Section 3.3 Second Derivatives: Bending and Acceleration (page 110)

3 $y = -1 - x^2$; no $\cdots$ **5** False **7** True **9** True (f' has 8 zeros, f'' has 7)
11 $x = 3$ is min: $f''(3) = 2$ **13** $x = 0$ not max or min; $x = \frac{9}{2}$ is min: $f''(\frac{9}{2}) = 81$
15 $x = \frac{3\pi}{4}$ is max: $f''(\frac{3\pi}{4}) = -\sqrt{2}$; $x = \frac{7\pi}{4}$ is min: $f''(\frac{7\pi}{4}) = \sqrt{2}$
17 Concave down for $x > \frac{1}{3}$ (inflection point)
19 $x = 3$ is max: $f''(3) = -4$; $x = 2, 4$ are min but $f'' = 0$ **21** $f(\Delta x) = f(-\Delta x)$ **23** $1 + x - \frac{x^2}{2}$
25 $1 - \frac{x^2}{6}$ **27** $1 - \frac{1}{2}x - \frac{1}{8}x^2$ **29** Error $\frac{1}{2}f''(x)\Delta x$ **31** Error $0\Delta x + \frac{1}{3}f'''(x)(\Delta x)^2$
37 $\frac{1}{.99} = 1.0101\overline{01}$; $\frac{1}{1.1} = .909\overline{09}$ **39** Inflection **41** 18 vs. 17 **43** Concave up; below

Section 3.4 Graphs (page 119)

1 $120; 150; \frac{60}{x}$ **3** Odd; $x = 0, y = x$ **5** Even; $x = 1, x = -1, y = 0$ **7** Even; $y = 1$ **9** Even
11 Even; $x = 1, x = -1, y = 0$ **13** $x = 0, x = -1, y = 0$ **15** $x = 1, y = 1$ **17** Odd **19** $\frac{2x}{x-1}$
21 $x + \frac{1}{x-4}$ **23** $\sqrt{x^2+1}$ **25** Of the same degree **27** Have degree $P <$ degree Q; none
29 $x = 1$ and $y = 3x + C$ if f is a polynomial; but $f(x) = (x-1)^{1/3} + 3x$ has no asymptote $x = 1$
31 $(x-3)^2$ **39** $x = \sqrt{2}, x = -\sqrt{2}, y = x$ **41** $Y = 100 \sin \frac{2\pi X}{360}$ **45** $c = 3, d = 10; c = 4, d = 20$
47 $x^* = \sqrt{5} = 2.236$ **49** $y = x - 2; Y = X; \mathrm{y} = 2\mathrm{x}$ **51** $x_{\max} = .281, x_{\min} = 6.339; x_{\text{infl}} = 4.724$
53 $x_{\min} = .393, x_{\max} = 1.53, x_{\min} = 3.33; x_{\text{infl}} = .896, 2.604$
55 $x_{\min} = -.7398, x_{\max} = .8135; x_{\text{infl}} = .04738; x_{\text{blowup}} = \pm 2.38$ **57** 8 digits

Section 3.5 Parabolas, Ellipses, and Hyperbolas (page 128)

1 $dy/dx = 0$ at $\frac{-b}{2a}$ **3** $V = (1, -4), F = (1, -3.75)$ **5** $V = (0, 0), F = (0, -1)$ **7** $F = (1, 1)$
9 $V = (0, \pm 3); F = (0, \pm\sqrt{8})$ **11** $V = (0, \pm 1); F = (0, \pm\sqrt{\frac{5}{4}})$ **13** Two lines, $a = b = c = 0; V = F = (0, 0)$
15 $y = 5x^2 - 4x$ **17** $y + p = \sqrt{x^2 + (y-p)^2} \rightarrow 4py = x^2; F = (0, \frac{1}{12}), y = -\frac{1}{12}; (\pm\frac{\sqrt{11}}{6}, \frac{11}{12})$
19 $x = ay^2$ with $a > 0; y = \frac{(x+p)^2}{4p}; y = -ax^2 + ax$ with $a > 0$
21 $\frac{x^2}{4} + y^2 = 1; \frac{(x-1)^2}{4} + (y-1)^2 = 1$ **23** $\frac{x^2}{25} + \frac{y^2}{9} = 1; \frac{(x-3)^2}{36} + \frac{(y-1)^2}{32} = 1; x^2 + y^2 = 25$
25 Circle, hyperbola, ellipse, parabola **27** $\frac{dy}{dx} = -\frac{4}{5}; y = -\frac{4}{5}x + 5$ **29** $\frac{5}{4}; \frac{9}{40} = \frac{1}{2}(\frac{5}{4} - \frac{4}{5})$
31 Circle; $(3, 1); 2; X = \frac{x-3}{2}, Y = \frac{y-1}{2}$ **33** $3x'^2 + y'^2 = 2$ **35** $y^2 - \frac{1}{3}x^2 = 1; \frac{y^2}{9} - \frac{4x^2}{9} = 1; y^2 - x^2 = 5$
37 $\frac{x^2}{25} - \frac{y^2}{39} = 1$ **39** $y^2 - 4y + 4, 2x^2 + 12x + 18; -14, (-3, 2)$, right-left
41 $F = (\pm\frac{\sqrt{5}}{2}, 0); y = \pm\frac{x}{2}$ **43** $(x + y + 1)^2 = 0$
45 $(a^2-1)x^2 + 2abxy + (b^2-1)y^2 + 2acx + 2bcy + c^2 = 0; 4(a^2 + b^2 - 1)$; if $a^2 + b^2 < 1$ then $B^2 - 4AC < 0$

Section 3.6 Iterations $x_{n+1} = F(x_n)$ (page 136)

1 $-.366; \infty$ **3** $1; 1$ **5** $\frac{2}{3}; \pm\infty$ **7** $-2; -2$
9 $\frac{1-\sqrt{3}}{2}$ attracts, $\frac{1+\sqrt{3}}{2}$ repels; $\frac{1}{2}$ attracts, 0 repels; 1 attracts, 0 repels; 1 attracts; $\frac{2}{3}$ attracts, 0 repels; $\pm\sqrt{2}$ repel
11 Negative **13** .900 **15** .679 **17** $|a| < 1$ **19** Unstable $|F'| > 1$ **21** $x^* = \frac{s}{1-a}; |a| < 1$

23 \$2000; \$2000 **25** $x_0, b/x_0, x_0, b/x_0, \cdots$ **27** $F' = -\frac{\sqrt{2}}{2}x^{-3/2} = -\frac{1}{2}$ at x^*
29 $F' = 1 - 2cx = 1 - 4c$ at $x^* = 2; 0 < c < \frac{1}{2}$ succeeds
31 $F' = 1 - 9c(x-2)^8 = 1 - 9c$ at $x^* = 3; 0 < c < \frac{2}{9}$ succeeds
33 $x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2}; x_{n+1} = x_n - \frac{\sin x_n - \frac{1}{2}}{\cos x_n}$ **35** $x^* = 4$ if $x_0 > 2.5; x^* = 1$ if $x_0 < 2.5$
37 $m = 1 + c$ at $x^* = 0, m = 1 - c$ at $x^* = 1$ (converges if $0 < c < 2$) **39** 0 **43** $F' = 1$ at $x^* = 0$

Section 3.7 Newton's Method and Chaos (page 145)

1 $x_{n+1} = x_n - \frac{x_n^3 - b}{3x_n^2} = \frac{2x_n}{3} + \frac{b}{3x_n^2}$ **5** $x_1 = x_0; x_1$ is not defined (∞) **7** $x^* = 1$ or 5 from $x_0 < 3, x_0 > 3$
11 $x_0 < \frac{1}{2}$ to $x^* = 0; x_0 > \frac{1}{2}$ to $x^* = 1$ **21** $x_{n+1} = x_n - \frac{x_n^k - 7}{kx_n^{k-1}}$ **23** $x_4 = \cot\pi = \infty; x_3 = \cot\frac{8\pi}{7} = \cot\frac{\pi}{7}$
25 π is not a fraction **27** $= \frac{1}{4}x_n^2 + \frac{1}{2} + \frac{1}{4x_n^2} = \frac{(x_n^2+1)^2}{4x_n^2} = \frac{y_n^2}{4(y_n - 1)}$ **29** $16z - 80z^2 + 128z^3 - 64z^4; 4; 2$
31 $|x_0| < 1$ **33** $\Delta x = 1$, one-step convergence for quadratics **35** $\frac{\Delta f}{\Delta x} = \frac{5.25}{1.5}; x_2 = 1.86$
37 $1.75 < x^* < 2.5; 1.75 < x^* < 2.125$ **39** $8; 3 < x^* < 4$ **41** Increases by 1; doubles for Newton
45 $x_1 = x_0 + \cot x_0 = x_0 + \pi$ gives $x_2 = x_1 + \cot x_1 = x_1 + \pi$ **49** $a = 2, Y'$s approach $\frac{1}{2}$

Section 3.8 The Mean Value Theorem and l'Hôpital's Rule (page 152)

1 $c = \sqrt{\frac{4}{3}}$ **3** No c **5** $c = 1$ **7** Corner at $\frac{1}{2}$ **9** Cusp at 0
11 $\sec^2 x - \tan^2 x =$ constant **13** 6 **15** -2 **17** -1 **19** n **21** $-\frac{1}{2}$ **23** Not $\frac{0}{0}$
25 -1 **27** 1; $\frac{1-\sin x}{1+\cos x}$ has no limit **29** $f'(c) = \frac{4^3 - 1^3}{4-1}; c = \sqrt{7}$
31 $0 = x^* - x_{n+1} + \frac{f''(c)}{2f'(x_n)}(x^* - x_n)^2$ gives $M \approx \frac{f''(x^*)}{2f'(x^*)}$ **33** $f'(0); \frac{f'(x)}{1}$; singularity **35** $\frac{f(x)}{g(x)} \to \frac{3}{4}$ **37** 1

CHAPTER 4 DERIVATIVES BY THE CHAIN RULE

Section 4.1 The Chain Rule (page 158)

1 $z = y^3, y = x^2 - 3, z' = 6x(x^2-3)^2$ **3** $z = \cos y, y = x^3, z' = -3x^2 \sin x^3$
5 $z = \sqrt{y}, y = \sin x, z' = \cos x / 2\sqrt{\sin x}$ **7** $z = \tan y + (1/\tan x), y = 1/x, z' = (\frac{-1}{x^2})\sec^2(\frac{1}{x}) - (\tan x)^{-2}\sec^2 x$
9 $z = \cos y, y = x^2 + x + 1, z' = -(2x+1)\sin(x^2 + x + 1)$ **11** $17\cos 17x$ **13** $\sin(\cos x)\sin x$
15 $x^2\cos x + 2x\sin x$ **17** $(\cos\sqrt{x+1})\frac{1}{2}(x+1)^{-1/2}$ **19** $\frac{1}{2}(1 + \sin x)^{-1/2}(\cos x)$ **21** $\cos(\frac{1}{\sin x})(\frac{-\cos x}{\sin^2 x})$
23 $8x^7 = 2(x^2)^2(2x^2)(2x)$ **25** $2(x+1) + \cos(x+\pi) = 2x + 2 - \cos x$
27 $(x^2+1)^2 + 1; \sin U$ from 0 to sin 1; $U(\sin x)$ is 1 and 0 with period 2π; R from 0 to x; $R(\sin x)$ is half-waves.
29 $g(x) = x + 2, h(x) = x^2 + 2; k(x) = 3$ **31** $f'(f(x))f'(x)$; no; $(-1/(1/x)^2)(-1/x^2) = 1$ and $f(f(x)) = x$
33 $\frac{1}{2}(\frac{1}{2}x + 8) + 8; \frac{1}{8}x + 14; \frac{1}{16}$ **35** $f(g(x)) = x, g(f(y)) = y$
37 $f(g(x)) = \frac{1}{1-x}, g(f(x)) = 1 - \frac{1}{x}, f(f(x)) = x = g(g(x)), g(f(g(x))) = \frac{x}{x-1} = f(g(f(x)))$
39 $f(y) = y - 1, g(x) = 1$ **43** $2\cos(x^2+1) - 4x^2\sin(x^2+1); -(x^2-1)^{-3/2}; -(\cos\sqrt{x})/4x + (\sin\sqrt{x})/4x^{3/2}$
45 $f'(u(t))u'(t)$ **47** $(\cos^2 u(x) - \sin^2 u(x))\frac{du}{dx}$ **49** $2xu(x) + x^2\frac{du}{dx}$ **51** $1/4\sqrt{1 - \sqrt{1-x}}\,\sqrt{1-x}$
53 df/dt **55** $f'(g(x))g'(x) = 4(x^3)^3 3x^2 = 12x^{11}$ **57** $3600; \frac{1}{2}; 18$ **59** $3; \frac{1}{3}$

Section 4.2 Implicit Differentiation and Related Rates (page 163)

1 $-x^{n-1}/y^{n-1}$ **3** $\frac{dy}{dx}=1$ **5** $\frac{dy}{dx}=\frac{1}{F'(y)}$ **7** $(y^2-2xy)/(x^2-2xy)$ or 1 **9** $\frac{1}{\sec^2 y}$ or $\frac{1}{1+x^2}$
11 First $\frac{dy}{dx}=-\frac{y}{x}$, second $\frac{dy}{dx}=\frac{x}{y}$ **13** Faster, faster **15** $2zz'=2yy'\rightarrow z'=\frac{y}{x}y'=y'\sin\theta$
17 $\sec^2\theta=\frac{c}{200\pi}$ **19** $500\frac{df}{dx}; 500\sqrt{1+(\frac{df}{dx})^2}$ **21** $\frac{dy}{dt}=-\frac{8}{3}; \frac{dy}{dt}=-2\sqrt{3}; \infty$ then 0
23 $V=\pi r^2h; \frac{dh}{dt}=\frac{1}{4\pi}\frac{dV}{dt}=-\frac{1}{4\pi}$ in/sec **25** $A=\frac{1}{2}ab\sin\theta, \frac{dA}{dt}=7$ **27** 1.6 m/sec; 9 m/sec; 12.8 m/sec
29 $-\frac{7}{5}$ **31** $\frac{dz}{dt}=\frac{\sqrt{2}}{2}\frac{dy}{dt}; \frac{d\theta}{dt}=\frac{1}{10}\cos^2\theta\frac{d\theta}{dt}; \theta''=\frac{\cos\theta}{10}y''-\frac{1}{50}\cos^3\theta\sin\theta(y')^2$

Section 4.3 Inverse Functions and Their Derivatives (page 170)

1 $x=\frac{y+6}{3}$ **3** $x=\sqrt{y+1}$ (x unrestricted $\rightarrow$ no inverse) **5** $x=\frac{1}{y-1}$ **7** $x=(1+y)^{1/3}$
9 (x unrestricted $\rightarrow$ no inverse) **11** $y=\frac{1}{x-a}$ **13** $2<f^{-1}(x)<3$ **15** f goes up and down
17 $f(x)g(x)$ and $\frac{1}{f(x)}$ **19** $m\neq 0; m\geq 0; |m|\geq 1$ **21** $\frac{dy}{dx}=5x^4, \frac{dx}{dy}=\frac{1}{5}y^{-4/5}$
23 $\frac{dy}{dx}=3x^2; \frac{dx}{dy}=\frac{1}{3}(1+y)^{-2/3}$ **25** $\frac{dy}{dx}=\frac{-1}{(x-1)^2}, \frac{dx}{dy}=\frac{-1}{(y-1)^2}$ **27** $y; \frac{1}{2}y^2+C$
29 $f(g(x))=-1/3x^3; g^{-1}(y)=\frac{-1}{y}; g(g^{-1}(x))=x$ **39** $2/\sqrt{3}$ **41** $1/6\cos 9$
43 Decreasing; $\frac{dx}{dy}=\frac{1}{dy/dx}<0$ **45** F; T; F **47** $g(x)=x^m, f(y)=y^n, x=(z^{1/n})^{1/m}$
49 $g(x)=x^3, f(y)=y+6, x=(z-6)^{1/3}$ **51** $g(x)=10^x, f(y)=\log y, x=\log(10^y)=y$
53 $y=x^3, y''=6x, d^2x/dy^2=-\frac{2}{9}y^{-5/3}$; m/sec^2, sec/m^2 **55** $p=\frac{1}{\sqrt{y}}-1; 0<y\leq 1$
57 $\max=G=\frac{3}{8}y^{4/3}, G'=\frac{1}{2}y^{1/3}$ **59** $y^2/100$

Section 4.4 Inverses of Trigonometric Functions (page 175)

1 $0, \frac{\pi}{2}, 0$ **3** $\frac{\pi}{2}, 0, \frac{\pi}{4}$ **5** π is outside $[-\frac{\pi}{2}, \frac{\pi}{2}]$ **7** $y=-\sqrt{3}/2$ and $\sqrt{3}/2$
9 $\sin x=\sqrt{1-y^2}; \sqrt{1-y^2}$ and 1 **11** $\frac{d(\sin^{-1}y)}{dy}\cos x=1\rightarrow\frac{d(\sin^{-1}y)}{dy}=\frac{1}{\cos x}=\frac{1}{\sqrt{1-y^2}}$
13 $y=0: 1, -1, 1; y=1: 0, 0, \frac{1}{2}$ **15** F; F; T; T; F; F **17** $\frac{du}{dx}=\frac{1}{\sqrt{1-x^2}}$ **19** $\frac{dz}{dx}=3$
21 $\frac{dz}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}$ **23** $1-\frac{y\sin^{-1}y}{\sqrt{1-y^2}}$ **25** $\frac{dx}{dy}=\frac{1}{|y+1|\sqrt{y^2+2y}}$ **27** $u=1$ so $\frac{du}{dy}=0$ **31** $\sec x=\sqrt{y^2+1}$
33 $\frac{1}{10}, 1, \frac{1}{2}$ **35** $-y/\sqrt{1-y^2}$ **37** $\frac{1}{2}\sec\frac{x}{2}\tan\frac{x}{2}$ **39** $\frac{nx^{n-1}}{|x^n|\sqrt{x^{2n}-1}}$ **41** $\frac{dy}{dx}=\frac{1}{1+x^2}$
43 $\frac{dy}{dx}=\pm\frac{1}{1+x^2}$ **47** $u=4\sin^{-1}y$ **49** π **51** $-\pi/4$

CHAPTER 5 INTEGRALS

Section 5.1 The Idea of the Integral (page 181)

1 1, 3, 7, 15, 127 **3** $-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}=\frac{1}{8}-1$ **5** $f_j-f_0=\frac{r^j-1}{r-1}$ **7** $3x$ for $x\leq 7$, $7x-4$ for $x\geq 1$
9 $\frac{1}{52}\frac{1}{\sqrt{52}}, \frac{2}{52}, \frac{1}{52}\sqrt{\frac{j}{52}}$ **11** Lower by 2 **13** Up, down; rectangle **15** $\sqrt{x+\Delta x}-\sqrt{x}; \Delta x; \frac{df}{dx}; \sqrt{x}$
17 6; 18; triangle **19** 18 rectangles **21** $6x-\frac{1}{2}x^2-10; 6-x$ **23** $\frac{14}{27}$ **25** $x^2; x^2; \frac{1}{3}x^3$

Section 5.2 Antiderivatives (page 186)

1 $x^5 + \frac{2}{3}x^6; \frac{5}{3}$ **3** $2\sqrt{x}; 2$ **5** $\frac{3}{4}x^{4/3}(1+2^{1/3}); \frac{3}{4}(1+2^{1/3})$ **7** $-2\cos x - \frac{1}{2}\cos 2x; \frac{5}{2} - 2\cos 1 - \frac{1}{2}\cos 2$
9 $x\sin x + \cos x; \sin 1 + \cos 1 - 1$ **11** $\frac{1}{2}\sin^2 x; \frac{1}{2}\sin^2 1$ **13** $f = C; 0$ **15** $f(b) - f(a); f_7 - f_2$
17 $8 + \frac{8}{N}$ **19** $\frac{\pi}{3}(1+\sqrt{3}); \frac{\pi}{6}(3+\sqrt{3}); 2$ **21** $\frac{5}{2}; \frac{205}{36}; \infty$ **23** $f(x) = 2\sqrt{x}$ **25** $\frac{1}{2}$, below -1; $\frac{1}{4}, \frac{5}{4}$
27 Increase - decrease; increase - decrease - increase
29 Area under B − area under D; time when $B = D$; time when $B - D$ is largest **33** T; F; F; T; F

Section 5.3 Summation Versus Integration (page 194)

1 $\frac{25}{12}; 16$ **3** 127; $2^{n+1} - 1$ **5** $\sum_{j=1}^{50} 2j = 2550; \sum_{i=1}^{100}(2j-1) = 10,000; \sum_{k=1}^{4}(-1)^{k+1}/k = \frac{7}{12}$
7 $\sum_{k=0}^{n} a_k x^k; \sum_{j=1}^{n} \sin\frac{2\pi j}{n}$ **9** 5.18738; 7.48547 **11** $2(a_i^2 + b_i^2)$ **13** $2^n - 1; \frac{1}{11} - \frac{1}{1}$ **15** F; T
17 $\frac{df}{dx} + C; f_9 - f_8 - f_1 + f_0$ **19** $f_1 = 1; n^2 + (2n+1) = (n+1)^2$
21 $a + b + c = 1, 2a + 4b + 8c = 5, 3a + 9b + 27c = 14$; sum of squares **23** $S_{400} = 80200; E_{400} = .0025 = \frac{1}{n}$
25 $S_{100,1/3} \approx 350, E_{100,1/3} \approx .00587; S_{100,3} = 25502500, E_{100,3} = .0201$ **27** v_1 and v_2 have the same sign
29 $v_1 = 9, v_2 = 12, \Sigma\Sigma = 21$ **31** At $N = 1, 2^{N-2}$ is not 1 **33** $0; \frac{1}{n}(v_1 + \cdots + v_n)$
35 $\Delta x \sum_{j=1}^{n} v(j\Delta x)$ **37** $f(1) - f(0) = \int_0^1 \frac{df}{dx}dx$

Section 5.4 Indefinite Integrals and Substitutions (page 200)

1 $\frac{2}{3}(2+x)^{3/2} + C$ **3** $(x+1)^{n+1}/(n+1) + C (n \neq -1)$ **5** $\frac{1}{12}(x^2+1)^6 + C$ **7** $-\frac{1}{4}\cos^4 x + C$
9 $-\frac{1}{8}\cos^4 2x + C$ **11** $\sin^{-1} t + C$ **13** $\frac{1}{3}(1+t^2)^{3/2} - (1+t^2)^{1/2} + C$ **15** $2\sqrt{x} + x + C$
17 $\sec x + C$ **19** $-\cos x + C$ **21** $\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}$ **23** $-\frac{1}{3}(1-2x)^{3/2}$ **25** $y = \sqrt{2x}$
27 $\frac{1}{2}x^2$ **29** $a\sin x + b\cos x$ **31** $\frac{4}{15}x^{5/2}$ **33** F; F; F; F **35** $f(x-1); 2f(\frac{x}{2})$
37 $x - \tan^{-1} x$ **39** $\int \frac{1}{u} du$ **41** $4.9t^2 + C_1 t + C_2$ **43** $f(t+3); f(t) + 3t; 3f(t); \frac{1}{3}f(3t)$

Section 5.5 The Definite Integral (page 205)

1 $C = -f(2)$ **3** $C = f(3)$ **5** $f(t)$ is wrong **7** $C = 0$ **9** $C = f(-a) - f(-b)$
11 $u = x^2 + 1; \int_1^2 u^{10}\frac{du}{2} = \frac{u^{11}}{22}]_1^2 = \frac{2^{11}-1}{22}$ **13** $u = \tan x; \int_0^1 u\, du = \frac{1}{2}$
15 $u = \sec x; \int_1^{\sqrt{2}} u\, du = \frac{1}{2}$ (same as 13) **17** $u = \frac{1}{x}, x = \frac{1}{u}, dx = \frac{-du}{u^2}; \int_1^{1/2} \frac{-du}{u}$
19 $S = \frac{1}{2}(\frac{1}{4}+1)^4 + \frac{1}{2}(1+1)^4; s = \frac{1}{2}(0) + \frac{1}{2}(\frac{1}{4}+1)^4$
21 $S = \frac{1}{2}[(\frac{1}{2})^3 + 1^3 + (\frac{3}{2})^3 + 2^3]; s = \frac{1}{2}[0^3 + (\frac{1}{2})^3 + 1^3 + (\frac{3}{2})^3]$
23 $S = \frac{1}{4}[(\frac{17}{16})^4 + (\frac{5}{4})^4 + (\frac{25}{16})^4 + 2^4]$ **25** Last rectangle minus first rectangle
27 $S = .07$ since 7 intervals have points where $W = 1$. The integral of $W(x)$ exists and equals zero.
29 M is increasing so Problem 25 gives $S - s = \Delta x(1-0)$; area from graph up to $y = 1$ is $\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot\frac{1}{2} + \cdots = \frac{1}{2}(1 + \frac{1}{4} + \frac{1}{16} + \cdots) = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$; area under graph is $\frac{1}{3}$.
31 $f(x) = 3 + \int_0^x v(x)dx; f(x) = \int_3^x v(x)dx$ **33** T;F;T;F;T;F;T

Section 5.6 Properties of the Integral and Average Value (page 212)

1 $\bar{v} = \frac{1}{2}\int_{-1}^{1} x^4 dx = \frac{1}{5}$ equals c^4 at $c = \pm(\frac{1}{5})^{1/4}$ **3** $\bar{v} = \frac{1}{\pi}\int_0^{\pi} \cos^2 x\, dx = \frac{1}{2}$ equals $\cos^2 c$ at $c = \frac{\pi}{4}$ and $\frac{3\pi}{4}$
5 $\bar{v} = \int_1^2 \frac{dx}{x^2} = \frac{1}{2}$ equals $\frac{1}{c^2}$ at $c = \sqrt{2}$ **7** $\int_3^5 v(x)dx$ **9** False, take $v(x) < 0$
11 True; $\frac{1}{3}\int_0^1 v(x)dx + \frac{2}{3}\cdot\frac{1}{2}\int_1^3 v(x)dx = \frac{1}{3}\int_0^3 v(x)dx$ **13** False; when $v(x) = x^2$ the function $x^2 - \frac{1}{3}$ is even
15 False; take $v(x) = 1$; factor $\frac{1}{2}$ is missing **17** $\bar{v} = \frac{1}{b-a}\int_a^b v(x)dx$ **19** 0 and $\frac{2}{\pi}$
21 $v(x) = Cx^2; v(x) = C$. This is "constant elasticity" in economics (Section 2.2) **23** $\overline{V} \to 0; \overline{V} \to 1$
25 $\frac{1}{2}\int_0^2 (a-x)dx = a+1$ if $a > 2$; $\frac{1}{2}\int_0^2 |a-x|dx = \frac{1}{2}$ area $= \frac{a^2}{2} - a + 1$ if $a < 2$; distance = absolute value
27 Small interval where $y = \sin\theta$ has probability $\frac{d\theta}{\pi}$; the average y is $\int_0^{\pi} \frac{\sin\theta\, d\theta}{\pi} = \frac{2}{\pi}$
29 Area under $\cos\theta$ is 1. Rectangle $0 \le \theta \le \frac{\pi}{2}, 0 \le y \le 1$ has area $\frac{\pi}{2}$. Chance of falling across a crack is $\frac{1}{\pi/2} = \frac{2}{\pi}$.
31 $\frac{1}{6^3}, \frac{3}{6^3}, \cdots, \frac{1}{6^3}; 10.5$ **33** $\frac{1}{t}\int_0^t 220\cos\frac{2\pi t}{60} dt = \frac{1}{t}\cdot 220\cdot\frac{60}{2\pi}\sin\frac{2\pi t}{60} = V_{\text{ave}}$
35 Any $v(x) = v_{\text{even}}(x) + v_{\text{odd}}(x)$; $(x+1)^3 = (3x^2+1) + (x^3+3x)$; $\frac{1}{x+1} = \frac{1}{1-x^2} - \frac{x}{1-x^2}$
37 16 per class; $\frac{6}{64}$; $E(x) = \frac{1800}{64} = \frac{225}{8}$ **39** F; F; T; T
41 $f(x) = \left\{ \begin{array}{ll} \frac{1}{2}(x-2)^2 & x \ge 2 \\ -\frac{1}{2}(x-2)^2 & x \le 2 \end{array} \right\} + C; f(5) - f(0) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$

Section 5.7 The Fundamental Theorem and Its Applications (page 219)

1 $\cos^2 x$ **3** 0 **5** $(x^2)^3(2x) = 2x^7$ **7** $v(x+1) - v(x)$ **9** $\frac{\sin^2 x}{x} - \frac{1}{x^2}\int_0^x \sin^2 t\, dt$
11 $\int_0^x v(u)du$ **13** 0 **15** $2\sin x^2$ **17** $u(x)v(x)$ **19** $\sin^{-1}(\sin x)\cos x = x\cos x$
21 F; F; F; T **23** Taking derivatives $v(x) = (x\cos x)' = \cos x - x\sin x$
25 Taking derivatives $-v(-x)(-1) = v(x)$ so v is even **27** F; T; T; F
29 $\int_1^x v(t)dt = \int_0^x v(t)dt - \int_0^1 v(t)dt = \frac{x}{x+2} - \frac{1}{1+2}$ (in revised printing)
31 $V = s^3; A = 3s^2$; half of hollow cube; $\Delta V \approx 3s^2 dS; 3s^2$ (which is A)
33 $dH/dr = 2\pi^2 r^3$ **35** Wedge has length $r \approx$ height of triangle; $\int_0^{\pi/2} \frac{1}{2}r^2 d\theta = \frac{\pi r^2}{4}$
37 $r = \frac{1}{\cos\theta}; \frac{d\theta}{2\cos^2\theta}; \int_0^{\pi/4} \frac{d\theta}{2\cos^2\theta} = \frac{\tan\theta}{2}\Big]_0^{\pi/4} = \frac{1}{2}$
39 $x = y^2; \int_0^2 y^2 dy = \frac{y^3}{3}\Big]_0^2 = \frac{8}{3}$; vertical strips have length $2 - \sqrt{x}$
41 Length $\sqrt{2}a$; width $\frac{da}{\sqrt{2}}$; $\int_0^1 a\,da = \frac{1}{2}$ **43** The differences of the sums $f_j = v_1 + v_2 + \cdots + v_j$ are $f_j - f_{j-1} = v_j$
45 No, $\int_0^x a(t)dt = \frac{df}{dx}(x) - \frac{df}{dx}(0)$ and $\int_0^1 (\int_0^x a(t)dt)dx = f(1) - f(0) - \frac{df}{dx}(0)$

Section 5.8 Numerical Integration (page 226)

1 $\frac{1}{2}\Delta x(v_0 - v_n)$ **3** $1, .5625, .3025; 0, .0625, .2025$ **5** $L_8 \approx .1427, T_8 \approx .2052, S_8 \approx .2000$
7 $p = 2$: for $y = x^2, \frac{1}{4}\cdot 0^2 + \frac{1}{2}\cdot(\frac{1}{2})^2 + \frac{1}{4}\cdot 1^2 \ne \frac{1}{3}$ **9** For $y = x^2$, error $\frac{1}{6}(\Delta x)^2$ from $\frac{1}{2} - \frac{1}{3}, y_1' = 2\Delta x$
13 8 intervals give $\frac{(\Delta x)^2}{12}[-\frac{1}{b^2} + \frac{1}{a^2}] = \frac{1}{1024} < .001$ **15** $f''(c)$ is $y'(c)$ **17** $\infty; .683, .749, .772 \to \frac{\pi}{4}$
19 $A + B + C = 1, \frac{1}{2}B + C = \frac{1}{2}, \frac{1}{4}B + C = \frac{1}{3}$; Simpson
21 $y = 1$ and x on [0,1]: $L_n = 1$ and $\frac{1}{2} - \frac{1}{2n}, R_n = 1$ and $\frac{1}{2} + \frac{1}{2n}$, so only $\frac{1}{2}L_n + \frac{1}{2}R_n$ gives 1 and $\frac{1}{2}$
23 $T_{10} \approx 500{,}000{,}000; T_{100} \approx 50{,}000{,}000; 25{,}000\pi$
25 $a = 4, b = 2, c = 1; \int_0^1 (4x^2 + 2x + 1)dx = \frac{10}{3}$; Simpson fits parabola **27** $c = \frac{1}{4320}$

CHAPTER 6 EXPONENTIALS AND LOGARITHMS

Section 6.1 An Overview (page 234)

1 $5; -5; -1; \frac{1}{5}; \frac{3}{2}; 2$ **5** $1; -10; 80; 1; 4; -1$ **7** $n \log_b x$ **9** $\frac{10}{3}; \frac{3}{10}$ **13** 10^5
15 $0;\ I_{SF} = 10^7 I_0;\ 8.3 + \log_{10} 4$ **17** $A = 7, b = 2.5$ **19** $A = 4, k = 1.5$
21 $\frac{1}{cx}; \frac{2}{cx}; \log 2$ **23** $y - 1 = cx; y - 10 = c(x-1)$ **25** $(.1^{-h} - 1)/(-h) = (10^h - 1)/(-h)$
27 $y'' = c^2 b^x;\ x'' = -1/cy^2$ **29** Logarithm

Section 6.2 The Exponential e^x (page 241)

1 $49e^{7x}$ **3** $8e^{8x}$ **5** $3^x \ln 3$ **7** $(\frac{2}{3})^x \ln \frac{2}{3}$ **9** $\frac{-e^x}{(1+e^x)^2}$ **11** 2 **13** xe^x **15** $\frac{4}{(e^x+e^{-x})^2}$
17 $e^{\sin x} \cos x + e^x \cos e^x$ **19** .1246, .0135, .0014 are close to $\frac{e}{2n}$ **21** $\frac{1}{e}; \frac{1}{e}$
23 $Y(h) = 1 + \frac{1}{10}; Y(1) = (1 + \frac{1}{10})^{10} = 2.59$ **25** $(1 + \frac{1}{x})^x < e < e^x < e^{3x/2} < e^{2x} < 10^x < x^x$
27 $\frac{e^{3x}}{3} + \frac{e^{7x}}{7}$ **29** $x + \frac{2^x}{\ln 2} + \frac{3^x}{\ln 3}$ **31** $\frac{(2e)^x}{\ln(2e)} + 2e^x$ **33** $\frac{e^{x^2}}{2} - \frac{e^{-x^2}}{2}$
35 $2e^{x/2} + \frac{e^{2x}}{2}$ **37** e^{-x} drops faster at $x = 0$ (slope -1); meet at $x = 1; e^{-x^2}/e^{-x} < e^{-9}/e^{-3} < \frac{1}{100}$ for $x > 3$
39 $y - e^a = e^a(x - a)$; need $-e^a = -ae^a$ or $a = 1$
41 $y' = x^x(\ln x + 1) = 0$ at $x_{\min} = \frac{1}{e}; y'' = x^x[(\ln x + 1)^2 + \frac{1}{x}] > 0$
43 $\frac{d}{dx}(e^{-x}y) = e^{-x}\frac{dy}{dx} - e^{-x}y = 0$ so $e^{-x}y =$ Constant or $y = Ce^x$
45 $\frac{e^{2x}}{2}]_0^1 = \frac{e^2-1}{2}$ **47** $\frac{2^x}{\ln 2}]_{-1}^1 = \frac{2-\frac{1}{2}}{\ln 2} = \frac{3}{2\ln 2}$ **49** $-e^{-x}]_0^\infty = 1$ **51** $e^{1+x}]_0^1 = e^2 - e$ **53** $\frac{2^{\sin x}}{\ln 2}]_0^\pi = 0$
55 $\int \frac{du/dx}{e^u} dx = -e^{-u} + C;\quad \int (e^u)^2 \frac{du}{dx} dx = \frac{1}{2}e^{2u} + C$ **57** $yy' = 1$ gives $\frac{1}{2}y^2 = x + C$ or $y = \sqrt{2x + 2C}$
59 $\frac{dF}{dx} = (n - x)x^{n-1}/e^x < 0$ for $x > n; F(2x) < \frac{\text{constant}}{e^x} \to 0$ **61** $\frac{6!}{\sqrt{12\pi}} \approx 117; (\frac{6}{e})^6 \approx 116;$ 7 digits

Section 6.3 Growth and Decay in Science and Economics (page 250)

1 $t^2 + y_0$ **3** $y_0 e^{2t}$ **5** $10\, e^{4t}; t = \frac{\ln 10}{4}$ **7** $\frac{1}{4}e^{4t} + 9.75;\ t = \frac{\ln 361}{4}$ **11** $c = \frac{\ln 2}{2}; t = \frac{\ln 10}{c}$
13 $\frac{5568}{-.7}\ln(\frac{1}{5})$ **15** $c = \frac{\ln 2}{20}; t = \frac{1}{c}\ln(\frac{8}{5})$ **17** $t = \frac{\ln(1/240)}{\ln(.98)}$ **19** $e^c = 3$ so $y_0 = e^{-3c}1000 = \frac{1000}{27}$
21 $p = 1013\, e^{ch};\ 50 = 1013\, e^{20c};\ c = \frac{1}{20}\ln(\frac{50}{1013});\ p(10) = 1013\, e^{10c} = 1013\sqrt{\frac{50}{1013}} = \sqrt{(1013)(50)}$
23 $c = \frac{\ln 2}{3}; (\frac{1}{2})^3 = \frac{1}{8}$ **25** $y = y_0 - at$ reaches y_1 at $t = \frac{y_0 - y_1}{a}$; then $y = Ae^{-at/y_1}$ **27** F; F; T; T
29 $A = \frac{1}{3}, B = -\frac{1}{3}$ **31** $e^t - 1$ **33** $1 - e^{-t}$ **35** $6; 6 + Ae^{-2t}; 6 - 6e^{-2t}, 6 + 4e^{-2t}; 6$
37 $4; 4 - \frac{1}{e}; 4$ **39** $ye^{-t}; y(t) = te^t$ **41** $A = 1, B = -1, C = -1$ **43** $e^{.0725} > .075$ **45** $s(e-1); \frac{s(e-1)}{e}$
47 $(1.02)(1.03) \to 5.06\%; 5\%$ by Problem 27 **49** $20{,}000\, e^{(20-T)(.05)} = 34{,}400$ (it grows for $20 - T$ years)
51 $s = -cy_0 e^{ct}/(e^{ct} - 1) = -(.01)(1000)e^{.60}/(e^{.60} - 1)$ **53** $y_0 = \frac{100}{.005}(1 - e^{-.005(48)})$
55 $e^{4c} = 1.20$ so $c = \frac{\ln 1.20}{4}$ **57** $24e^{36.5} = ?$ **59** To $-\infty$; constant; to $+\infty$
61 $\frac{dY}{dT} = 60cY; \frac{dY}{dT} = 60(-Y + 5)$; still $Y_\infty = 5$
63 $y = 60e^{ct} + 20, 60 = 60e^{12c} + 20, c = \frac{1}{12}\ln(\frac{40}{60}); 100 = 60e^{ct} + 20$ at $t = \frac{1}{c}\ln(\frac{80}{60})$ **65** 0

Section 6.4 Logarithms (page 258)

1 $\frac{1}{x}$ **3** $\frac{-1}{x(\ln x)^2}$ **5** $\ln x$ **7** $\frac{\cos x}{\sin x} = \cot x$ **9** $\frac{7}{x}$ **11** $\frac{1}{3}\ln t + C$ **13** $\ln\frac{4}{3}$
15 $\frac{1}{2}\ln 5$ **17** $-\ln(\ln 2)$ **19** $\ln(\sin x) + C$ **21** $-\frac{1}{3}\ln(\cos 3x) + C$ **23** $\frac{1}{3}(\ln x)^3 + C$
27 $\ln y = \frac{1}{2}\ln(x^2+1); \frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$ **29** $\frac{dy}{dx} = e^{\sin x}\cos x$
31 $\frac{dy}{dx} = e^x e^{e^x}$ **33** $\ln y = e^x \ln x; \frac{dy}{dx} = ye^x(\ln x + \frac{1}{x})$ **35** $\ln y = -1$ so $y = \frac{1}{e}, \frac{dy}{dx} = 0$ **37** 0
39 $-\frac{1}{x}$ **41** $\sec x$ **47** .1; .095; .095310179 **49** $-.01; -.01005; -.010050335$
51 l'Hôpital: 1 **53** $\frac{1}{\ln b}$ **55** $3 - 2\ln 2$ **57** Rectangular area $\frac{1}{2} + \cdots + \frac{1}{n} < \int_1^n \frac{dt}{t} = \ln n$
59 Maximum at e **61** 0 **63** $\log_{10} e$ or $\frac{1}{\ln 10}$ **65** $1 - x; 1 + x\ln 2$
67 Fraction is $y = 1$ when $\ln(T+2) - \ln 2 = 1$ or $T = 2e - 2$ **69** $y' = \frac{2}{(t+2)^2} \to y = 1 - \frac{2}{t+2}$ never equals 1
71 $\ln p = x\ln 2$; **LD** $2^x \ln 2$; **ED** $p = e^{x\ln 2}, p' = \ln 2\, e^{x\ln 2}$
75 $2^4 = 4^2; y\ln x = x\ln y \to \frac{\ln x}{x} = \frac{\ln y}{y}; \frac{\ln x}{x}$ decreases after $x = e$, and the only integers before e are 1 and 2.

Section 6.5 Separable Equations Including the Logistic Equation (page 266)

1 $7e^t - 5$ **3** $(\frac{3}{2}x^2 + 1)^{1/3}$ **5** x **7** $e^{1-\cos t}$ **9** $(\frac{ct}{2} + \sqrt{y_0})^2$ **11** $y_\infty = 0; t = \frac{1}{by_0}$
15 $z = 1 + e^{-t}$, y is in **13** **17** $ct = \ln 3, ct = \ln 9$
19 $b = 10^{-9}, c = 13 \cdot 10^{-3}; y_\infty = 13 \cdot 10^6$; at $y = \frac{c}{2b}$ (10) gives $\ln\frac{1}{b} = ct + \ln\frac{10^6}{c - 10^6 b}$ so $t = 1900 + \frac{\ln 12}{c} = 2091$
21 y^2 dips down and up (a valley) **23** $sc = 1 = sbr$ so $s = \frac{1}{c}, r = \frac{c}{b}$
25 $y = \frac{N}{1 + e^{-Nt}(N-1)}; T = \frac{\ln(N-1)}{N} \to 0$ **27** Dividing cy by $y + K > 1$ slows down y'
29 $\frac{dR}{dy} = \frac{cK}{(y+K)^2} > 0, \frac{cy}{y+K} \to c$
31 $\frac{dY}{dT} = \frac{-Y}{Y+1}$; multiply $e^{y/K}\frac{y}{K} = e^{-ct/K}e^{y_0/K}(\frac{y_0}{K})$ by K and take the Kth power to reach (19)
33 $y' = (3-y)^2; \frac{1}{3-y} = t + \frac{1}{3}; y = 2$ at $t = \frac{2}{3}$
35 $Ae^t + D = Ae^t + B + Dt + t \to D = -1, B = -1; y_0 = A + B$ gives $A = 1$
37 $y \to 1$ from $y_0 > 0$, $y \to -\infty$ from $y_0 < 0; y \to 1$ from $y_0 > 0, y \to -1$ from $y_0 < 0$
39 $\int \frac{\cos y\, dy}{\sin y} = \int dt \to \ln(\sin y) = t + C = t + \ln\frac{1}{2}$. Then $\sin y = \frac{1}{2}e^t$ stops at 1 when $t = \ln 2$

Section 6.6 Powers Instead of Exponentials (page 276)

1 $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$ **3** $1 \pm x + \frac{x^2}{2} \pm \frac{x^3}{6} + \cdots$ **5** 1050.62; 1050.95; 1051.25
7 $1 + n(\frac{-1}{n}) + \frac{n(n+1)}{2}(\frac{-1}{n})^2 \to 1 - 1 + \frac{1}{2}$ **9** square of $(1 + \frac{1}{n})^n$; set $N = 2n$
11 Increases; $\ln(1 + \frac{1}{x}) - \frac{1}{x+1} > 0$ **13** $y(3) = 8$ **15** $y(t) = 4(3^t)$ **17** $y(t) = t$
19 $y(t) = \frac{1}{2}(3^t - 1)$ **21** $s(\frac{a^t - 1}{a - 1})$ if $a \neq 1$; st if $a = 1$ **23** $y_0 = 6$ **25** $y_0 = 3$
27 $-2, -10, -26 \to -\infty; -5, -\frac{17}{2}, -\frac{41}{4} \to -12$ **29** $P = \frac{b}{c+d}$ **31** 10.38% **33** $100(1.1)^{20} = \$673$
35 $\frac{100{,}000(.1/12)}{1 - (1 + .1/12)^{-240}} = 965$ **37** $\frac{1000}{.1}(1.1^{20} - 1) = 57{,}275$ **39** $y_\infty = 1500$ **41** $2; (\frac{53}{52})^{52} = 2.69; e$
43 $1.0142^{12} = 1.184 \to$ Visa charges 18.4%

Section 6.7 Hyperbolic Functions (page 280)

1 $e^x, e^{-x}, \frac{e^{2x} - e^{-2x}}{4} = \frac{1}{2}\sinh 2x$ **7** $\sinh nx$ **9** $3\sinh(3x+1)$ **11** $\frac{-\sinh x}{\cosh^2 x} = -\tanh x \operatorname{sech} x$
13 $4\cosh x \sinh x$ **15** $\frac{x}{\sqrt{x^2+1}}(\operatorname{sech}\sqrt{x^2+1})^2$ **17** $6\sinh^5 x \cosh x$
19 $\cosh(\ln x) = \frac{1}{2}(x + \frac{1}{x}) = 1$ at $x = 1$ **21** $\frac{5}{13}, \frac{13}{5}, -\frac{12}{5}, -\frac{13}{12}, -\frac{5}{12}$ **23** $0, 0, 1, \infty, \infty$
25 $\frac{1}{2}\sinh(2x+1)$ **27** $\frac{1}{3}\cosh^3 x$ **29** $\ln(1 + \cosh x)$ **31** e^x

33 $\int y\,dx = \int \sinh t(\sinh t\,dt)$; $A = \frac{1}{2}\sinh t\cosh t - \int y\,dx$; $A' = \frac{1}{2}$; $A = 0$ at $t = 0$ so $A = \frac{1}{2}t$.
41 $e^y = x + \sqrt{x^2+1}, y = \ln[x + \sqrt{x^2+1}]$ **47** $\frac{1}{4}\ln\left|\frac{2+x}{2-x}\right|$ **49** $\sinh^{-1} x$ (see 41) **51** $-\text{sech}^{-1}x$
53 $\frac{1}{2}\ln 3; \infty$ **55** $y(x) = \frac{1}{c}\cosh cx; \frac{1}{c}\cosh cL - \frac{1}{c}$
57 $y'' = y - 3y^2; \frac{1}{2}(y')^2 = \frac{1}{2}y^2 - y^3$ is satisfied by $y = \frac{1}{2}\text{sech}^2\frac{x}{2}$

CHAPTER 7 TECHNIQUES OF INTEGRATION

Section 7.1 Integration by Parts (page 287)

1 $-x\cos x + \sin x + C$ **3** $-xe^{-x} - e^{-x} + C$ **5** $x^2\sin x + 2x\cos x - 2\sin x + C$
7 $\frac{1}{2}(2x+1)\ln(2x+1) - x + C$ **9** $\frac{1}{2}e^x(\sin x - \cos x) + C$ **11** $\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$
13 $\frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C$ **15** $x(\ln x)^2 - 2x\ln x + 2x + C$ **17** $x\sin^{-1}x + \sqrt{1-x^2} + C$
19 $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{x}{2} + C$ **21** $x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$
23 $e^x(x^3 - 3x^2 + 6x - 6) + C$ **25** $x\tan x + \ln(\cos x) + C$ **27** -1 **29** $-\frac{3}{4}e^{-2} + \frac{1}{4}$ **31** -2
33 $3\ln 10 - 6 + 2\tan^{-1}3$ **35** $u = x^n, v = e^x$ **37** $u = x^n, v = \sin x$ **39** $u = (\ln x)^n, v = x$
41 $u = x\sin x, v = e^x \to \int e^x\sin x\,dx$ in **9** and $-\int x\cos x\, e^x dx$. Then $u = -x\cos x, v = e^x \to \int e^x\cos x\,dx$ in **10** and $-\int x\sin x\, e^x dx$ (move to left side): $\frac{e^x}{2}(x\sin x - x\cos x + \cos x)$. Also try $u = xe^x, v = -\cos x$.
43 $\int \frac{1}{2}u\sin u\,du = \frac{1}{2}(\sin u - u\cos u) = \frac{1}{2}(\sin x^2 - x^2\cos x^2)$; odd
45 3· step function; $3e^x$· step function **49** $0; x\delta(x)] - \int \delta(x)dx = -1; v(x)\delta(x)] - \int v(x)\delta(x)dx$
51 $v(x) = \int_x^1 f(x)dx$
53 $u(x) = \frac{1}{k}\int_0^x v(x)dx; \frac{1}{k}(\frac{x}{2} - \frac{x^3}{6}); \frac{x}{k}$ for $x \le \frac{1}{2}, \frac{1}{k}(2x - x^2 - \frac{1}{4})$ for $x \ge \frac{1}{2}; \frac{x}{k}$ for $x \le \frac{1}{2}, \frac{1}{2k}$ for $x \ge \frac{1}{2}$.
55 $u = x^2, v = -\cos x \to -x^2\cos x + (2x)\sin x - \int 2\sin x\,dx$ **57** Compare **23**
59 $uw']_0^1 - \int_0^1 u'w' - u'w]_0^1 + \int_0^1 u'w' = [uw' - u'w]_0^1$
61 No mistake: $e^x\cosh x - e^x\sin hx = 1$ is part of the constant C

Section 7.2 Trigonometric Integrals (page 293)

1 $\int(1-\cos^2 x)\sin x\,dx = -\cos x + \frac{1}{3}\cos^3 x + C$ **3** $\frac{1}{2}\sin^2 x + C$
5 $\int(1-u^2)^2u^2(-du) = -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$ **7** $\frac{2}{3}(\sin x)^{3/2} + C$
9 $\frac{1}{8}\int\sin^3 2x\,dx = \frac{1}{16}(-\cos 2x + \frac{1}{3}\cos^3 2x) + C$ **11** $\frac{\pi}{2}$ **13** $\frac{1}{3}(\frac{3x}{2} + \frac{\sin 6x}{4}) + C$
15 $x + C$ **17** $\frac{1}{6}\cos^5 x\sin x + \frac{5}{6}\int\cos^4 x\,dx$; use equation (5)
19 $\int_0^{\pi/2}\cos^n x\,dx = \frac{n-1}{n}\int_0^{\pi/2}\cos^{n-2}x\,dx = \cdots = \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{1}{2}\int_0^{\pi/2}dx$
21 $I = -\sin^{n-1}x\cos x + (n-1)\int\sin^{n-2}x\cos^2 x\,dx = -\sin^{n-1}x\cos x + (n-1)\int\sin^{n-2}x\,dx - (n-1)I$. So $nI = -\sin^{n-1}x\,\cos x + (n-1)\int\sin^{n-2}x\,dx$.
23 $0, +, 0, 0, 0, -$ **25** $-\frac{2}{3}\cos^3 x, 0$ **27** $-\frac{1}{2}(\frac{\cos 2x}{2} + \frac{\cos 200x}{200}), 0$ **29** $\frac{1}{2}(\frac{\sin 200x}{200} + \frac{\sin 2x}{2}), 0$
31 $-\frac{1}{2}\cos x, 0$ **33** $\int_0^\pi x\sin x\,dx = \int_0^\pi A\sin^2 x\,dx \to A = 2$ **35** Sum = zero = $\frac{1}{2}$ (left + right)
37 p is even **39** $p - q$ is even **41** $\sec x + C$ **43** $\frac{1}{3}\tan^3 x + C$ **45** $\frac{1}{3}\sec^3 x + C$
47 $\frac{1}{3}\tan^3 x - \tan x + x + C$ **49** $\ln|\sin x| + C$ **51** $\frac{1}{2\cos^2 x} + C$ **53** $A = \sqrt{2}, -\sqrt{2}\sin(x + \frac{\pi}{4})$
55 $4\sqrt{2}$ **57** $\frac{1000}{\sqrt{3}}$ **59** $\frac{1-\cos x+\sin x}{1+\cos x+\sin x} + C$ **61** p and q are 10 and 1

Section 7.3 Trigonometric Substitutions (page 299)

1 $x = 2\sin\theta; \int d\theta = \sin^{-1}\frac{x}{2} + C$ **3** $x = 2\sin\theta; \int 4\cos^2\theta\, d\theta = 2\sin^{-1}\frac{x}{2} + x\sqrt{1-\frac{x^2}{4}} + C$

5 $x = \sin\theta; \int \sin^2\theta\, d\theta = \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2} + C$

7 $x = \tan\theta; \int \cos^2\theta\, d\theta = \frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{1+x^2} + C$

9 $x = 5\sec\theta; \int 5(\sec^2\theta - 1)d\theta = \sqrt{x^2-25} - 5\sec^{-1}\frac{x}{5} + C$

11 $x = \sec\theta; \int \cos\theta\, d\theta = \frac{\sqrt{x^2-1}}{x} + C$ **13** $x = \tan\theta; \int \cos\theta\, d\theta = \frac{x}{\sqrt{1+x^2}} + C$

15 $x = 3\sec\theta; \int \frac{\cos\theta\, d\theta}{9\sin^2\theta} = \frac{-1}{9\sin\theta} + C = \frac{-x}{9\sqrt{x^2-9}} + C$

17 $x = \sec\theta; \int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln(\sec\theta + \tan\theta) + C = \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\ln(x+\sqrt{x^2-1}) + C$

19 $x = \tan\theta; \int \frac{\cos\theta\, d\theta}{\sin^2\theta} = -\frac{1}{\sin\theta} + C = \frac{-\sqrt{x^2+1}}{x} + C$

21 $\int \frac{-\sin\theta\, d\theta}{\sin\theta} = -\theta + C = -\cos^{-1}x + C$; with $C = \frac{\pi}{2}$ this is $\sin^{-1}x$

23 $\int \frac{\tan\theta\sec^2\theta\, d\theta}{\sec^2\theta} = -\ln(\cos\theta) + C = \ln\sqrt{x^2+1} + C$ which is $\frac{1}{2}\ln(x^2+1) + C$

25 $x = a\sin\theta; \int_{-\pi/2}^{\pi/2} a^2\cos^2\theta\, d\theta = \frac{a^2\pi}{2}$ = area of semicircle **27** $\sin^{-1}x]_{.5}^{1} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

29 Like Example 6: $x = \sin\theta$ with $\theta = \frac{\pi}{2}$ when $x = \infty, \theta = \frac{\pi}{3}$ when $x = 2$, $\int_{\pi/3}^{\pi/2}\frac{\cos\theta\, d\theta}{\sin^2\theta} = -1 + \frac{2}{\sqrt{3}}$

31 $x = 3\tan\theta; \int_{-\pi/2}^{\pi/2}\frac{3\sec^2\theta\, d\theta}{9\sec^2\theta} = \frac{\theta}{3}\Big]_{-\pi/2}^{\pi/2} = \frac{\pi}{3}$ **33** $\int \frac{x^{n+1}+x^{n-1}}{x^2+1}dx = \int x^{n-1}dx = \frac{x^n}{n}$

35 $x = \sec\theta; \frac{1}{2}(e^f + e^{-f}) = \frac{1}{2}(x + \sqrt{x^2-1} + \frac{1}{x+\sqrt{x^2-1}}) = \frac{1}{2}(x + \sqrt{x^2-1} + x - \sqrt{x^2-1}) = x$

37 $x = \cosh\theta; \int d\theta = \cosh^{-1}x + C$

39 $x = \cosh\theta; \int \sinh^2\theta\, d\theta = \frac{1}{2}(\sinh\theta\cosh\theta - \theta) + C = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln(x+\sqrt{x^2-1}) + C$

41 $x = \tanh\theta; \int d\theta = \tanh^{-1}x + C$ **43** $(x-2)^2 + 4$ **45** $(x-3)^2 - 9$ **47** $(x+1)^2$

49 $u = x-2, \int \frac{du}{u^2+4} = \frac{1}{2}\tan^{-1}\frac{u}{2} = \frac{1}{2}\tan^{-1}(\frac{x-2}{2}) + C; u = x-3, \int \frac{du}{u^2-9} = \frac{1}{6}\ln\frac{u-3}{u+3} = \frac{1}{6}\ln\frac{x-6}{x} + C;$
$u = x+1, \int \frac{du}{u^2} = \frac{-1}{u} = \frac{-1}{x+1} + C$

51 $u = x+b; \int \frac{du}{u^2-b^2+c}$ uses $u = a\sec\theta$ if $b^2 > c, u = a\tan\theta$ if $b^2 < c$, equals $-\frac{1}{u} = \frac{-1}{x+b}$ if $b^2 = c$

53 $\cos\theta$ is negative $(-\sqrt{1-x^2})$ from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$; then $\int_0^1 - \int_1^{-1} + \int_{-1}^0 \sqrt{1-x^2}dx = \pi$ = area of unit circle

55 Divide y by 4, multiply dx by 4, same $\int y\, dx$

57 No $\sin^{-1}x$ for $x > 1$; the square root is imaginary. All correct with complex numbers.

Section 7.4 Partial Fractions (page 304)

1 $A = -1, B = 1, -\ln x + \ln(x-1) + C$ **3** $\frac{1}{x-3} - \frac{1}{x-2}$ **5** $\frac{1}{2x} - \frac{2}{x+1} + \frac{5/2}{x+2}$

7 $\frac{3}{x} + \frac{1}{x^2}$ **9** $3 - \frac{3}{x^2+1}$ **11** $-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}$ **13** $-\frac{1/6}{x} + \frac{1/2}{x-1} - \frac{1/2}{x-2} + \frac{1/6}{x-3}$

15 $\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}; A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, D = -\frac{1}{2}$

17 Coefficients of $y: 0 = -Ab + B$; match constants $1 = Ac; A = \frac{1}{c}, B = \frac{b}{c}$

19 $A = 1$, then $B = 2$ and $C = 1; \int \frac{dx}{x-1} + \int \frac{(2x+1)dx}{x^2+x+1} =$
$\ln(x-1) + \ln(x^2+x+1) = \ln(x-1)(x^2+x+1) = \ln(x^3-1)$

21 $u = e^x; \int \frac{du}{u^2-u} = \int \frac{du}{u-1} - \int \frac{du}{u} = \ln(\frac{u-1}{u}) + C = \ln(\frac{e^x-1}{e^x}) + C$

23 $u = \cos\theta; \int \frac{-du}{1-u^2} = -\frac{1}{2}\int\frac{du}{1-u} - \frac{1}{2}\int\frac{du}{1+u} = \frac{1}{2}\ln(1-u) - \frac{1}{2}\ln(1+u) = \frac{1}{2}\ln\frac{1-\cos\theta}{1+\cos\theta} + C$. We can reach $\frac{1}{2}\ln\frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \ln\frac{1-\cos\theta}{\sin\theta} = \ln(\csc\theta - \cot\theta)$ or a different way $\frac{1}{2}\ln\frac{1-\cos^2\theta}{(1+\cos\theta)^2} = \ln\frac{\sin\theta}{1+\cos\theta} = -\ln\frac{1+\cos\theta}{\sin\theta} =$ $-\ln(\csc\theta + \cot\theta)$

25 $u = e^x; du = e^x dx = u\, dx; \int \frac{1+u}{(1-u)u}du = \int\frac{2du}{1-u} + \int\frac{du}{u} = -2\ln(1-e^x) + \ln e^x + C = -2\ln(1-e^x) + x + C$

27 $x+1=u^2, dx = 2u\,du; \int \frac{2u\,du}{1+u} = \int[2-\frac{2}{1+u}]du = 2u - 2\ln(1+u) + C =$ $2\sqrt{x+1} - 2\ln(1+\sqrt{x+1}) + C$

29 Note $Q(a)=0$. Then $\frac{x-a}{Q(x)} = \frac{x-a}{Q(x)-Q(a)} \to \frac{1}{Q'(a)}$ by definition of derivative. At a double root $Q'(a)=0$.

Section 7.5 Improper Integrals (page 309)

1 $\frac{x^{1-e}}{1-e}]_1^\infty = \frac{1}{e-1}$ **3** $-2(1-x)^{1/2}]_0^1 = 2$ **5** $\tan^{-1} x]_{-\pi/2}^0 = \frac{\pi}{2}$ **7** $\frac{1}{2}(\ln x)^2]_0^1 = -\infty$

9 $x\ln x - x]_0^e = 0$ **11** $\ln(\ln(\ln x))]_{100}^\infty = \infty$ **13** $\frac{1}{2}(x+\sin x\cos x)]_0^\infty = \infty$

15 $\frac{x^{1-p}}{1-p}]_0^\infty$ diverges for every p! **17** Less than $\int_1^\infty \frac{dx}{x^6} = \frac{1}{5}$

19 Less than $\int_0^1 \frac{dx}{x^2+1} + \int_1^\infty \frac{\sqrt{x}\,dx}{x^2} = \tan^{-1} x]_0^1 - \frac{2}{\sqrt{x}}]_1^\infty = \frac{\pi}{4} + 2$

21 Less than $\int_1^\infty e^{-x}dx = \frac{1}{e}$, greater than $-\frac{1}{e}$

23 Less than $\int_0^1 e^2 dx + e\int_1^\infty e^{-(x-1)^2}dx = e^2 + e\int_1^\infty e^{-u^2}du = e^2 + \frac{e}{\sqrt{\pi}}$

25 $\int_0^1 \frac{\sin^2 x\,dx}{x^2} + \int_1^\infty \frac{\sin^2 x\,dx}{x^2}$ less than $1 + \int_1^\infty \frac{dx}{x^2} = 2$ **27** $p! = p$ times $(p-1)!; 1 = 1$ times $0!$

29 $u = x, dv = xe^{-x^2}dx : -x\frac{e^{-x^2}}{2}]_0^\infty + \int_0^\infty \frac{e^{-x^2}}{2}dx = \frac{1}{4}\sqrt{\pi}$ **31** $\int_0^\infty 1000e^{-.1t}dt = -10,000e^{-.1t}]_0^\infty = \$10,000$

33 $W = \frac{-GMm}{x}]_R^\infty = \frac{GMm}{R} = \frac{1}{2}mv_0^2$ if $v_0 = \sqrt{\frac{2GM}{R}}$

35 $\int_0^\infty \frac{dx}{2^x} = \int_0^\infty e^{-x\ln 2}dx = \frac{e^{-x\ln 2}}{-\ln 2}]_0^\infty = \frac{1}{\ln 2}$

37 $\int_0^{\pi/2}(\sec x - \tan x)dx = [\ln(\sec x + \tan x) + \ln(\cos x)]_0^{\pi/2} = [\ln(1+\sin x)]_0^{\pi/2} = \ln 2$. The areas under $\sec x$ and $\tan x$ separately are infinite **39** Only $p = 0$

CHAPTER 8 APPLICATIONS OF THE INTEGRAL

Section 8.1 Areas and Volumes by Slices (page 318)

1 $x^2 - 3 = 1$ gives $x = \pm 2; \int_{-2}^2[(1-(x^2-3)]dx = \frac{32}{3}$

3 $y^2 = x = 9$ gives $y = \pm 3; \int_{-3}^3[9-y^2]dy = 36$

5 $x^4 - 2x^2 = 2x^2$ gives $x = \pm 2$ (or $x = 0$); $\int_{-2}^2[2x^2 - (x^4 - 2x^2)]dx = \frac{128}{15}$

7 $y = x^2 = -x^2 + 18x$ gives $x = 0, 9; \int_0^9[(-x^2+18x) - x^2]dx = 243$

9 $y = \cos x = \cos^2 x$ when $\cos x = 1$ or 0, $x = 0$ or $\frac{\pi}{2}$ or $\cdots$ $\int_0^{\pi/2}(\cos x - \cos^2 x)dx = 1 - \frac{\pi}{4}$

11 $e^x = e^{2x-1}$ gives $x = 1; \int_0^1[e^x - e^{2x-1}]dx = (e-1) - (\frac{e-e^{-1}}{2})$

13 Intersections $(0,0), (1,3), (2,2)$; $\int_0^1[3x-x]dx + \int_1^2[4-x-x]dx = 2$

15 Inside, since $1 - x^2 < \sqrt{1-x^2}$; $\int_{-1}^1[\sqrt{1-x^2} - (1-x^2)]dx = \frac{\pi}{2} - \frac{4}{3}$

17 $V = \int_{-a}^a \pi y^2 dx = \int_{-a}^a \pi b^2(1-\frac{x^2}{a^2})dx = \frac{4\pi b^2 a}{3}$; around y axis $V = \frac{4\pi a^2 b}{3}$; rotating $x = 2, y = 0$ around y axis gives a circle not in the first football

19 $V = \int_0^\pi 2\pi x \sin x\,dx = 2\pi^2$ **21** $\int_0^8 \pi(8-x)^2 dx = \frac{512\pi}{3}; \int_0^8 2\pi x(8-x)dx = \frac{512\pi}{3}$ (same cone tipped over)

23 $\int_0^1 \pi\cdot 1^2 dx - \int_0^1 \pi(x^4)^2 dx = \frac{8\pi}{9}; \int_0^1 2\pi(1-x^4)x\,dx = \frac{2\pi}{3}$

25 $\int_{1/3}^2 \pi(3^2)dx - \int_{1/3}^2 \pi(\frac{1}{x})^2 dx = \frac{25\pi}{2}; \int_{1/3}^2 2\pi x(3-\frac{1}{x})dx = \frac{25\pi}{3}$

27 $\int_0^1 \pi[(x^{2/3})^2 - (x^{3/2})^2]dx = \frac{5\pi}{28}; \int_0^1 2\pi x(x^{2/3} - x^{3/2})dx = \frac{5\pi}{28}$ (notice xy symmetry)

29 $x^2 = R^2 - y^2, V = \int_{R-h}^R \pi(R^2-y^2)dy = \pi(Rh^2 - \frac{h^3}{3})$

31 $\int_{-a}^a(2\sqrt{a^2-x^2})^2 dx = \frac{16}{3}a^3$ **33** $\int_0^1(2\sqrt{1-y})^2 dy = 2$ **37** $\int A(x)dx$ or in this case $\int a(y)dy$

39 Ellipse; $\sqrt{1-x^2}\tan\theta; \frac{1}{2}(1-x^2)\tan\theta; \frac{2}{3}\tan\theta$

41 Half of $\pi r^2 h$; rectangles **43** $\int_1^3 \pi(5^2-2^2)dx = 42\pi$ **45** $\int_1^3 \pi(4^2-1^2)dx = 30\pi$

47 $\int_0^{b-a} \pi((b-y)^2 - a^2)dy = \frac{\pi}{3}(b^3 - 3a^2b + 2a^3)$ **49** $\int_0^2 \pi(3-x)^2dx; \int_0^1 2\pi y(2)dy + \int_1^3 2\pi y(3-y)dy$
51 $\int_a^b \pi(\frac{y}{m})^2dy = \frac{\pi(b^3-a^3)}{3m^2}$ **53** $960\,\pi$ cm **55** $\frac{\pi}{2}$ **57** $\frac{2\pi}{3}$
59 2π **61** $\int_0^4 2\pi y(2-\sqrt{y})dy = \frac{32\pi}{5}$ **63** $3\pi e$ **65** Height 1; $\int_0^a 2\pi x\,dx = \pi a^2$; cylinder
67 Length of hole is $2\sqrt{b^2-a^2} = 2$, so $b^2 - a^2 = 1$ and volume is $\frac{4\pi}{3}$ **69** F; T(?); F; T

Section 8.2 Length of a Plane Curve (page 324)

1 $\int_0^1 \sqrt{1+(\frac{3}{2}x^{1/2})^2}dx = \frac{8}{27}[(\frac{13}{4})^{3/2} - 1] = \frac{13\sqrt{13}-8}{27}$ **3** $\int_0^1 \sqrt{1+x^2(x^2+2)}dx = \int_0^1(1+x^2)dx = \frac{4}{3}$
5 $\int_1^3 \sqrt{1+(x^2-\frac{1}{4x^2})^2}dx = \int_1^3(x^2+\frac{1}{4x^2})dx = \frac{53}{6}$
7 $\int_1^4 \sqrt{1+(x^{1/2}-\frac{1}{4}x^{-1/2})^2}dx = \int_1^4(x^{1/2}+\frac{1}{4}x^{-1/2})dx = \frac{31}{6}$
9 $\int_0^{\pi/2} \sqrt{9\cos^4 t\ \sin^2 t + 9\sin^4 t\ \cos^2\ t}\ dt = \int_0^{\pi/2} 3\cos t \sin t\ dt = \frac{3}{2}$
11 $\int_0^{\pi/2} \sqrt{\sin^2 t + (1-\cos t)^2}dt = \int_0^{\pi/2} \sqrt{2-2\cos t}\ dt = \int_0^{\pi/2} 2\sin\frac{t}{2}\ dt = 4 - 2\sqrt{2}$
13 $\int_0^1 \sqrt{t^2+2t+1}\ dt = \int_0^1(t+1)dt = \frac{3}{2}$ **15** $\int_0^\pi \sqrt{1+\cos^2 x}\ dx = 3.820$ **17** $\int_1^e \sqrt{1+\frac{1}{x^2}}dx = 2.003$
19 Graphs are flat toward (1,0) then steep up to (1,1); limiting length is 2
21 $\frac{ds}{dt} = \sqrt{36\sin^2 3t + 36\cos^2 3t} = 6$ **23** $\int_0^1 \sqrt{26}\ dy = \sqrt{26}$
25 $\int_{-1}^1 \sqrt{\frac{1}{4}(e^y - e^{-y})^2 + 1}\ dy = \int_{-1}^1 \frac{1}{2}(e^y + e^{-y})dy = \frac{1}{2}(e^y - e^{-y})]_{-1}^1 = e - \frac{1}{e}$.
Using $x = \cosh y$ this is $\int \sqrt{1+\sinh^2 y}\ dy = \int \cosh y\ dy = \sinh y]_{-1}^1 = 2\sinh 1$
27 Ellipse; two y's for the same x **29** Carpet length $2 \neq$ straight distance $\sqrt{2}$
31 $(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2; ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2}dt;$
$ds = \sqrt{\sin^2 t + \cos^2 t + 1}dt = \sqrt{2}dt; 2\pi\sqrt{2}$; curve = helix, shadow = circle
33 $L = \int_0^1 \sqrt{1+4x^2}dx; \int_0^2 \sqrt{1+x^2}dx = \int_0^1 \sqrt{1+4u^2}\ 2du = 2L$; stretch xy plane by 2 ($y = x^2$ becomes $\frac{y}{2} = (\frac{x}{2})^2$)

Section 8.3 Area of a Surface of Revolution (page 327)

1 $\int_2^6 2\pi\sqrt{x}\sqrt{1+(\frac{1}{2\sqrt{x}})^2}dx = \int_2^6 2\pi\sqrt{x+\frac{1}{4}}dx = \frac{49\pi}{3}$ **3** $2\int_0^1 2\pi(7x)\sqrt{50}dx = 14\pi\sqrt{50}$
5 $\int_{-1}^1 2\pi\sqrt{4-x^2}\sqrt{1+\frac{x^2}{4-x^2}}dx = \int_{-1}^1 4\pi dx = 8\pi$ **7** $\int_0^2 2\pi x\sqrt{1+(2x)^2}dx = \frac{\pi}{6}(1+4x^2)^{3/2}]_0^2 = \frac{\pi}{6}[17^{3/2}-1]$
9 $\int_0^3 2\pi x\sqrt{2}dx = 9\pi\sqrt{2}$ **11** Figure shows radius s times angle θ = arc $2\pi R$
13 $2\pi r\Delta s = \pi(R+R')(s-s') = \pi Rs - \pi R's'$ because $R's - Rs' = 0$
15 Radius a, center at $(0,b)$; $(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 = a^2$, surface area $\int_0^{2\pi} 2\pi(b + a\sin t)a\ dt = 4\pi^2ab$
17 $\int_1^2 2\pi x\sqrt{1+\frac{(1-x)^2}{2x-x^2}}dx = \int_1^2 \frac{2\pi x\ dx}{\sqrt{2x-x^2}} = \pi^2 + 2\pi$ (write $2x - x^2 = 1 - (x-1)^2$ and set $x - 1 = \sin\theta$)
19 $\int_{1/2}^1 2\pi x\sqrt{1+\frac{1}{x^4}}dx$ (can be done)
21 Surface area $= \int_1^\infty 2\pi\frac{1}{x}\sqrt{1+\frac{1}{x^4}}dx > \int_1^\infty \frac{2\pi dx}{x} = 2\pi\ln x]_1^\infty = \infty$ but volume $= \int_1^\infty \pi(\frac{1}{x})^2dx = \pi$
23 $\int_0^\pi 2\pi\sin t\sqrt{2\sin^2 t + \cos^2 t}\ dt = \int_0^\pi 2\pi\sin t\sqrt{2-\cos^2 t}\ dt = \int_{-1}^1 2\pi\sqrt{2-u^2}du =$
$\pi u\sqrt{2-u^2} + 2\pi\sin^{-1}\frac{u}{\sqrt{2}}]_{-1}^1 = 2\pi + \pi^2$

Section 8.4 Probability and Calculus (page 334)

1 $P(X<4) = \frac{7}{8}, P(X=4) = \frac{1}{16}, P(X>4) = \frac{1}{16}$ **3** $\int_0^\infty p(x)dx$ is not 1; $p(x)$ is negative for large x
5 $\int_2^\infty e^{-x}dx = \frac{1}{e^2}; \int_1^{1.01} e^{-x}dx \approx (.01)\frac{1}{e}$ **7** $p(x) = \frac{1}{\pi}; F(x) = \frac{x}{\pi}$ for $0 \le x \le \pi$ ($F = 1$ for $x > \pi$)

9 $\mu = \frac{1}{7}\cdot 1 + \frac{1}{7}\cdot 2 + \cdots + \frac{1}{7}\cdot 7 = 4$ **11** $\int_0^\infty \frac{2x\,dx}{\pi(1+x^2)} = \frac{1}{\pi}\ln(1+x^2)]_0^\infty = +\infty$

13 $\int_0^\infty axe^{-ax}dx = [-xe^{-ax}]_0^\infty + \int_0^\infty e^{-ax}dx = \frac{1}{a}$

15 $\int_0^x \frac{2dx}{\pi(1+x^2)} = \frac{2}{\pi}\tan^{-1}x; \int_0^x e^{-x}dx = 1-e^{-x}; \int_0^x ae^{-ax}dx = 1-e^{-ax}$ **17** $\int_{10}^\infty \frac{1}{10}e^{-x/10}dx = -e^{-x/10}]_{10}^\infty = \frac{1}{e}$

19 *Exponential* better than Poisson: 60 years $\rightarrow \int_0^{60} .01e^{-.01x}\,dx = 1-e^{-.6} = .45$

21 $y = \frac{x-\mu}{\sigma}$; three areas $\approx \frac{1}{3}$ each because $\mu-\sigma$ to μ is the same as μ to $\mu+\sigma$ and areas add to 1

23 $-2\mu\int xp(x)dx + \mu^2\int p(x)dx = -2\mu\cdot\mu + \mu^2\cdot 1 = -\mu^2$

25 $\mu = 0\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 2\cdot\frac{1}{3} = 1;\ \sigma^2 = (0-1)^2\cdot\frac{1}{3} + (1-1)^2\cdot\frac{1}{3} + (2-1)^2\cdot\frac{1}{3} = \frac{2}{3}$.
Also $\sum n^2p_n - \mu^2 = 0\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 4\cdot\frac{1}{3} - 1 = \frac{2}{3}$

27 $\mu = \int_0^\infty \frac{xe^{-x/2}dx}{2} = 2; 1 - \int_0^4 \frac{e^{-x/2}dx}{2} = 1 + [e^{-x/2}]_0^4 = e^{-2}$

29 Standard deviation (yes – no poll) $\le \frac{1}{2\sqrt{N}} = \frac{1}{2\sqrt{900}} = \frac{1}{60}$ Poll showed $\frac{870}{900} = \frac{29}{30}$ peaceful.
95% confidence interval is from $\frac{29}{30} - \frac{2}{60}$ to $\frac{29}{30} + \frac{2}{60}$, or 93% to 100% peaceful.

31 95% confidence of unfair if more than $\frac{2\sigma}{\sqrt{N}} = \frac{1}{\sqrt{2500}} = 2\%$ away from 50% heads.
2% of 2500 = 50. So unfair if more than 1300 or less than 1200.

33 55 is 1.5σ below the mean, and the area up to $\mu - 1.5\sigma$ is about 8% so 24 students fail.
A grade of 57 is 1.3σ below the mean and the area up to $\mu - 1.3\sigma$ is about 10%.

35 .999; $.999^{1000} = (1 - \frac{1}{1000})^{1000} \approx \frac{1}{e}$ because $(1-\frac{1}{n})^n \rightarrow \frac{1}{e}$.

Section 8.5 Masses and Moments (page 340)

1 $\overline{x} = \frac{10}{6}$ **3** $\overline{x} = \frac{4}{4}$ **5** $\overline{x} = \frac{3.5}{3}$ **7** $\overline{x} = \frac{2}{3} = \overline{y}$ **9** $\overline{x} = \frac{7/2}{7} = \overline{y}$ **11** $\overline{x} = \frac{1/3}{\pi/4} = \overline{y}$ **13** $\overline{x} = \frac{1/4}{1/2}, \overline{y} = \frac{1/8}{1/2}$

15 $\overline{x} = \frac{0}{3\pi} = \overline{y}$ **21** $I = \int x^2\rho\,dx - 2t\int x\rho\,dx + t^2\int\rho\,dx; \frac{dI}{dt} = -2\int x\rho\,dx + 2t\int\rho\,dx = 0$ for $t = \overline{x}$

23 South Dakota **25** $2\pi^2a^2b$ **27** $M_x = 0, M_y = \frac{\pi}{2}$ **29** $\frac{2}{\pi}$ **31** Moment

33 $I = \sum m_nr_n^2; \frac{1}{2}\sum m_nr_n^2\omega_n^2; 0$ **35** $14\pi\ell\frac{r^2}{2}; 14\pi\ell\frac{r^4}{4}; \frac{1}{2}$

37 $\frac{2}{3}$; solid ball, solid cylinder, hallow ball, hollow cylinder **39** No

41 $T \approx \sqrt{1+J}$ by Problem **40** so $T \approx \sqrt{1.4}, \sqrt{1.5}, \sqrt{5/3}, \sqrt{2}$

Section 8.6 Force, Work, and Energy (page 346)

1 2.4 ft lb; 2.424 ... ft lb **3** 24000 lb/ft; $83\frac{1}{3}$ ft lb **5** $10x$ ft lb; $10x$ ft lb **7** 25000 ft lb; 20000 ft lb

9 864,000 Nkm **11** $5.6\cdot 10^7$ Nkm **13** $k = 10$ lb/ft; $W = 25$ ft lb **15** $\int 60wh\,dh = 48000w, 12000w$

17 $\frac{1}{2}wAH^2; \frac{3}{8}wAH^2$ **19** $9600w$ **21** $(1-\frac{v^2}{c^2})^{-3/2}$ **23** (800) (9800) kg **25** ± force

CHAPTER 9 POLAR COORDINATES AND COMPLEX NUMBERS

Section 9.1 Polar Coordinates (page 350)

1 $(1, \frac{\pi}{2})$ **3** $(2, \frac{\pi}{4})$ **5** $(\sqrt{2}, \frac{5\pi}{4})$ **7** (0,2) **9** $(\sqrt{10}, \sqrt{10})$ **11** $(\sqrt{3}, -1)$ **13** $2\sqrt{2}$

15 $\sqrt{r^2 + R^2 - 2rR\cos(\theta - \phi)}$

17 $0 < r < \infty, -\frac{\pi}{2} < \theta < \frac{\pi}{2}; 0 \le r < \infty, \pi \le \theta \le 2\pi; \sqrt{4} < r < \sqrt{5}, 0 \le \theta < 2\pi; 0 \le r < \infty, -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

19 $y = x\tan\theta, r = x\sec\theta$ **21** $\theta = \frac{\pi}{4}$, all r; $r = \frac{1}{\sin\theta + \cos\theta}$; $r = \cos\theta + \sin\theta$

23 $x^2 + y^2 = y$ **25** $x = r\sin\theta\cos\theta, y = r\sin^2\theta, x^2 + y^2 = y$

27 $x^2 + y^2 = x + y, (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = (\frac{\sqrt{2}}{2})^2$ **29** $x = \frac{\cos\theta}{\cos\theta + \sin\theta}, y = \frac{\sin\theta}{\cos\theta + \sin\theta}$ **31** $(x^2 + y^2)^3 = x^4$

Section 9.2 Polar Equations and Graphs (page 355)

1 Line $y=1$ **3** Circle $x^2+y^2=2x$ **5** Ellipse $3x^2+4y^2=1-2x$ **7** x, y, r symmetries
9 x symmetry only **11** No symmetry **13** x, y, r symmetries!
15 $x^2+y^2=6y+8x \rightarrow (x-4)^2+(y-3)^2=5^2$, center (4,3) **17** (2,0), (0,0)
19 $r=1-\frac{\sqrt{2}}{2}, \theta=\frac{3\pi}{4}; r=1+\frac{\sqrt{2}}{2}, \theta=\frac{7\pi}{4}; (0,0)$ **21** $r=2, \theta=\pm\frac{\pi}{12}, \pm\frac{5\pi}{12}, \pm\frac{7\pi}{12}, \pm\frac{11\pi}{12}$
23 $(x,y)=(1,1)$ **25** $r=\cos 5\theta$ has 5 petals **27** $(x^2+y^2-x)^2=x^2+y^2$
29 $(x^2+y^2)^3=(x^2-y^2)^2$ **31** $\cos\theta=-\frac{1}{2}, \sin\theta=\frac{\sqrt{3}}{2} \rightarrow y=\frac{2\sqrt{3}}{3}, x=-\frac{2}{3}$ **33** $x=\frac{4}{3}, r=-\frac{5}{3}$ **35** .967

Section 9.3 Slope, Length, and Area for Polar Curves (page 359)

1 Area $\frac{3\pi}{2}$ **3** Area $\frac{9\pi}{2}$ **5** Area $\frac{\pi}{8}$ **7** Area $\frac{\pi}{8}-\frac{1}{4}$ **9** $\int_{-\pi/3}^{\pi/3}(\frac{9}{2}\cos^2\theta-\frac{(1+\cos\theta)^2}{2})d\theta=\pi$
11 Area 8π **13** Only allow $r^2>0$, then $4\int_0^{\pi/4}\frac{1}{2}\cos 2\theta\, d\theta=1$ **15** $2+\frac{\pi}{4}$
17 $\theta=0$; left points $r=\frac{1}{2}, \theta=\pm\frac{2\pi}{3}, x=-\frac{1}{4}, y=\pm\frac{\sqrt{3}}{4}$
19 $\frac{r^2}{2c}]_6^{14}=40,000; \frac{1}{2c}[r\sqrt{r^2+c^2}+c^2\ln(r+\sqrt{r^2+c^2})]_6^{14}=40,000.001$
21 $\tan\psi=\tan\theta$ **23** $x=0, y=1$ is on limacon but not circle **25** $\frac{1}{2}\ln(2\pi+\sqrt{1+4\pi^2})+\pi\sqrt{1+4\pi^2}$
27 $\frac{3\pi}{2}$ **29** $\frac{1}{2}$ (base)(height) $\approx\frac{1}{2}(r\Delta\theta)r$ **31** $\frac{4\pi}{5}\sqrt{2}$ **33** $2\pi(2-\sqrt{2})$ **35** $\frac{8\pi}{3}$ **39** $\sec\theta$

Section 9.4 Complex Numbers (page 364)

1 Sum = 4, product = 5 **5** Angles $\frac{3\pi}{4}, \frac{3\pi}{2}, \frac{9\pi}{4}$ **7** Real axis; imaginary axis; $\frac{1}{2}$ axis $x\geq 0$; unit circle
9 $cd=5+10i, \frac{c}{d}=\frac{11-2i}{25}$ **11** $2\cos\theta, 1; -1, 1$ **13** Sum = 0, product = -1 **15** $r^4e^{4i\theta}, \frac{1}{r}e^{-i\theta}, \frac{1}{r^4}e^{-4i\theta}$
17 Evenly spaced on circle around origin **19** e^{it}, e^{-it} **21** e^t, e^{-t}, e^0 **23** $\cos 7t, \sin 7t$
29 $t=-\frac{2\pi}{\sqrt{3}}, y=-e^{\pi/\sqrt{3}}$ **31** F; T; at most 2; Re $c<0$ **33** $\frac{1}{r}e^{-i\theta}, x=\frac{1}{r}\cos\theta, y=-\frac{1}{r}\sin\theta; \pm\frac{1}{\sqrt{r}}e^{-i\theta/2}$

CHAPTER 10 INFINITE SERIES

Section 10.1 The Geometric Series (page 373)

1 Subtraction leaves $G-xG=1$ or $G=\frac{1}{1-x}$ **3** $\frac{1}{2}; \frac{4}{5}; \frac{100}{11}; 3\frac{4}{99}$ **5** $2\cdot 1+3\cdot 2x+4\cdot 3x^2+\cdots=\frac{2}{(1-x)^3}$
7 .142857 repeats because the next step divides 7 into 1 again
9 If q (prime, not 2 or 5) divides 10^N-10^M then it divides $10^{N-M}-1$ **11** This decimal does not repeat
13 $\frac{87}{99}; \frac{123}{999}$ **15** $\frac{x}{1-x^2}$ **17** $\frac{x^3}{1-x^3}$ **19** $\frac{\ln x}{1-\ln x}$ **21** $\frac{1}{x-1}$ **23** $\tan^{-1}(\tan x)=x$
25 $(1+x+x^2+x^3\cdots)(1-x+x^2-x^3\cdots)=1+x^2+x^4+\cdots$
27 $2(.1234\ldots)$ is $2\cdot\frac{1}{10}\cdot\frac{1}{(1-\frac{1}{10})^2}=\frac{20}{81}$; $1-.0123\ldots$ is $1-\frac{1}{100}\frac{1}{(1-\frac{1}{10})^2}=\frac{80}{81}$ **29** $\frac{2}{3}\frac{1}{1-\frac{1}{3}}=1$
31 $-\ln(1-.1)=-\ln .9$ **33** $\frac{1}{2}\ln\frac{1.1}{.9}$ **35** $(n+1)!$ **37** $y=\frac{b}{1-bx}$
39 All products like a_1b_2 are missed; $(1+1)(1+1)\neq 1+1$ **41** Take $x=\frac{1}{2}$ in (13): ln 3 = 1.0986
43 In 3 seconds the ball goes 78 feet **45** $\tan z=\frac{2}{3}$; (18) is slower with $x=\frac{2}{3}$

Section 10.2 Convergence Tests: Positive Series (page 380)

1 $\frac{1}{2}+\frac{1}{4}+\cdots$ is smaller than $1+\frac{1}{3}+\cdots$
3 $a_n = s_n - s_{n-1} = \frac{1}{n^2-n}, s=1; a_n = 4, s=\infty; a_n = \ln\frac{2n}{n+1} - \ln\frac{2(n-1)}{n} = \ln\frac{n^2}{n-1}, s = \ln 2$
5 No decision on $\sum b_n$ **7** Diverges: $\frac{1}{100}(1+\frac{1}{2}+\cdots)$ **9** $\sum\frac{1}{100+n^2}$ converges: $\sum\frac{1}{n^2}$ is larger
11 Converges: $\sum\frac{1}{n^2}$ is larger **13** Diverges: $\sum\frac{1}{2n}$ is smaller **15** Diverges: $\sum\frac{1}{2n}$ is smaller
17 Converges: $\sum\frac{2}{2^n}$ is larger **19** Converges: $\sum\frac{3}{3^n}$ is larger **21** $L=0$ **23** $L=0$ **25** $L=\frac{1}{2}$
27 root $(\frac{n-1}{n})^n \to L = \frac{1}{e}$ **29** $s=1$ (only survivor) **31** If y decreases, $\sum_2^n y(i) \le \int_1^n y(x)dx \le \sum_1^{n-1} y(i)$
33 $\sum_1^\infty e^{-n} \le \int^{\infty_0} e^{-x}dx = 1; \frac{1}{e}+\frac{1}{e^2}+\cdots = \frac{1}{e-1}$ **35** Converges faster than $\int\frac{dx}{x^2+1}$
37 Diverges because $\int_0^\infty \frac{x\,dx}{x^2+1} = \frac{1}{2}\ln(x^2+1)]_0^\infty = \infty$ **39** Diverges because $\int_1^\infty x^{e-\pi}dx = \frac{x^{e-\pi+1}}{e-\pi+1}]_0^\infty = \infty$
41 Converges (geometric) because $\int_1^\infty (\frac{e}{\pi})^x dx < \infty$ **43** (b) $\int_n^{n+1}\frac{dx}{x} >$ (base 1) (height $\frac{1}{n+1}$)
45 After adding we have $1+\frac{1}{2}+\cdots+\frac{1}{2n}$ (close to $\ln 2n$); thus originally close to $\ln 2n - \ln n = \ln\frac{2n}{n} = \ln 2$
47 $\int_{100}^{1000}\frac{dx}{x^2} = \frac{1}{100} - \frac{1}{1000} = .009$ **49** Comparison test: $\sin a_n < a_n$; if $a_n = \pi n$ then $\sin a_n = 0$ but $\sum a_n = \infty$
51 $a_n = n^{-5/2}$ **53** $a_n = \frac{2^n}{n^n}$ **55** Ratios are $1, \frac{1}{2}, 1, \frac{1}{2}, \cdots$ (no limit L); $(\frac{1}{2^k})^{1/2k} = \frac{1}{\sqrt{2}}$; yes
57 Root test $\frac{1}{\ln n} \to L = 0$ **59** Root test $L = \frac{1}{10}$ **61** Divergence: N terms add to $\ln\frac{N+2}{2} \to \infty$
63 Diverge (compare $\sum\frac{1}{n}$) **65** Root test $L = \frac{3}{4}$ **67** Beyond some point $\frac{a_n}{b_n} < 1$ or $a_n < b_n$

Section 10.3 Convergence Tests: All Series (page 384)

1 Terms don't approach zero **3** Absolutely **5** Conditionally not absolutely **7** No convergence
9 Absolutely **11** No convergence **13** By comparison with $\sum|a_n|$
15 Even sums $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots$ diverge; a_n's are not decreasing **17** (b) If $a_n > 0$ then s_n is too large so $s - s_n < 0$
19 $s = 1 - \frac{1}{e}$; below by less than $\frac{1}{5!}$
21 Subtract $2(\frac{1}{2^2}+\frac{1}{4^2}+\cdots) = \frac{2}{4}(\frac{1}{1^2}+\frac{1}{2^2}+\cdots) = \frac{\pi^2}{12}$ from positive series to get alternating series
23 Text proves: If $\sum|a_n|$ converges so does $\sum a_n$
25 New series $= (\frac{1}{2}) - \frac{1}{4} + (\frac{1}{6}) - \frac{1}{8}\cdots = \frac{1}{2}(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}\cdots)$ **27** $\frac{3}{2}\ln 2$: add $\ln 2$ series to $\frac{1}{2}$ ($\ln 2$ series)
29 Terms alternate and decrease to zero; partial sums are $1+\frac{1}{2}+\cdots+\frac{1}{n} - \ln n \to \gamma$
31 .5403? **33** Hint + comparison test **35** Partial sums $a_n - a_0$; sum $-a_0$ if $a_n \to 0$
37 $\frac{1}{1-\frac{1}{2}}\frac{1}{1-\frac{1}{3}} = 3$ but product is not $1+\frac{2}{3}+\cdots$
39 Write x to base 2, as in 1.0010 which keeps $1+\frac{1}{8}$ and deletes $\frac{1}{2}, \frac{1}{4}, \cdots$
41 $\frac{1}{9}+\frac{1}{27}+\cdots$ adds to $\frac{1/9}{1-1/3} = \frac{1}{6}$ and can't cancel $\frac{1}{3}$
43 $\frac{\sin 1}{1-\cos 1} = \cot\frac{1}{2}$ (trig identity) $= \tan(\frac{\pi}{2} - \frac{1}{2}); s = \sum\frac{e^{in}}{n} = -\log(1-e^i)$ by 10a in Section 10.1; take imaginary part

Section 10.4 The Taylor Series for $e^x, \sin x$, and $\cos x$ (page 390)

1 $1+2x+\frac{(2x)^2}{2!}+\cdots$; derivatives $2^n; 1+2+\frac{2^2}{2!}+\cdots$ **3** Derivatives $i^n; 1+ix+\cdots$
5 Derivatives $2^n n!; 1+2x+4x^2+\cdots$ **7** Derivatives $-(n-1)!; -x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots$
9 $y = 2-e^x = 1-x-\frac{x^2}{2!}-\cdots$ **11** $y = x - \frac{x^3}{6}+\cdots = \sin x$ **13** $y = xe^x = x+x^2+\frac{x^3}{2!}+\cdots$
15 $1+2x+x^2; 4+4(x-1)+(x-1)^2$ **17** $-(x-1)^5$ **19** $1-(x-1)+(x-1)^2-\cdots$
21 $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots = \ln(1+(x-1))$ **23** $e^{-1}e^{1-x} = e^{-1}(1-(x-1)+\frac{(x-1)^2}{2!}-\cdots)$
25 $x+2x^2+2x^3$ **27** $\frac{1}{2} - \frac{x^2}{24} + \frac{x^4}{720}$ **29** $x - \frac{x^3}{18} + \frac{x^5}{600}$ **31** $1+x^2+\frac{x^4}{2}$ **33** $1+x-\frac{x^3}{2}$

35 ∞ slope; $1+\frac{1}{2}(x-1)$ **37** $x-\frac{x^3}{3}+\frac{x^5}{5}$ **39** $x+\frac{x^3}{3}+\frac{2x^5}{15}$ **41** $1+x+\frac{x^2}{2}$ **43** $1+0x-x^2$
45 $\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}, \sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}$ **47** 99th powers $-1,-i,e^{3\pi i/4},-i$
49 $e^{-i\pi/3}$ and -1; sum zero, product -1 **53** $i\frac{\pi}{2}, i\frac{\pi}{2}+2\pi i$ **55** $2e^x$

Section 10.5 Power Series (page 395)

1 $1+4x+(4x)^2+\cdots; r=\frac{1}{4}; x=\frac{1}{4}$ **3** $e(1-x+\frac{x^2}{2!}-\cdots); r=\infty$
5 $\ln e+\ln(1+\frac{x}{e})=1+\frac{x}{e}-\frac{1}{2}(\frac{x}{e})^2+\cdots; r=e; x=-e$
7 $|\frac{x-1}{2}|<1$ or $(-1,3)$; $\frac{2}{3-x}$ **9** $|x-a|<1; -\ln(1-(x-a))$
11 $1+\frac{x}{2!}+\frac{x^2}{3!}+\cdots$; add to 1 at $x=0$ **13** $a_1, a_3, \cdots$ are all zero **15** $\frac{1-(1-\frac{1}{2}x^2\cdots)}{x^2}\to\frac{1}{2}$
17 $f^{(8)}(c)=\cos c<1$; alternating terms might not decrease (as required)
19 $f=\frac{1}{1-x}, |R_n|\le\frac{x^{n+1}}{(1-c)^{n+2}}; R_n=\frac{x^{n+1}}{1-x}; (1-c)^4=1-\frac{1}{2}$
21 $f^{(n+1)}(x)=\frac{n!}{(1-x)^{n+1}}, |R_n|\le\frac{x^{n+1}}{(1-c)^{n+1}}(\frac{1}{n+1})\to 0$ when $x=\frac{1}{2}$ and $1-c>\frac{1}{2}$
23 $R_2=f(x)-f(a)-f'(a)(x-a)-\frac{1}{2}f''(a)(x-a)^2$ so $R_2=R_2'=R_2''=0$ at $x=a, R_2'''=f'''$;
Generalized Mean Value Theorem in 3.8 gives $a<c<c_2<c_1<x$
25 $1+\frac{1}{2}x^2+\frac{3}{8}(x^2)^2$ **27** $(-1)^n; (-1)^n(n+1)$
29 (a) one friend k times, the other $n-k$ times, $0\le k\le n$; 21 **33** $(16-1)^{1/4}\approx 1.968$
35 $(1+.1)^{1.1}=1(1.1)(.1)+\frac{(1.1)(.1)}{2}(.1)^2\approx 1.1105$ **37** $1+\frac{x^2}{2}+\frac{5x^4}{24}; r=\frac{\pi}{2}$ **41** $x+x^2+\frac{5}{6}x^3+\frac{5}{6}x^4$
43 $x^2-\frac{1}{3}x^4+\frac{2}{45}x^6$ **45** $1+\frac{x}{2}+\frac{3x}{8}+\frac{5x}{16}$ **47** .2727 **49** $-\frac{1}{6}-\frac{1}{3}=-\frac{1}{2}$ **51** $r=1, r=\frac{\pi}{2}-1$

CHAPTER 11 VECTORS AND MATRICES

Section 11.1 Vectors and Dot Products (page 405)

1 $(0,0,0); (5,5,5); 3; -3; \cos\theta=-1$ **3** $2\mathbf{i}-\mathbf{j}-\mathbf{k}; -\mathbf{i}-7\mathbf{j}+8\mathbf{k}; 6; -3; \cos\theta=-\frac{1}{2}$
5 $(v_2,-v_1); (v_2,-v_1,0), (v_3,0,-v_1)$ **7** (0,0);(0,0,0) **9** Cosine of θ; projection of $\mathbf{w}$ on $\mathbf{v}$
11 F;T;F **13** Zero; sum = 10 o'clock vector; sum = 8 o'clock vector times $\frac{1+\sqrt{3}}{2}$
15 45° **17** Circle $x^2+y^2=4$; $(x-1)^2+y^2=4$; vertical line $x=2$; half-line $x\ge 0$
19 $\mathbf{v}=-3\mathbf{i}+2\mathbf{j}, \mathbf{w}=2\mathbf{i}-\mathbf{j}; \mathbf{i}=4\mathbf{v}-\mathbf{w}$ **21** $d=-6; C=\mathbf{i}-2\mathbf{j}+\mathbf{k}$
23 $\cos\theta=\frac{1}{\sqrt{3}}; \cos\theta=\frac{2}{\sqrt{6}}; \cos\theta=\frac{1}{3}$ **25** $\mathbf{A}\cdot(\mathbf{A}+\mathbf{B})=1+\mathbf{A}\cdot\mathbf{B}=1+\mathbf{B}\cdot\mathbf{A}=\mathbf{B}\cdot(\mathbf{A}+\mathbf{B})$; equilateral, 60°
27 $a=\mathbf{A}\cdot\mathbf{I}, b=\mathbf{A}\cdot\mathbf{J}$ **29** $(\cos t,\sin t)$ and $(-\sin t,\cos t)$; $(\cos 2t,\sin 2t)$ and $(-2\sin 2t, 2\cos 2t)$
31 $\mathbf{C}=\mathbf{A}+\mathbf{B}, \mathbf{D}=\mathbf{A}-\mathbf{B}; \mathbf{C}\cdot\mathbf{D}=\mathbf{A}\cdot\mathbf{A}+\mathbf{B}\cdot\mathbf{A}-\mathbf{A}\cdot\mathbf{B}-\mathbf{B}\cdot\mathbf{B}=r^2-r^2=0$
33 $\mathbf{U}+\mathbf{V}-\mathbf{W}=(2,5,8), \mathbf{U}-\mathbf{V}+\mathbf{W}=(0,-1,-2), -\mathbf{U}+\mathbf{V}+\mathbf{W}=(4,3,6)$
35 c and $\sqrt{a^2+b^2}$; b/a and $\sqrt{a^2+b^2+c^2}$
37 $\mathbf{M}_1=\frac{1}{2}\mathbf{A}+\mathbf{C}, \mathbf{M}_2=\mathbf{A}+\frac{1}{2}\mathbf{B}, \mathbf{M}_3=\mathbf{B}+\frac{1}{2}\mathbf{C}; \mathbf{M}_1+\mathbf{M}_2+\mathbf{M}_3=\frac{3}{2}(\mathbf{A}+\mathbf{B}+\mathbf{C})=\mathbf{0}$
39 $8\le 3\cdot 3; 2\sqrt{xy}\le x+y$ **41** Cancel a^2c^2 and b^2d^2; then $b^2c^2+a^2d^2\ge 2abcd$ because $(bc-ad)^2\ge 0$
43 F; T; T; F **45** all $2\sqrt{2}$; $\cos\theta=-\frac{1}{3}$

Section 11.2 Planes and Projections (page 414)

1 (0,0,0) and $(2,-1,0)$; $\mathbf{N}=(1,2,3)$ **3** $(0,5,6)$ and (0,6,7); $\mathbf{N}=(1,0,0)$
5 (1,1,1) and (1,2,2); $\mathbf{N}=(1,1,-1)$ **7** $x+y=3$ **9** $x+2y+z=2$

11 Parallel if $\mathbf{N}\cdot\mathbf{V}=0$; perpendicular if $\mathbf{V}=$ multiple of $\mathbf{N}$

13 $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (vector between points) is not perpendicular to $\mathbf{N}$; $\mathbf{V}\cdot\mathbf{N}$ is not zero; plane through first three is $x+y+z=1$; $x+y-z=3$ succeeds; right side must be zero

15 $ax+by+cz=0; a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ **17** $\cos\theta=\frac{\sqrt{5}}{3},\frac{\sqrt{5}}{3},\frac{1}{3}$

19 $\frac{2}{36}\mathbf{A}$ has length $\frac{1}{3}$ **21** $\mathbf{P}=\frac{1}{2}\mathbf{A}$ has length $\frac{1}{2}|\mathbf{A}|$ **23** $\mathbf{P}=-\mathbf{A}$ has length $|\mathbf{A}|$ **25** $\mathbf{P}=\mathbf{O}$

27 Projection on $\mathbf{A}=(1,2,2)$ has length $\frac{5}{3}$; force down is 4; mass moves in the direction of $\mathbf{F}$

29 $|\mathbf{P}|_{\min}=\frac{5}{|\mathbf{N}|}=$ distance from plane to origin **31** Distances $\frac{1}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$ both reached at $(\frac{1}{3},\frac{1}{3},-\frac{1}{3})$

33 $\mathbf{i}+\mathbf{j}+\mathbf{k}; t=-\frac{4}{3}; (\frac{2}{3},-\frac{1}{3},-\frac{1}{3}); \frac{4}{\sqrt{3}}$

35 Same $\mathbf{N}=(2,-2,1)$; for example $\mathbf{Q}=(0,0,1)$; then $\mathbf{Q}+\frac{2}{9}\mathbf{N}=(\frac{4}{9},-\frac{4}{9},\frac{11}{9})$ is on second plane; $\frac{2}{9}|\mathbf{N}|=\frac{2}{3}$

37 $3\mathbf{i}+4\mathbf{j}$; $(3t,4t)$ is on the line if $3(3t)+4(4t)=10$ or $t=\frac{10}{25}$; $P=(\frac{30}{25},\frac{40}{25}), |P|=2$

39 $2x+2(\frac{10}{4}-\frac{3}{4}x)(-\frac{3}{4})=0$ so $x=\frac{30}{25}=\frac{6}{5}$; $3x+4y=10$ gives $y=\frac{8}{5}$

41 Use equations (8) and (9) with $\mathbf{N}=(a,b)$ and $\mathbf{Q}=(x_1,y_1)$ **43** $t=\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|^2}$; $\mathbf{B}$ onto $\mathbf{A}$

45 $aVL=\frac{1}{2}\mathbf{L}_I-\frac{1}{2}\mathbf{L}_{III}; aVF=\frac{1}{2}\mathbf{L}_{II}+\frac{1}{2}\mathbf{L}_{III}$

47 $\mathbf{V}\cdot\mathbf{L}_I=2-1; \mathbf{V}\cdot\mathbf{L}_{II}=-3-1, \mathbf{V}\cdot\mathbf{L}_{III}=-3-2$; thus $\mathbf{V}\cdot 2\mathbf{i}=1, \mathbf{V}\cdot(\mathbf{i}-\sqrt{3}\mathbf{j})=-4$, and $\mathbf{V}=\frac{1}{2}\mathbf{i}+\frac{3\sqrt{3}}{2}\mathbf{j}$

Section 11.3 Cross Products and Determinants (page 423)

1 $\mathbf{O}$ **3** $3\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ **5** $-2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$ **7** $27\,\mathbf{i}+12\,\mathbf{j}-17\,\mathbf{k}$

9 $\mathbf{A}$ perpendicular to $\mathbf{B}$; $\mathbf{A},\mathbf{B},\mathbf{C}$ mutually perpendicular **11** $|\mathbf{A}\times\mathbf{B}|=\sqrt{2}, \mathbf{A}\times\mathbf{B}=\mathbf{j}-\mathbf{k}$ **13** $\mathbf{A}\times\mathbf{B}=\mathbf{O}$

15 $|\mathbf{A}\times\mathbf{B}|^2=(a_1^2+a_2^2)(b_1^2+b_2^2)-(a_1b_1+a_2b_2)^2=(a_1b_2-a_2b_1)^2$; $\mathbf{A}\times\mathbf{B}=(a_1b_2-a_2b_1)\mathbf{k}$

17 F; T; F; T **19** $\mathbf{N}=(2,1,0)$ or $2\mathbf{i}+\mathbf{j}$ **21** $x-y+z=2$ so $\mathbf{N}=\mathbf{i}-\mathbf{j}+\mathbf{k}$

23 $[(1,2,1)-(2,1,1)]\times[(1,1,2)-(2,1,1)]=\mathbf{N}=\mathbf{i}+\mathbf{j}+\mathbf{k}; x+y+z=4$

25 $(1,1,1)\times(a,b,c)=\mathbf{N}=(c-b)\mathbf{i}+(a-c)\mathbf{j}+(b-a)\mathbf{k}$; points on a line if $a=b=c$ (many planes)

27 $\mathbf{N}=\mathbf{i}+\mathbf{j}$, plane $x+y=$ constant **29** $\mathbf{N}=\mathbf{k}$, plane $z=$ constant

31 $\begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}=x-y+z=0$ **33** $\mathbf{i}-3\mathbf{j}; -\mathbf{i}+3\mathbf{j}; -3\mathbf{i}-\mathbf{j}$ **35** $-1,4,-9$

39 $+c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}-c_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}+c_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

41 $\text{area}^2=(\frac{1}{2}ab)^2+(\frac{1}{2}ac)^2+(\frac{1}{2}bc)^2=(\frac{1}{2}|\mathbf{A}\times\mathbf{B}|)^2$ when $\mathbf{A}=a\mathbf{i}-b\mathbf{j}, \mathbf{B}=a\mathbf{i}-c\mathbf{k}$

43 $\mathbf{A}=\frac{1}{2}(2\cdot 1-(-1)1)=\frac{3}{2}$; fourth corner can be (3,3)

45 $a_1\mathbf{i}+a_2\mathbf{j}$ and $b_1\mathbf{i}+b_2\mathbf{j}$; $|a_1b_2-a_2b_1|$; $\mathbf{A}\times\mathbf{B}=\cdots+(a_1b_2-a_2b_1)\mathbf{k}$

47 $\mathbf{A}\times\mathbf{B}$; from Eq. (6), $(\mathbf{A}\times\mathbf{B})\times\mathbf{i}=-(a_3b_1-a_1b_3)\mathbf{k}+(a_1b_2-a_2b_1)\mathbf{j}$; $(\mathbf{A}\cdot\mathbf{i})\mathbf{B}-(\mathbf{B}\cdot\mathbf{i})\mathbf{A}=a_1(b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k})-b_1(a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k})$

49 $\mathbf{N}=(Q-P)\times(R-P)=\mathbf{i}+\mathbf{j}+\mathbf{k}$; area $\frac{1}{2}\sqrt{3}$; $x+y+z=2$

Section 11.4 Matrices and Linear Equations (page 433)

1 $x=5, y=2, D=-2, \begin{bmatrix} 7 \\ 3 \end{bmatrix}=5\begin{bmatrix} 1 \\ 1 \end{bmatrix}+2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ **3** $x=3, y=1, \begin{bmatrix} 8 \\ 0 \end{bmatrix}=3\begin{bmatrix} 3 \\ 1 \end{bmatrix}+\begin{bmatrix} -1 \\ -3 \end{bmatrix}, D=-8$

5 $x=2y, y=$ anything, $D=0, 2y\begin{bmatrix} 2 \\ 1 \end{bmatrix}+y\begin{bmatrix} -4 \\ -2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **7** no solution, $D=0$

9 $x=\frac{1}{D}\begin{vmatrix} 8 & -1 \\ 0 & -3 \end{vmatrix}=\frac{-24}{-8}=3, y=\frac{1}{D}\begin{bmatrix} 3 & 8 \\ 1 & 0 \end{bmatrix}=\frac{-8}{-8}=1$ **11** $\frac{0}{0}$

15 $ad - bc = -2$ so $A^{-1} = \begin{bmatrix} .5 & .5 \\ .5 & -.5 \end{bmatrix}$ **17** Are parallel; multiple; the same; infinite

19 Multiples of each other; in the same direction as the columns; infinite

21 $d_1 = .34, d_2 = 4.91$ **23** $.96x + .02y = .58, .04x + .98y = 4.92; D = .94, x = .5, y = 5$

25 $a = 1$ gives any $x = -y; a = -1$ gives any $x = y$

27 $D = -2, A^{-1} = -\frac{1}{2}\begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}; D = -8, (2A)^{-1} = \frac{1}{2}A^{-1}; D = \frac{1}{-2}, (A^{-1})^{-1} =$ original A;
$D = -2$ (not $+2$), $(-A)^{-1} = -A^{-1}; D = 1, I^{-1} = I$

29 $AB = \begin{bmatrix} 7 & 5 \\ 5 & 1 \end{bmatrix}, BA = \begin{bmatrix} 5 & 11 \\ 3 & 3 \end{bmatrix}, BC = \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}, CB = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$

31 $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}, \begin{matrix} aecf+aedh & +bgcf+bgdh \\ -afce-afdg & -bhce-bhdg \end{matrix} = (ad-bc)(eh-fg)$

33 $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & \frac{1}{2} \end{bmatrix}, B^{-1} = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & 1 \end{bmatrix}$ **35** Identity; $B^{-1}A^{-1}$ **37** Perpendicular; $\mathbf{u} = \mathbf{v} \times \mathbf{w}$

39 Line $4 + t$, errors $-1, 2, -1$ **41** $d_1 - 2d_2 + d_3 = 0$ **43** A^{-1} can't multiply $\mathbf{O}$ and produce $\mathbf{u}$

Section 11.5 Linear Algebra (page 443)

1 $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 5 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$ **3** $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

5 det $A = 0$, add 3 equations $\rightarrow 0 = 1$ **7** $5\mathbf{a} + 1\mathbf{b} + 0\mathbf{c} = \mathbf{d},\ A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

9 $\mathbf{b} \times \mathbf{c}; \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = 0$; determinant is zero **11** 6, 2, 0; product of diagonal entries

13 $A^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ 0 & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}, B^{-1} = \begin{bmatrix} 0 & 2 & -\frac{1}{2} \\ 0 & -3 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ **15** Zero; same plane; D is zero

17 $\mathbf{d} = (1, -1, 0); \mathbf{u} = (1, 0, 0)$ or $(7,3,1)$ **19** $AB = \begin{bmatrix} 8 & 4 & 1 \\ 40 & 26 & 0 \\ 18 & 12 & 0 \end{bmatrix}$, det $AB = 12 =$ (det A) times (det B)

21 $A + C = \begin{bmatrix} 2 & 3 & -3 \\ -1 & 4 & 6 \\ 0 & -1 & 6 \end{bmatrix}$, det$(A + C)$ is not det A + det C

23 $p = \frac{(2)(3)-(0)(6)}{6} = 1, q = \frac{-(4)(3)+(0)(0)}{6} = -2$ **25** $(A^{-1})^{-1}$ is always A

27 $-1, -1, 1, 1, ; (y, x, z), (z, y, x), (y, z, x), (z, x, y)$ **29** $2! = 2, 4! = 24$

31 $z = \frac{1}{2}, y = -\frac{3}{2}, x = 3; z = \frac{7}{2}, y = \frac{3}{2}, x = -\frac{1}{2}$

33 New second equation $3z = 0$ doesn't contain y; exchange with third equation; there is a solution

35 Pivots 1,2,4, $D = 8$; pivots $1, -1, 2, D = -2$ **37** $a_{12} = 1, a_{21} = 0, \sum a_{ij}b_{jk} =$ row i, column k in AB

39 $a_{11}a_{22} - a_{12}a_{21} \neq 0; D = 0$

CHAPTER 12 MOTION ALONG A CURVE

Section 12.1 The Position Vector (page 452)

1 $\mathbf{v}(1) = \mathbf{i} + 3\mathbf{j}$; speed $\sqrt{10}$; **3** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$; tangent to circle is perpendicular to $\frac{x}{y} = \frac{\cos t}{\sin t}$

5 $\mathbf{v} = e^t\,\mathbf{i} - e^{-t}\,\mathbf{j} = \mathbf{i} - \mathbf{j}; y - 1 = -(x - 1); xy = 1$

7 $\mathbf{R} = (1,2,4) + (4,3,0)t; \mathbf{R} = (1,2,4) + (8,6,0)t; \mathbf{R} = (5,5,4) + (8,6,0)t$
9 $\mathbf{R} = (2+t, 3, 4-t); \mathbf{R} = (2+\frac{t^2}{2}, 3, 4-\frac{t^2}{2})$; the same line
11 Line; $y = 2+2t, z = 2+3t; y = 2+4t, z = 2+6t$
13 Line; $\sqrt{36+9+4} = 7; (6,3,2)$; line segment **15** $\frac{\sqrt{2}}{2}; 1; \frac{\sqrt{2}}{2}$ **17** $x = t, y = mt + b$
19 $\mathbf{v} = \mathbf{i} - \frac{1}{t^2}\mathbf{j}, |\mathbf{v}| = \sqrt{1+t^{-4}}, \mathbf{T} = \mathbf{v}/|\mathbf{v}|; \mathbf{v} = (\cos t - t\sin t)\mathbf{i} + (\sin t + t\cos t)\mathbf{j}; |\mathbf{v}| = \sqrt{1+t^2}$;
$\mathbf{T} = \mathbf{v}/|\mathbf{v}|; \mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, |\mathbf{v}| = 3, \mathbf{T} = \frac{1}{3}\mathbf{v}$
21 $\mathbf{R} = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$ + any $\mathbf{R}_0$; same $\mathbf{R}$ plus any $\mathbf{w}t$
23 $\mathbf{v} = (1-\sin t)\mathbf{i} + (1-\cos t)\mathbf{j}; |\mathbf{v}| = \sqrt{3 - 2\sin t - 2\cos t}, |\mathbf{v}|_{\min} = \sqrt{3-2\sqrt{2}}, |\mathbf{v}|_{\max} = \sqrt{3+2\sqrt{2}}$;
$\mathbf{a} = -\cos t\,\mathbf{i} + \sin t\,\mathbf{j}, |\mathbf{a}| = 1$; center is on $x = t, y = t$
25 Leaves at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}); \mathbf{v} = (-\sqrt{2}, \sqrt{2}); \mathbf{R} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) + v(t - \frac{\pi}{8})$
27 $\mathbf{R} = \cos\frac{s}{\sqrt{2}}\mathbf{i} + \sin\frac{s}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$
29 $\mathbf{v} = \sec^2 t\,\mathbf{i} + \sec t\tan t\,\mathbf{j}; |\mathbf{v}| = \sec^2 t\sqrt{1+\sin^2 t}; \mathbf{a} = 2\sec^2 t\tan t\,\mathbf{i} + (\sec^3 t + \sec t\tan^2 t)\,\mathbf{j}$;
curve is $y^2 - x^2 = 1$; hyperbola has asymptote $y = x$
31 If $\mathbf{T} = \mathbf{v}$ then $|\mathbf{v}| = 1$; line $\mathbf{R} = t\mathbf{i}$ or helix in Problem 27
33 $(x(t), y(t)) = \begin{matrix} (2t,0) & 0 \le t \le \frac{1}{2} & (3-2t,1) & 1 \le t \le \frac{3}{2} \\ (1,2t-1) & \frac{1}{2} \le t \le 1 & (0,4-2t) & \frac{3}{2} \le t \le 2 \end{matrix}$
35 $x(t) = 4\cos\frac{t}{2}, y(t) = 4\sin\frac{t}{2}$ **37** F; F; T; T; F **39** $\frac{y}{x} = \tan\theta$ but $\frac{y}{x} \ne \tan t$
41 $\mathbf{v}$ and $\mathbf{w}$; $\mathbf{v}$ and $\mathbf{w}$ and $\mathbf{u}$; $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v}$ and $\mathbf{w}$ and $\mathbf{u}$; not zero
43 $\mathbf{u} = (8,3,2)$; projection perpendicular to $\mathbf{v} = (1,2,2)$ is $(6,-1,-2)$ which has length $\sqrt{41}$
45 $x = G(t), y = F(t); y = x^{2/3}; t = 1$ and $t = -1$ give the same x so they would give the same y; $y = G(F^{-1}(x))$

Section 12.2 Plane Motion: Projectiles and Cycloids (page 457)

1 (a) $T = 16/g$ sec, $R = 128\sqrt{3}/g$ ft, $Y = 32/g$ ft (b) $\frac{16\sqrt{2}}{g}; \frac{128\sqrt{3}}{g}, \frac{96}{g}$ (c) $\frac{32}{g}, 0, \frac{128}{g}$ **3** $x = 1.2$ or 33.5
5 $y = x - \frac{1}{2}x^2 = 0$ at $x = 2$; $y = x\tan x - \frac{g}{2}(\frac{x}{v_0\cos\alpha})^2 = 0$ at $x = R$ **7** $x = v_0\sqrt{\frac{2h}{g}}$
9 $v_0 \approx 11.2, \tan\alpha \approx 4.32$ **11** $v_0 = \sqrt{gR} = \sqrt{980}$ m/sec; larger **13** $v_0^2/2g = 40$ meters
15 Multiply R and H by 4; $dR = 2v_0^2\cos 2\alpha d\alpha/g, dH = v_0^2\sin\alpha\cos\ \alpha\ d\alpha/g$
17 $t = \frac{12\sqrt{2}}{10}$ sec; $y = 12 - \frac{144g}{100} \approx -2.1$ m; + 2.1m **19** $\mathbf{T} = \frac{(1-\cos\theta)\mathbf{i} + \sin\theta\mathbf{j}}{\sqrt{2-2\cos\theta}}$
21 Top of circle **25** $ca(1-\cos\theta), ca\sin\theta; \theta = \pi, \frac{\pi}{2}$ **27** After $\theta = \pi : x = \pi a + v_0 t$ and $y = 2a - \frac{1}{2}gt^2$ **29** 2; 3
31 $\frac{64\pi a^2}{3}; 5\pi^2 a^3$ **33** $x = \cos\theta + \theta\sin\theta, y = \sin\theta - \theta\cos\theta$ **35** $(a = 4)\ 6\pi$
37 $y = 2\sin\theta - \sin 2\theta = 2\sin\theta(1-\cos\theta); x^2 + y^2 = 4(1-\cos\theta)^2; r = 2(1-\cos\theta)$

Section 12.3 Curvature and Normal Vector (page 463)

1 $\frac{e^x}{(1+e^{2x})^{3/2}}$ **3** $\frac{1}{2}$ **5** 0 (line) **7** $\frac{2+t^2}{(1+t^2)^{3/2}}$ **9** $(-\sin t^2, \cos t^2); (-\cos t^2, -\sin t^2)$
11 $(\cos t, \sin t); (-\sin t, -\cos t)$ **13** $(-\frac{3}{5}\sin t, \frac{3}{5}\cos t, \frac{4}{5}); |\mathbf{v}| = 5, \kappa = \frac{3}{25}; \frac{5}{3}$ longer; $\tan\theta = \frac{4}{3}$
15 $\frac{1}{2\sqrt{2a}\sqrt{1-\cos\theta}}$ **17** $\kappa = \frac{3}{16}, \mathbf{N} = \mathbf{i}$ **19** $(0,0); (-3,0)$ with $\frac{1}{\kappa} = 4$; $(-1,2)$ with $\frac{1}{\kappa} = 2\sqrt{2}$
21 Radius $\frac{1}{\kappa}$, center $(1, \pm\sqrt{\frac{1}{\kappa^2} - 1})$ for $\kappa \le 1$ **23** $\mathbf{U}\cdot\mathbf{V}'$ **25** $\frac{1}{\sqrt{2}}(\sin t\,\mathbf{i} - \cos t\,\mathbf{j} + \mathbf{k})$ **27** $\frac{1}{2}$
29 $\mathbf{N}$ in the plane, $\mathbf{B} = \mathbf{k}, \tau = 0$ **31** $\frac{d^2y/dx^2}{1+(dy/dx)^2}$ **33** $\mathbf{a} = 0\,\mathbf{T} + 5\omega^2\mathbf{N}$ **35** $\mathbf{a} = \frac{t}{\sqrt{1+t^2}}\mathbf{T} + \frac{2+t^2}{\sqrt{1+t^2}}\mathbf{N}$
37 $\mathbf{a} = \frac{4t}{\sqrt{1+4t^2}}\mathbf{T} + \frac{2}{\sqrt{1+4t^2}}\mathbf{N}$ **39** $|F^2 + 2(F')^2 - FF''|/(F^2 + F'^2)^{3/2}$

Section 12.4 Polar Coordinates and Planetary Motion (page 468)

1 $\mathbf{j}, -\mathbf{i}; \mathbf{i}+\mathbf{j} = \mathbf{u}_r - \mathbf{u}_\theta$ **3** $(2,-1); (1,2)$ **5** $\mathbf{v} = 3e^3(\mathbf{u}_r + \mathbf{u}_\theta) = 3e^3(\cos 3 - \sin 3)\mathbf{i} + 3e^3(\sin 3 + \cos 3)\mathbf{j}$
7 $\mathbf{v} = -20\sin 5t\,\mathbf{i} + 20\cos 5t\,\mathbf{j} = 20\,\mathbf{T} = 20\,\mathbf{u}_\theta; \mathbf{a} = -100\cos 5t\,\mathbf{i} - 100\sin 5t\,\mathbf{j} = 100\,\mathbf{N} = -100\,\mathbf{u}_r$
9 $r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} = 0 = \frac{1}{r}\frac{d}{dt}(r^2\frac{d\theta}{dt})$ **11** $\frac{d\theta}{dt} = .0004$ radians/sec; $h = r^2\frac{d\theta}{dt} = 40{,}000$
13 $m\mathbf{R}\times\mathbf{a}$; torque **15** $T^{2/3}(GM/4\pi^2)^{1/3}$ **17** $4\pi^2a^3/T^2G$ **19** $\frac{4\pi^2(150)^3 10^{27}}{(365\frac{1}{4})^2(24)^2(3600)^2(6.67)10^{-11}}$ kg
23 Use Problem **15** **25** $a + c = \frac{1}{C-D}, a - c = \frac{1}{C+D}$, solve for C, D
27 Kepler measures area from focus (sun) **29** Line; $x = 1$
33 $r = 20 - 2t, \theta = \frac{2\pi t}{10}, \mathbf{v} = -2\mathbf{u}_r + (20-2t)\frac{2\pi}{10}\mathbf{u}_\theta; \mathbf{a} = (2t-20)(\frac{2\pi}{10})^2\mathbf{u}_r - 4(\frac{2\pi}{10})\mathbf{u}_\theta; \int_0^{10}|\mathbf{v}|dt$

CHAPTER 13 PARTIAL DERIVATIVES

Section 13.1 Surfaces and Level Curves (page 475)

3 x derivatives $\infty, -1, -2, -4e^{-4}$ (flattest) **5** Straight lines **7** Logarithm curves
9 Parabolas **11** No: $f = (x+y)^n$ or $(ax+by)^n$ or any function of $ax+by$ **13** $f(x,y) = 1 - x^2 - y^2$
15 Saddle **17** Ellipses $4x^2 + y^2 = c^2$ **19** Ellipses $5x^2 + y^2 = c^2 + 4cx + x^2$
21 Straight lines not reaching (1,2) **23** Center $(1,1); f = x^2 + y^2 - 1$ **25** Four, three, planes, spheres
27 Less than 1, equal to 1, greater than 1 **29** Parallel lines, hyperbolas, parabolas
31 $\frac{d}{dx} : 48x - 3x^2 = 0, x = 16$ hours **33** Plane; planes; 4 left and 3 right (3 pairs)

Section 13.2 Partial Derivatives (page 479)

1 $3 + 2xy^2; -1 + 2yx^2$ **3** $3x^2y^2 - 2x; 2x^3y - e^y$ **5** $\frac{-2y}{(x-y)^2}; \frac{2x}{(x-y)^2}$ **7** $\frac{-2x}{(x^2+y^2)^2}; \frac{-2y}{(x^2+y^2)^2}$
9 $\frac{x}{x^2+y^2}; \frac{y}{x^2+y^2}$ **11** $\frac{-y}{x^2+y^2}; \frac{x}{x^2+y^2}$ **13** 2, 3, 4 **15** $6(x+iy), 6i(x+iy), -6(x+iy)$
17 $(f = \frac{1}{r})f_{xx} = \frac{2x^2-y^2}{r^5}; f_{xy} = \frac{3xy}{r^5}; f_{yy} = \frac{2y^2-x^2}{r^5}$ **19** $-a^2\cos ax\cos by, ab\sin ax\sin by, -b^2\cos ax\cos by$
21 Omit line $x = y$; all positive numbers; $f_x = -2(x-y)^{-3}, f_y = 2(x-y)^{-3}$
23 Omit $z = t$; all numbers; $\frac{-1}{z-t}, \frac{1}{z-t}, \frac{(x-y)}{(z-t)^2}, \frac{(y-x)}{(z-t)^2}$
25 $x > 0, t > 0$ and $x = 0, t > 1$ and $x = -1, -2, \cdots, t = e, e^2, \cdots; f_x = (\ln t)x^{\ln t - 1}, f_t = (\ln x)t^{\ln x - 1}$
27 $y, x; f = G(x) + H(y)$ **29** $\frac{\partial f}{\partial x} = \frac{\partial(xy)}{\partial x}v(xy) = yv(xy)$
31 $f_{xxx} = 6y^3, f_{yyy} = 6x^3, f_{xxy} = f_{xyx} = f_{yxx} = 18xy^2, f_{yyx} = f_{yxy} = f_{xyy} = 18x^2y$
33 $g(y) = Ae^{cy/7}$ **35** $g(y) = Ae^{cy/2} + Be^{-cy/2}$
37 $f_t = -2f, f_{xx} = f_{yy} = -e^{-2t}\sin x\sin y; e^{-13t}\sin 2x\sin 3y$
39 $\sin(x+t)$ moves left **41** $\sin(x-ct), \cos(x+ct), e^{x-ct}$
43 $(B-A)h_y(C^*) = (B-A)[f_y(b,C^*) - f_y(a,C^*)] = (B-A)(b-a)f_{yx}(c^*,C^*)$; continuous f_{xy} and f_{yx}
45 y converges to b; inside and stay inside; $d_n = \sqrt{(x_n-a)^2 + (y_n-b)^2} \to$ zero; $d_n < \epsilon$ for $n > N$
47 ϵ, less than δ **49** $f(a,b); \frac{1}{x-1}$ or $\frac{1}{(x-1)(y-2)}$ **51** $f(0,0) = 1; f(0,0) = 1$; not defined for $x < 0$

Section 13.3 Tangent Planes and Linear Approximations (page 488)

1 $z-1=y-1; \mathbf{N}=\mathbf{j}-\mathbf{k}$ **3** $z-2=\frac{1}{3}(x-6)-\frac{2}{3}(y-3); \mathbf{N}=\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\mathbf{k}$
5 $2(x-1)+4(y-2)+2(z-1)=0; \mathbf{N}=2\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ **7** $z-1=x-1; \mathbf{N}=\mathbf{i}-\mathbf{k}$
9 Tangent plane $2z_0(z-z_0)-2x_0(x-x_0)-2y_0(y-y_0)=0$; $(0,0,0)$ satisfies this equation because $z_0^2-x_0^2-y_0^2=0$ on the surface; $\cos\theta=\frac{\mathbf{N}\cdot\mathbf{k}}{|\mathbf{N}||\mathbf{k}|}=\frac{-z_0}{\sqrt{x_0^2+y_0^2+z_0^2}}=\frac{-1}{\sqrt{2}}$ (surface is the 45° cone)
11 $dz=3dx-2dy$ for both; $dz=0$ for both; $\Delta z=0$ for $3x-2y$, $\Delta z=.00029$ for x^3/y^2; tangent plane
13 $z=z_0+F_z t$; plane $6(x-4)+12(y-2)+8(z-3)=0$; normal line $x=4+6t, y=2+12t, z=3+8t$
15 Tangent plane $4(x-2)+2(y-1)+4(z-2)=0$; normal line $x=2+4t, y=1+2t, z=2+4t$; $(0,0,0)$ at $t=-\frac{1}{2}$
17 $dw=y_0dx+x_0dy$; product rule; $\Delta w-dw=(x-x_0)(y-y_0)$
19 $dI=4000dR+.08dP; dP=\$100; I=(.78)(4100)=\$319.80$
21 Increase $=\frac{26}{101}-\frac{25}{100}=\frac{3}{404}$, decrease $=\frac{25}{100}-\frac{25}{101}=\frac{1}{404}; dA=\frac{1}{y}dx-\frac{x}{y^2}dy; 3$ **23** $\Delta\theta\approx\frac{-y\Delta x+x\Delta y}{\sqrt{x^2+y^2}}$
25 Q increases; $Q_s=-\frac{250}{3}, Q_t=\frac{-5}{3}, P_s=-.2Q_s=\frac{50}{3}, P_t=-.2Q_t=\frac{1}{3}; Q=50-\frac{250}{3}(s-.4)-\frac{5}{3}(t-10)$
27 $s=1, t=10$ gives $Q=40$: $\begin{array}{rcl} P_s &=& -Q_s=sQ_s+Q=Q_s+40 \\ P_t &=& -Q_t=sQ_t+1=Q_t+1\end{array}$; $Q_s=-20, Q_t=-\frac{1}{2}, P_s=20, P_t=\frac{1}{2}$
29 $z-2=x-2+2(y-1)$ and $z-3=4(x-2)-2(y-1); x=1, y=\frac{1}{2}, z=0$
31 $\Delta x=-\frac{1}{2}, \Delta y=\frac{1}{2}; x_1=\frac{1}{2}, y_1=-\frac{1}{2}$; line $x+y=0$
33 $\begin{array}{l} 3a^2\Delta x-\Delta y=-a-a^3 \\ -\Delta x+3a^2\Delta y=a+a^3\end{array}$ gives $\Delta y=-\Delta x=\frac{a+a^3}{1+3a^2}$; lemon starts at $(1/\sqrt{3},-1/\sqrt{3})$
35 If $x^3=y$ then $y^3=x^9$. Then $x^9=x$ only if $x=0$ or 1 or -1 (or complex number)
37 $\Delta x=-x_0+1, \Delta y=-y_0+2, (x_1,y_1)=(1,2)=$ solution
39 $G=H=\frac{x_n^2}{2x_n-1}$ **41** $J=\begin{bmatrix} e^x & 0 \\ 1 & e^y\end{bmatrix}, \Delta x=-1+e^{-x_n}, \Delta y=-1-(x_n-1+e^{-x_n})e^{-y_n}$
43 $(x_1,y_1)=(0,\frac{5}{4}),(-\frac{3}{4},\frac{5}{4}),(\frac{3}{4},0)$

Section 13.4 Directional Derivatives and Gradients (page 495)

1 grad $f=2x\mathbf{i}-2y\mathbf{j}, D_\mathbf{u}f=\sqrt{3}x-y, D_\mathbf{u}f(P)=\sqrt{3}$
3 grad $f=e^x\cos y\,\mathbf{i}-e^x\sin y\,\mathbf{j}, D_\mathbf{u}f=-e^x\sin y, D_\mathbf{u}f(P)=-1$
5 $f=\sqrt{x^2+(y-3)^2}$, grad $f=\frac{x}{f}\mathbf{i}+\frac{y-3}{f}\mathbf{j}, D_\mathbf{u}f=\frac{x}{f}, D_\mathbf{u}f(P)=\frac{1}{\sqrt{5}}$ **7** grad $f=\frac{2x}{x^2+y^2}\mathbf{i}+\frac{2y}{x^2+y^2}\mathbf{j}$
9 grad $f=6x\mathbf{i}+4y\mathbf{j}=6\mathbf{i}+8\mathbf{j}=$ steepest direction at P; level direction $-8\mathbf{i}+6\mathbf{j}$ is perpendicular; 10, 0
11 T; F (grad f is a vector); F;T **13** $\mathbf{u}=(\frac{a}{\sqrt{a^2+b^2}},\frac{b}{\sqrt{a^2+b^2}}), D_\mathbf{u}f=\sqrt{a^2+b^2}$
15 grad $f=(e^{x-y},-e^{x-y})=(e^{-1},-e^{-1})$ at P; $\mathbf{u}=(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}), D_\mathbf{u}f=\sqrt{2}e^{-1}$
17 grad $f=\mathbf{O}$ at maximum; level curve is one point **19** $\mathbf{N}=(-1,1,-1), \mathbf{U}=(-1,1,2), \mathbf{L}=(1,1,0)$
21 Direction $-\mathbf{U}=(-2,0,-4)$ **23** $-\mathbf{U}=(\frac{x}{\sqrt{1-x^2-y^2}},\frac{y}{\sqrt{1-x^2-y^2}},\frac{-x^2-y^2}{1-x^2-y^2})$
25 $f=(x+2y)$ and $(x+2y)^2; \mathbf{i}+2\mathbf{j}$; straight lines $x+2y=$ constant (perpendicular to $\mathbf{i}+2\mathbf{j}$)
27 grad $f=\pm(\frac{1}{\sqrt{5}},\frac{-2}{\sqrt{5}})$; grad $g=\pm(2\sqrt{5},\sqrt{5}), f=\pm(\frac{x}{\sqrt{5}}-\frac{2y}{\sqrt{5}})+C, g=\pm(2\sqrt{5}x+\sqrt{5}y)+C$
29 $\theta=$ constant along ray in direction $\mathbf{u}=\frac{3\mathbf{i}+4\mathbf{j}}{5}$; grad $\theta=\frac{-y\mathbf{i}+x\mathbf{j}}{x^2+y^2}=\frac{-4\mathbf{i}+3\mathbf{j}}{25}$; $\mathbf{u}\cdot$ grad $\theta=0$
31 $\mathbf{U}=(f_x,f_y,f_x^2+f_y^2)=(-1,-2,5); -\mathbf{U}=(-1,-2,5)$; tangent at the point (2,1,6)
33 grad f toward $2\mathbf{i}+\mathbf{j}$ at P, $\mathbf{j}$ at Q, $-2\mathbf{i}+\mathbf{j}$ at R; $(2,\frac{1}{2})$ and $(2\frac{1}{2},2)$; largest upper left, smallest lower right; $z_{\max}>9$; z goes from 2 to 8 and back to 6

35 $f = \frac{1}{2}\sqrt{(x-1)^2+(y-1)^2}$; $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})_{0,0} = (\frac{-1}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}})$
37 Figure C now shows level curves; $|\text{grad } f|$ is varying; f could be xy
39 x^2+xy; e^{x-y}; no function has $\frac{\partial f}{\partial x} = y$ and $\frac{\partial f}{\partial y} = -x$ because then $f_{xy} \neq f_{yx}$
41 $\mathbf{v} = (1, 2t)$; $\mathbf{T} = \mathbf{v}/\sqrt{1+4t^2}$; $\frac{df}{dt} = \mathbf{v} \cdot (2t, 2t^2) = 2t + 4t^3$; $\frac{df}{ds} = (2t+4t^3)/\sqrt{1+4t^2}$
43 $\mathbf{v} = (2,3)$; $\mathbf{T} = \frac{\mathbf{v}}{\sqrt{13}}$; $\frac{df}{dt} = \mathbf{v} \cdot (2x_0 + 4t, -2y_0 - 6t) = 4x_0 - 6y_0 - 10t$; $\frac{df}{ds} = \frac{df/dt}{\sqrt{13}}$
45 $\mathbf{v} = (e^t, 2e^{2t}, -e^{-t})$; $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$; grad $f = (\frac{1}{x}, \frac{1}{y}, \frac{1}{z}) = (e^{-t}, e^{-2t}, e^t)$, $\frac{df}{dt} = 1 + 2 - 1$, $\frac{df}{ds} = \frac{2}{|\mathbf{v}|}$
47 $\mathbf{v} = (-2\sin 2t, 2\cos 2t)$, $\mathbf{T} = (-\sin 2t, \cos 2t)$; grad $f = (y, x)$, $\frac{df}{ds} = -2\sin^2 2t + 2\cos^2 2t$, $\frac{df}{dt} = \frac{1}{2}\frac{df}{ds}$; zero slope because $f = 1$ on this path
49 $z - 1 = 2(x-4) + 3(y-5)$; $f = 1 + 2(x-4) + 3(y-5)$ **51** grad $f \cdot \mathbf{T} = 0$; $\mathbf{T}$

Section 13.5 The Chain Rule (page 503)

1 $f_y = cf_x = c\cos(x+cy)$ **3** $f_y = 7f_x = 7e^{x+7y}$ **5** $3g^2 \frac{\partial g}{\partial x}\frac{dx}{dt} + 3g^2 \frac{\partial g}{\partial y}\frac{dy}{dt}$ **7** Moves left at speed 2
9 $\frac{dx}{dt} = 1$ (wave moves at speed 1)
11 $\frac{\partial^2}{\partial x^2} f(x+iy) = f''(x+iy)$, $\frac{\partial^2}{\partial y^2} f(x+iy) = i^2 f''(x+iy)$ so $f_{xx} + f_{yy} = 0$; $(x+iy)^2 = (x^2-y^2) + i(2xy)$
13 $\frac{df}{dt} = 2x(1) + 2y(2t) = 2t + 4t^3$ **15** $\frac{df}{dt} = y\frac{dx}{dt} + x\frac{dy}{dt} = -1$ **17** $\frac{df}{dt} = \frac{1}{x+y}\frac{dx}{dt} + \frac{1}{x+y}\frac{dy}{dt} = 1$
19 $V = \frac{1}{3}\pi r^2 h$, $\frac{dV}{dt} = \frac{2\pi rh}{3}\frac{dr}{dt} + \frac{\pi r^2}{3}\frac{dh}{dt} = 36\pi$
21 $\frac{dD}{dt} = \frac{90}{\sqrt{90^2+90^2}}(60) + \frac{90}{\sqrt{90^2+90^2}}(45) = \frac{105}{\sqrt{2}}$ mph; $\frac{dD}{dt} = \frac{60}{\sqrt{45^2+60^2}}(60) + \frac{45}{\sqrt{45^2+60^2}}(45) \approx 74$ mph
23 $\frac{df}{dt} = u_1\frac{\partial f}{\partial x} + u_2\frac{\partial f}{\partial y} + u_3\frac{\partial f}{\partial z}$ **25** $\frac{\partial f}{\partial t} = 1$ with x and y fixed; $\frac{df}{dt} = 6$
27 $f_t = f_x t + f_y(2t)$; $f_{tt} = f_{xt}t + f_x + 2f_{yt}t + 2f_y = (f_{xx}t + f_{yx}(2t))t + f_x + 2(f_{xy}t + f_{yy}(2t))t + 2f_y$
29 $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$, θ is fixed
31 $r_{xx} = \frac{1}{\sqrt{x^2+y^2}} - \frac{x^2}{(x^2+y^2)^{3/2}} = \frac{y^2}{(x^2+y^2)^{3/2}}$; $\frac{\partial}{\partial x}(\frac{x}{r}) = \frac{1}{r} - xr^{-2}\frac{\partial r}{\partial x} = \frac{1}{r} - \frac{x^2}{r^3} = \frac{y^2}{r^3}$
33 $(\frac{\partial z}{\partial x})_y = \frac{1}{\sqrt{1-(x+y)^2}}$; $(\cos z)(\frac{\partial z}{\partial x})_y = 1$; first answer is also $\frac{1}{\sqrt{1-\sin^2 z}} = \frac{1}{\cos z}$
35 $f_r = f_x\cos\theta + f_y\sin\theta$, $f_{r\theta} = -f_x\sin\theta + f_y\cos\theta + f_{xx}(-r\sin\theta\cos\theta) + f_{xy}(-r\sin^2\theta + r\cos^2\theta) + f_{yy}(r\cos\theta\sin\theta)$
37 Yes (with y constant): $\frac{\partial z}{\partial x} = ye^{xy}$, $\frac{\partial x}{\partial z} = \frac{1}{zy} = \frac{1}{ye^{xy}}$ **39** $f_t = f_x x_t + f_y y_t$; $f_{tt} = f_{xx}x_t^2 + 2f_{xy}x_t y_t + f_{yy}y_t^2$
41 $(\frac{\partial f}{\partial x})_z = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x} = a - \frac{3}{5}b$; $(\frac{\partial f}{\partial x})_y = a$; $(\frac{\partial f}{\partial z})_x = \frac{\partial f}{\partial y}\frac{\partial y}{\partial z} = \frac{1}{5}b$
43 1 **45** $f = y^2$ so $f_x = 0$, $f_y = 2y = 2r\sin\theta$; $f = r^2$ so $f_r = 2r = 2\sqrt{x^2+y^2}$, $f_\theta = 0$
47 $g_u = f_x x_u + f_y y_u = f_x + f_y$; $g_v = f_x x_v + f_y y_v = f_x - f_y$; $g_{uu} = f_{xx}x_u + f_{xy}y_u + f_{yx}x_u + f_{yy}y_u$ $= f_{xx} + 2f_{xy} + f_{yy}$; $g_{vv} = f_{xx}x_v + f_{xy}y_v - f_{yx}x_v - f_{yy}y_v = f_{xx} - 2f_{xy} + f_{yy}$. Add $g_{uu} + g_{vv}$ **49** False

Section 13.6 Maxima, Minima, and Saddle Points (page 512)

1 (0,0) is a minimum **3** (3,0) is a saddle point **5** No stationary points **7** (0,0) is a maximum
9 (0,0,2) is a minimum **11** All points on the line $x = y$ are minima **13** (0,0) is a saddle point
15 (0,0) is a saddle point; (2,0) is a minimum; $(0,-2)$ is a maximum; $(2,-2)$ is a saddle point
17 Maximum of area $(12-3y)y$ is 12
19 $\begin{matrix} 2(x+y) + 2(x+2y-5) + 2(x+3y-4) = 0 \\ 2(x+y) + 4(x+2y-5) + 6(x+3y-4) = 0 \end{matrix}$ gives $\begin{matrix} x = 2; \\ y = -1 \end{matrix}$ min because $E_{xx}E_{yy} = (6)(28) > E_{xy}^2 = 12^2$
21 Minimum at $(0, \frac{1}{2})$; $(0,1)$; $(0,1)$

23 $\frac{df}{dt}=0$ when $\tan t=\sqrt{3}$; $f_{\max}=2$ at $(\frac{1}{2},\frac{\sqrt{3}}{2})$, $f_{\min}=-2$ at $(-\frac{1}{2},-\frac{\sqrt{3}}{2})$

25 $(ax+by)_{\max}=\sqrt{a^2+b^2}$; $(x^2+y^2)_{\min}=\frac{1}{a^2+b^2}$ **27** $0<c<\frac{1}{4}$

29 The vectors head-to-tail form a 60-60-60 triangle. The outer angle is 120° **31** $2+\sqrt{3}; 1+\sqrt{3}; 1+\frac{\sqrt{3}}{10}$

35 Steiner point where the arcs meet **39** Best point for $p=\infty$ is equidistant from corners

41 grad $f=(\sqrt{2}\,\frac{x-x_1}{d_1}+\frac{x-x_2}{d_2}+\frac{x-x_3}{d_3},\sqrt{2}\,\frac{y-y_1}{d_1}+\frac{y-y_2}{d_2}+\frac{y-y_3}{d_3})$; angles are 90-135-135

43 Third derivatives all 6; $f=\frac{6}{3!}x^3+\frac{6}{2!}x^2y+\frac{6}{2!}xy^2+\frac{6}{3!}y^3$

45 $(\frac{\partial}{\partial x})^n(\frac{\partial}{\partial y})^m\ln(1-xy)]_{0,0}=n!(n-1)!$ for $m=n>0$, other derivatives zero; $f=-xy-\frac{x^2y^2}{2}-\frac{x^3y^3}{3}-\cdots$

47 All derivatives are e^2 at (1,1); $f\approx e^2[1+(x-1)+(y-1)+\frac{1}{2}(x-1)^2+(x-1)(y-1)+\frac{1}{2}(y-1)^2]$

49 $x=1,y=-1: f_x=2, f_y=-2, f_{xx}=2, f_{xy}=0, f_{yy}=2$; series must recover x^2+y^2

51 Line $x-2y=$ constant; $x+y=$ constant

53 $\frac{x^2}{2}f_{xx}+xyf_{xy}+\frac{y^2}{2}f_{yy}]_{0,0}$; $f_{xx}>0$ and $f_{xx}f_{yy}>f_{xy}^2$ at (0,0); $f_x=f_y=0$ **55** $\Delta x=-1,\Delta y=-1$

57 $f=x^2(12-4x)$ has $f_{\max}=16$ at (2,4); line has slope -4, $y=\frac{16}{x^2}$ has slope $\frac{-32}{8}=-4$

59 If the fence were not perpendicular, a point to the left or right would be closer

Section 13.7 Constraints and Lagrange Multipliers (page 519)

1 $f=x^2+(k-2x)^2$; $\frac{df}{dx}=2x-4(k-2x)=0$; $(\frac{2k}{5},\frac{k}{5}),\frac{k^2}{5}$ **3** $\lambda=-4, x_{\min}=2, y_{\min}=2$

5 $\lambda=\frac{1}{3(4)^{1/3}}:(x,y)=(\pm 2^{1/6},0)$ or $(0,\pm 2^{1/6})$, $f_{\min}=2^{1/3}$; $\lambda=\frac{1}{3}:(x,y)=(\pm 1,\pm 1)$, $f_{\max}=2$

7 $\lambda=\frac{1}{2},(x,y)=(2,-3)$; tangent line is $2x-3y=13$

9 $(1-c)^2+(-a-c)^2+(2-a-b-c)^2+(2-b-c)^2$ is minimized at $a=-\frac{1}{2}, b=\frac{3}{2}, c=\frac{3}{4}$

11 $(1,-1)$ and $(-1,1)$; $\lambda=-\frac{1}{2}$

13 f is not a minimum when C crosses to lower level curve; stationary point when C is tangent to level curve

15 Substituting $\frac{\partial L}{\partial x}=\frac{\partial L}{\partial y}=\frac{\partial L}{\partial \lambda}=0$ and $L=f_{\min}$ leaves $\frac{df_{\min}}{dk}=\lambda$

17 x^2 is never negative; (0,0); $1=\lambda(-3y^2)$ but $y=0$; $g=0$ has a cusp at $(0,0)$

19 $2x=\lambda_1+\lambda_2, 4y=\lambda_1, 2z=\lambda_1-\lambda_2, x+y+z=0, x-z=1$ gives $\lambda_1=0,\lambda_2=1, f_{\min}=\frac{1}{2}$ at $(\frac{1}{2},0,-\frac{1}{2})$

21 $(1,0,0);(0,1,0);(\lambda_1,\lambda_2,0); x=y=0$ **23** $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$; $\lambda=0$

25 $(1,0,0),(0,1,0),(0,0,1)$; at these points $f=4$ and -2 (min) and 5(max)

27 By increasing k, more points are available so $f_{\max}$ goes up. Then $\lambda=\frac{df_{\min}}{dk}\geq 0$

29 $(0,0);\lambda=0; f_{\min}$ stays at 0

31 $5=\lambda_1+\lambda_2, 6=\lambda_1+\lambda_3, \lambda_2\geq 0, \lambda_3\leq 0$; subtraction $5-6=\lambda_2-\lambda_3$ or $-1\geq 0$ (impossible);
$x=2004, y=-2000$ gives $5x+6y=-1980$

33 $2x=4\lambda_1+\lambda_2, 2y=4\lambda_1+\lambda_3, \lambda_2\geq 0, \lambda_3\geq 0, 4x+4y=40$; max area 100 at (10,0)(0,10); min 25 at (5,5)

CHAPTER 14 MULTIPLE INTEGRALS

Section 14.1 Double Integrals (page 526)

1 $\frac{8}{3};\frac{2}{3}$ **3** $1;\ln\frac{3}{2}$ **5** 2 **7** $\frac{1}{2}$ **9** $\frac{4}{3}$ **11** $\int_{y=1}^{2}\int_{x=1}^{2}dx\,dy+\int_{y=2}^{4}\int_{y/2}^{2}dx\,dy$

13 $\int_{y=0}^{1}\int_{x=-\frac{1}{2}\ln y}^{-\ln y}dx\,dy$ **15** $\int_{x=0}^{1}\int_{y=-\sqrt{x}}^{\sqrt{x}}dy\,dx$ **17** $\int_0^1\int_0^{y/2}dx\,dy=\int_0^{1/2}\int_{2x}^{1}dy\,dx=\frac{1}{4}$

19 $\int_0^3\int_{-y}^{y}dx\,dy=\int_{-1}^{0}\int_{-x}^{3}dy\,dx+\int_0^1\int_x^3 dy\,dx=9$ **21** $\int_0^4\int_{y/2}^{y}dx\,dy+\int_4^8\int_{y/2}^{4}dx\,dy=\int_0^4\int_x^{2x}dy\,dx=8$

23 $\int_0^1\int_0^{bx}dy\,dx+\int_1^2\int_0^{b(2-x)}dy\,dx=\int_0^b\int_{y/b}^{2-(y/b)}dx\,dy=b$ **25** $f(a,b)-f(a,0)-f(0,b)+f(0,0)$

27 $\int_0^1 \int_0^1 (2x - 3y + 1)dx\,dy = \frac{1}{2}$ **29** $\int_a^b f(x)dx = \int_a^b \int_0^{f(x)} 1dy\,dx$ **31** $50,000\pi$

33 $\int_1^3 \int_1^2 x^2\,dx\,dy = \frac{14}{3}$ **35** $2\int_0^{1/\sqrt{2}} \int_0^{\sqrt{1-y^2}} 1dx\,dy = \frac{\pi}{4}$

37 $\frac{1}{n^2}\Sigma_{j=1}^n \Sigma_{i=1}^n f(\frac{i-\frac{1}{2}}{n}, \frac{j-\frac{1}{2}}{n})$ is exact for $f = 1, x, y, xy$ **39** Volume 8.5 **41** Volumes $\ln 2, 2\ln(1+\sqrt{2})$

43 $\int_0^1 \int_0^1 x^y dx\,dy = \int_0^1 \frac{1}{y+1}dy = \ln 2; \int_0^1 \int_0^1 x^y dy\,dx = \int_0^1 \frac{x-1}{\ln x}dx = \ln 2$

45 With long rectangles $\sum y_i \Delta A = \sum \Delta A = 1$ but $\int\int y\,dA = \frac{1}{2}$

Section 14.2 Change to Better Coordinates (page 534)

1 $\int_{\pi/4}^{3\pi/4} \int_0^1 r\,dr\,d\theta = \frac{\pi}{4}$ **3** S = quarter-circle with $u \geq 0$ and $v \geq 0$; $\int_0^1 \int_0^{\sqrt{1-v^2}} du\,dv$

5 R is symmetric across the y axis; $\int_0^1 \int_0^{\sqrt{1-v^2}} u\,du\,dv = \frac{1}{3}$ divided by area gives $(\bar{u}, \bar{v}) = (4/3\pi, 4/3\pi)$

7 $2\int_0^{1/\sqrt{2}} \int_{1+x}^{1+\sqrt{1-x^2}} dy\,dx$; xy region R^* becomes R in the x^*y^* plane; $dx\,dy = dx^*dy^*$ when region moves

9 $J = \begin{vmatrix} \partial x/\partial r^* & \partial x/\partial \theta^* \\ \partial y/\partial r^* & \partial y/\partial \theta^* \end{vmatrix} = \begin{vmatrix} \cos\theta^* & -r^*\sin\theta^* \\ \sin\theta^* & r^*\cos\theta^* \end{vmatrix} = r^*; \int_{\pi/4}^{3\pi/4} \int_0^1 r^*dr^*d\theta^*$

11 $I_y = \int\int_R x^2dx\,dy = \int_{\pi/4}^{3\pi/4} \int_0^1 r^2\cos^2\theta\,r\,dr\,d\theta = \frac{\pi}{16} - \frac{1}{8}; I_x = \frac{\pi}{16} + \frac{1}{8}; I_0 = \frac{\pi}{8}$

13 (0,0), (1,2), (1,3), (0,1); area of parallelogram is 1

15 $x = u, y = u + 3v + uv$; then $(u,v) = (1,0), (1,1), (0,1)$ give corners $(x,y) = (1,0), (1,5), (0,3)$

17 Corners (0,0), (2,1), (3,3), (1,2); sides $y = \frac{1}{2}x, y = 2x - 3, y = \frac{1}{2}x + \frac{3}{2}, y = 2x$

19 Corners (1,1), $(e^2, e), (e^3, e^3), (e, e^2)$; sides $x = y^2, y = x^2/e^3, x = y^2/e^3, y = x^2$

21 Corners (0,0), (1,0), (1,2), (0,1); sides $y = 0, x = 1, y = 1 + x^2, x = 0$

23 $J = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$, area $\int_0^1 \int_0^1 3du\,dv = 3$; $J = \begin{vmatrix} 2e^{2u+v} & e^{2u+v} \\ e^{u+2v} & 2e^{u+2v} \end{vmatrix} = 3e^{3u+3v}, \int_0^1 \int_0^1 3e^{3u+3v}du\,dv =$
$\int_0^1 (e^{3+3v} - e^{3v})dv = \frac{1}{3}(e^6 - 2e^3 + 1)$

25 Corners $(x,y) = (0,0), (1,0), (1, f(1)), (0, f(0))$; $(\frac{1}{2}, 1)$ gives $x = \frac{1}{2}, y = f(\frac{1}{2})$; $J = \begin{vmatrix} 1 & 0 \\ vf'(u) & f(u) \end{vmatrix} = f(u)$

27 $B^2 = 2\int_0^{\pi/4} \int_0^{1/\sin\theta} e^{-r^2} r\,dr\,d\theta = \int_0^{\pi/4} (e^{-1/\sin^2\theta} - 1)d\theta$

29 $\bar{r} = \int\int r^2dr\,d\theta / \int\int r\,dr\,d\theta = \int_0^\pi \frac{8}{3}a^3\cos^3\theta\,d\theta/\pi a^2 = \frac{32a}{9\pi}$ **31** $\int_0^{2\pi} \int_0^1 r^2 r\,dr\,d\theta = \frac{\pi}{2}$

33 Along the right side; along the bottom; at the bottom right corner

35 $\int\int xy\,dx\,dy = \int_0^1 \int_0^1 (u\cos\alpha - v\sin\alpha)(u\sin\alpha + v\cos\alpha)du\,dv = \frac{1}{4}(\cos^2\alpha - \sin^2\alpha)$

37 $\int_0^{2\pi} \int_4^5 r^2 r^2 r\,dr\,d\theta = \frac{2\pi}{6}(5^6 - 4^6)$ **39** $x = \cos\alpha - \sin\alpha, y = \sin\alpha + \cos\alpha$ goes to $u = 1, v = 1$

Section 14.3 Triple Integrals (page 540)

1 $\int_0^1 \int_0^z \int_0^y dx\,dy\,dz = \frac{1}{6}$

3 $0 \leq y \leq x \leq z \leq 1$ and all other orders xzy, yzx, zxy, zyx; all six contain $(0,0,0)$; to contain (1,0,1)

5 $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 dx\,dy\,dz = 8$ **7** $\int_{-1}^1 \int_{-1}^z \int_{-1}^1 dx\,dy\,dz = 4$ **9** $\int_{-1}^1 \int_z^1 \int_1^z dx\,dy\,dz = \frac{4}{3}$

11 $\int_0^1 \int_0^{2-2z} \int_0^{2-y-2z} dx\,dy\,dz = \frac{2}{3}$ **13** $\int_0^{1/2} \int_0^{2-2z} \int_0^{2-y-2z} dx\,dy\,dz = \frac{7}{12}$

15 $\int_0^1 \int_0^{1-z} \int_0^{\sqrt{(1-z)^2-y^2}} dx\,dy\,dz = \frac{\pi}{3}$ **17** $\int_0^6 \int_0^1 \int_0^{\sqrt{1-y^2}} dx\,dy\,dz = 6\pi$ **19** $\int_0^1 \int_0^1 \int_0^{\sqrt{1-y^2}} dx\,dy\,dz = \pi$

21 Corner of cube at $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$; sides $\frac{2}{\sqrt{3}}$; area $\frac{8}{3\sqrt{3}}$

23 Horizontal slices are circles of area $\pi r^2 = \pi(4 - z)$; volume $= \int_0^4 \pi(4 - z)dz = 8\pi$; centroid has $\bar{x} = 0, \bar{y} = 0, \bar{z} = \int_0^4 z\pi(4 - z)dz/8\pi = \frac{4}{3}$

25 $I = \frac{z^2}{2}$ gives zeros; $\frac{\partial I}{\partial x} = \int_0^z \int_0^y f\,dy\,dz$, $\frac{\partial I}{\partial y} = \int_0^z \int_0^x f\,dx\,dz$, $\frac{\partial^2 I}{\partial y\,\partial z} = \int_0^x f\,dx$
27 $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (y^2+z^2)dx\,dy\,dz = \frac{16}{3}$; $\iiint x^2 dV = \frac{8}{3}$; $3\iiint (x - \frac{x+y+z}{3})^2 dV = \frac{16}{3}$
29 $\int_0^3 \int_0^2 \int_0^y dx\,dy\,dz = 6$ **31** Trapezoidal rule is second-order; correct for $1, x, y, z, xy, xz, yz, xyz$

Section 14.4 Cylindrical and Spherical Coordinates (page 547)

1 $(r,\theta,z) = (D,0,0)$; $(\rho,\phi,\theta) = (D,\frac{\pi}{2},0)$ **3** $(r,\theta,z) = (0$, any angle, $D)$; $(\rho,\phi,\theta) = (D, 0$, any angle$)$
5 $(x,y,z) = (2,-2,2\sqrt{2})$; $(r,\theta,z) = (2\sqrt{2}, -\frac{\pi}{4}, 2\sqrt{2})$ **7** $(x,y,z) = (0,0,-1)$; $(r,\theta,z) = (0$, any angle, $-1)$
9 $\phi = \tan^{-1}(\frac{r}{z})$ **11** 45° cone in unit sphere: $\frac{2\pi}{3}(1 - \frac{1}{\sqrt{2}})$ **13** cone without top: $\frac{7\pi}{3}$
15 $\frac{1}{4}$ hemisphere: $\frac{\pi}{6}$ **17** $\frac{\pi^2}{8}$ **19** Hemisphere of radius π : $\frac{2}{3}\pi^4$ **21** $\pi(R^2 - z^2)$; $4\pi r\sqrt{R^2 - r^2}$
23 $\frac{2}{3}a^3 \tan\alpha$ (see 8.1.39) **27** $\frac{\partial q}{\partial D} = \frac{\rho - D\cos\phi}{q} = \frac{\text{near side}}{\text{hypotenuse}} = \cos\alpha$
31 Wedges are not exactly similar; the error is higher order ⇒ proof is correct
33 Proportional to $1 + \frac{1}{h}(\sqrt{a^2 + (D-h)^2} - \sqrt{a^2 + D^2})$
35 $J = \begin{vmatrix} a & & \\ & b & \\ & & c \end{vmatrix} = abc$; straight edges at right angles **37** $\begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$
39 $\frac{8\pi\rho^4}{3}$; $\frac{2}{3}$ **41** ρ^3; ρ^2; force $= 0$ inside hollow sphere

CHAPTER 15 VECTOR CALCULUS

Section 15.1 Vector Fields (page 554)

1 $f(x,y) = x + 2y$ **3** $f(x,y) = \sin(x+y)$ **5** $f(x,y) = \ln(x^2+y^2) = 2\ln r$
7 $\mathbf{F} = xy\mathbf{i} + \frac{x^2}{2}\mathbf{j}$, $f(x,y) = \frac{x^2 y}{2}$ **9** $\frac{\partial f}{\partial x} = 0$ so f cannot depend on x; streamlines are vertical ($y =$ constant)
11 $\mathbf{F} = 3\mathbf{i} + \mathbf{j}$ **13** $\mathbf{F} = \mathbf{i} + 2y\,\mathbf{j}$ **15** $\mathbf{F} = 2x\,\mathbf{i} - 2y\,\mathbf{j}$ **17** $\mathbf{F} = e^{x-y}\,\mathbf{i} - e^{x-y}\,\mathbf{j}$
19 $\frac{dy}{dx} = -1$; $y = -x + C$ **21** $\frac{dy}{dx} = -\frac{x}{y}$; $x^2 + y^2 = C$ **23** $\frac{dy}{dx} = \frac{-x/y^2}{1/y} = \frac{-x}{y}$; $x^2 + y^2 = C$ **25** parallel
27 $\mathbf{F} = \frac{5x}{r}\mathbf{i} + \frac{5y}{r}\mathbf{j}$ **29** $\mathbf{F} = \frac{-mMG}{r^3}(x\mathbf{i} + y\,\mathbf{j}) - \frac{mMG}{((x-1)^2+y^2)^{3/2}}((x-1)\,\mathbf{i} + y\,\mathbf{j})$
31 $\mathbf{F} = \frac{\sqrt{2}}{2}y\,\mathbf{i} - \frac{\sqrt{2}}{2}x\,\mathbf{j}$ **33** $\frac{dy}{dx} = \frac{-2}{x^2} = -\frac{1}{2}$; $\frac{dy}{dx} = \frac{x}{\sqrt{x^2-3}} = 2$
35 $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} = \frac{\partial f}{\partial r}\frac{x}{r}$; $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r}\frac{y}{r}$; $f(r) = C$ gives circles
37 T; F (no equipotentials); T; F (not multiple of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$)
39 $\mathbf{F}$ and $\mathbf{F} + \mathbf{i}$ and $2\mathbf{F}$ have the same streamlines (different velocities) and equipotentials (different potentials). But if f is given, $\mathbf{F}$ must be grad f.

Section 15.2 Line Integrals (page 562)

1 $\int_0^1 \sqrt{1^2 + 2^2}dt = \sqrt{5}$; $\int_0^1 2\,dt = 2$ **3** $\int_0^1 t^2\sqrt{2}dt + \int_1^2 1\cdot(2-t)dt = \frac{\sqrt{2}}{3} + \frac{1}{2}$
5 $\int_0^{2\pi}(-3\sin t)dt = 0$ (gradient field); $\int_0^{2\pi} -9\sin^2 t\,dt = -9\pi = -$ area
7 No, $xy\,\mathbf{j}$ is not a gradient field; take line $x = t, y = t$ from (0,0) to (1,1) and $\int t^2 dt \neq \frac{1}{2}$
9 No, for a circle $(2\pi r)^2 \neq 0^2 + 0^2$ **11** $f = x + \frac{1}{2}y^2$; $f(0,1) - f(1,0) = -\frac{1}{2}$
13 $f = \frac{1}{2}x^2y^2$; $f(0,1) - f(1,0) = 0$ **15** $f = r = \sqrt{x^2+y^2}$; $f(0,1) - f(1,0) = 0$
17 Gradient for $n = 2$; after calculation $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{n-2}{r^n}$
19 $x = a\cos t, z = a\sin t, ds = a\,dt, M = \int_0^{2\pi}(a + a\sin t)a\,dt = 2\pi a^2$

21 $x = a\cos t, y = a\sin t, ds = a\,dt, M = \int_0^{2\pi} a^3\cos^2 t\,dt = \pi a^3, (\bar{x},\bar{y}) = (0,0)$ by symmetry

23 $\mathbf{T} = \frac{2\mathbf{i}+2t\mathbf{j}}{\sqrt{4+4t^2}} = \frac{\mathbf{i}+t\mathbf{j}}{\sqrt{1+t^2}}; \mathbf{F} = 3x\,\mathbf{i}+4\,\mathbf{j} = 6t\,\mathbf{i}+4\,\mathbf{j}, ds = 2\sqrt{1+t^2}dt, \mathbf{F}\cdot\mathbf{T}ds = (6t\,\mathbf{i}+4\,\mathbf{j})\cdot(\frac{\mathbf{i}+t\mathbf{j}}{\sqrt{1+t^2}})2\sqrt{1+t^2}dt =$ $20t\,dt; \mathbf{F}\cdot d\mathbf{R} = (6t\mathbf{i}+4\mathbf{j})\cdot(2\,dt\mathbf{i}+2t\,dt\,\mathbf{j}) = 20t\,dt$; work $= \int_1^2 20t\,dt = 30$

25 If $\frac{\partial M(y)}{\partial y} = \frac{\partial N(x)}{\partial x}$ then $M = ay+b, N = ax+c$, constants a,b,c

27 $\mathbf{F} = 4x\mathbf{j}$ (work $= 4$ from (1,0) up to (1,1)) **29** $f = [x-2y]_{(0,0)}^{(1,1)} = -1$ **31** $f = [xy^2]_{(0,0)}^{(1,1)} = 1$

33 Not conservative; $\int_0^1(t\mathbf{i}-t\mathbf{j})\cdot(dt\,\mathbf{i}+dt\,\mathbf{j}) = \int 0\,dt = 0; \int_0^1(t^2\mathbf{i}-t\mathbf{j})\cdot(dt\,\mathbf{i}+2t\,dt\,\mathbf{j}) = \int_0^1 -t^2dt = -\frac{1}{3}$

35 $\frac{\partial M}{\partial y} = ax, \frac{\partial N}{\partial x} = 2x+b$, so $a = 2, b$ is arbitrary **37** $\frac{\partial M}{\partial y} = 2ye^{-x} = \frac{\partial N}{\partial x}; f = -y^2e^{-x}$

39 $\frac{\partial M}{\partial y} = \frac{-xy}{r^3} = \frac{\partial N}{\partial x}; f = r = \sqrt{x^2+y^2} = |x\mathbf{i}+y\mathbf{j}|$

41 $\mathbf{F} = (x-y)\mathbf{i}+(x+y)\mathbf{j}$ has $\frac{\partial M}{\partial y} = -1, \frac{\partial N}{\partial x} = 1$, no f **43** $2\pi; 0; 0$

Section 15.3 Green's Theorem (page 571)

1 $\int_0^{2\pi}(a\cos t)a\cos t\,dt = \pi a^2; N_x - M_y = 1, \iint dx\,dy =$ area πa^2

3 $\int_0^1 x\,dx + \int_1^0 x\,dx = 0, N_x - M_y = 0, \iint 0\,dx\,dy = 0$

5 $\int x^2y\,dx = \int_0^{2\pi}(a\cos t)^2(a\sin t)(-a\sin t\,dt) = -\frac{a^4}{4}\int_0^{2\pi}(\sin 2t)^2dt = -\frac{\pi a^4}{4}$;
$N_x - M_y = -x^2, \iint(-x^2)dx\,dy = \int_0^{2\pi}\int_0^a -r^2\cos^2\theta\,r\,dr\,d\theta = -\frac{\pi a^4}{4}$

7 $\int x\,dy - y\,dx = \int_0^\pi(\cos^2 t+\sin^2 t)\,dt = \pi; \iint(1+1)dx\,dy = 2$ (area) $= \pi; \int x^2dy - xy\,dx = \frac{1}{2}+1$;
$\int_0^1\int_0^1(2x+x)dx\,dy = \frac{3}{2}$

9 $\frac{1}{2}\int_0^{2\pi}(3\cos^4 t\sin^2 t + 3\sin^4 t\cos^2 t)dt = \frac{1}{2}\int_0^{2\pi}3\cos^2 t\sin^2 t\,dt = \frac{3}{2}\frac{\pi}{4}$ (see Answer 5)

11 $\int\mathbf{F}\cdot d\mathbf{R} = 0$ around any loop; $\mathbf{F} = \frac{x}{r}\mathbf{i}+\frac{y}{r}\mathbf{j}$ and $\int\mathbf{F}\cdot d\mathbf{R} = \int_0^{2\pi}[-\sin t\cos t + \sin t\cos t]dt = 0$;
$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ gives $\iint 0\,dx\,dy$

13 $x = \cos 2t, y = \sin 2t, t$ from 0 to $2\pi; \int_0^{2\pi} -2\sin^2 2t\,dt = -2\pi = -2$ (area);
$\int_0^{2\pi} -2dt = -4\pi = -2$ times Example 7

15 $\int M dy - N dx = \int_0^{2\pi} 2\sin t\cos t\,dt = 0; \iint(M_x+N_y)dx\,dy = \iint 0\,dx\,dy = 0$

17 $M = \frac{x}{r}, N = \frac{y}{r}, \int M dy - N dx = \int_0^{2\pi}(\cos^2 t+\sin^2 t)dt = 2\pi; \iint(M_x+N_y)dx\,dy = \iint(\frac{1}{r}-\frac{x^2}{r^3}+\frac{1}{r}-\frac{y^2}{r^3})dx\,dy =$
$\iint\frac{1}{r}dx\,dy = \iint dr\,d\theta = 2\pi$

19 $\int M dy - N dx = \int -x^2y\,dx = \int_1^0 -x^2(1-x)dx = \frac{1}{12}; \int_0^1\int_0^{1-y}x^2dx\,dy = \frac{1}{12}$

21 $\iint(M_x+N_y)dx\,dy = \iint \text{div}\,\mathbf{F}\,dx\,dy = 0$ between the circles

23 Work: $\int a\,dx + b\,dy = \iint(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y})dx\,dy$; Flux: *same integral*

25 $g = \tan^{-1}(\frac{y}{x}) = \theta$ is undefined at (0,0) **27** Test $M_y = N_x : x^2dx + y^2dy$ is exact $= d(\frac{1}{3}x^3 + \frac{1}{3}y^3)$

29 div $\mathbf{F} = 2y - 2y = 0; g = xy^2$ **31** div $\mathbf{F} = 2x+2y$; no g **33** div $\mathbf{F} = 0; g = e^x\sin y$

35 div $\mathbf{F} = 0; g = \frac{y^2}{x}$

37 $N_x - M_y = -2x, -6xy, 0, 2x-2y, 0, -2e^{x+y}$; in **31** and **33** $f = \frac{1}{3}(x^3+y^3)$ and $f = e^x\cos y$

39 $\mathbf{F} = (3x^2-3y^2)\mathbf{i} - 6xy\,\mathbf{j}$; div $\mathbf{F} = 0$ **41** $f = x^4 - 6x^2y^2 + y^4; g = 4x^3y - 4xy^3$

43 $\mathbf{F} = e^x\cos y\,\mathbf{i} - e^x\sin y\,\mathbf{j}; g = e^x\sin y$

45 $N = f(x), \int M dx + N dy = \int_0^1 f(1)dy + \int_1^0 f(0)dy = f(1) - f(0); \iint(N_x - M_y)dx\,dy =$
$\iint\frac{\partial f}{\partial x}dx\,dy = \int_0^1\frac{\partial f}{\partial x}dx$ (Fundamental Theorem of Calculus)

Section 15.4 Surface Integrals (page 581)

1 $\mathbf{N} = -2x\mathbf{i} - 2y\mathbf{j} + \mathbf{k}; dS = \sqrt{1+4x^2+4y^2}\,dx\,dy; \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\,r\,dr\,d\theta = \frac{\pi}{6}(17^{3/2}-1)$
3 $\mathbf{N} = -\mathbf{i}+\mathbf{j}+\mathbf{k}; dS = \sqrt{3}\,dx\,dy$; area $\sqrt{3}\pi$
5 $\mathbf{N} = \frac{-x\mathbf{i}-y\mathbf{j}}{\sqrt{1-x^2-y^2}} + \mathbf{k}; dS = \frac{dx\,dy}{\sqrt{1-x^2-y^2}}; \int_0^{2\pi}\int_0^{1/\sqrt{2}} \frac{r\,dr d\theta}{\sqrt{1-r^2}} = \pi(2-\sqrt{2})$
7 $\mathbf{N} = -7\mathbf{j}+\mathbf{k}; dS = 5\sqrt{2}\,dx\,dy$; area $5\sqrt{2}A$
9 $\mathbf{N} = (y^2-x^2)\mathbf{i} - 2xy\mathbf{j} + \mathbf{k}; dS = \sqrt{1+(y^2-x^2)^2+4x^2y^2}dx\,dy = \sqrt{1+(y^2+x^2)^2}dx\,dy;$
$\int_0^{2\pi}\int_0^1 \sqrt{1+r^4}\,r\,dr\,d\theta = \frac{\pi}{\sqrt{2}} + \frac{\pi\ln(1+\sqrt{2})}{2}$
11 $\mathbf{N} = 2\mathbf{i}+2\mathbf{j}+\mathbf{k}; dS = 3dx\,dy$; 3(area of triangle with $2x+2y \le 1$) $= \frac{3}{8}$
13 $\pi a\sqrt{a^2+h^2}$ **15** $\int_0^1\int_0^{1-y} xy(\sqrt{3}\,dx\,dy) = \frac{\sqrt{3}}{24}$
17 $\int_0^{2\pi}\int_0^{\pi/4} \sin^2\phi\cos\phi\sin\theta\cos\theta(\sin\phi\,d\phi\,d\theta) = 0$ **19** $\mathbf{A} = \mathbf{i}+\mathbf{j}+2\mathbf{k}; \mathbf{B} = \mathbf{j}+\mathbf{k}; \mathbf{N} = -\mathbf{i}-\mathbf{j}+\mathbf{k}; dS = \sqrt{3}\,du\,dv$
21 $\mathbf{A} = -\sin u(\cos v\,\mathbf{i} + \sin v\,\mathbf{j}) + \cos u\,\mathbf{k}; \mathbf{B} = -(3+\cos u)\sin v\,\mathbf{i} + (3+\cos u)\cos v\,\mathbf{j};$
$\mathbf{N} = -(3+\cos u)(\cos u\cos v\,\mathbf{i} + \cos u\sin v\,\mathbf{j} + \sin u\,\mathbf{k}); dS = (3+\cos u)du\,dv$
23 $\iint(-M\frac{\partial f}{\partial x} - N\frac{\partial f}{\partial y} + P)dx\,dy = \iint(-2x^2-2y^2+z)dx\,dy = \iint -r^2(r\,dr\,d\theta) = -8\pi$
25 $\mathbf{F}\cdot\mathbf{N} = -x+y+z = 0$ on plane
27 $\mathbf{N} = -\mathbf{i}-\mathbf{j}+\mathbf{k}, \mathbf{F} = (v+u)\,\mathbf{i} - u\,\mathbf{j}, \iint \mathbf{F}\cdot\mathbf{N}dS = \iint -v\,du\,dv = 0$
29 $\iint dS = \int_0^{2\pi}\int_0^{2\pi}(3+\cos u)du\,dv = 12\pi^2$ **31** Yes **33** No
35 $\mathbf{A} = \mathbf{i} + f'\cos\theta\,\mathbf{j} + f'\sin\theta\,\mathbf{k}; \mathbf{B} = -f\sin\theta\,\mathbf{j} + f\cos\theta\,\mathbf{k}; \mathbf{N} = ff'\mathbf{i} - f\cos\theta\,\mathbf{j} - f\sin\theta\,\mathbf{k}; dS = |\mathbf{N}|dx\,d\theta =$
$f(x)\sqrt{1+f'^2}\,dx\,d\theta$

Section 15.5 The Divergence Theorem (page 588)

1 div $\mathbf{F} = 1, \iiint dV = \frac{4\pi}{3}$ **3** div $\mathbf{F} = 2x+2y+2z, \iiint$ div $\mathbf{F}\,dV = 0$ **5** div $\mathbf{F} = 3, \iint 3dV = \frac{3}{6} = \frac{1}{2}$
7 $\mathbf{F}\cdot\mathbf{N} = \rho^2, \iint_{\rho=a}\rho^2 dS = 4\pi a^4$ **9** div $\mathbf{F} = 2z, \int_0^{2\pi}\int_0^{\pi/2}\int_0^a 2\rho\cos\phi(\rho^2\sin\phi\,d\rho\,d\phi\,d\theta) = \frac{1}{2}\pi a^4$
11 $\int_0^a\int_0^a\int_0^a(2x+1)dx\,dy\,dz = a^4+a^3; -2a^2+2a^2+0+a^4+0+a^3$
13 div $\mathbf{F} = \frac{x}{\rho}, \iiint \frac{x}{\rho}dV = 0; \mathbf{F}\cdot\mathbf{n} = x, \iint x dS = 0$ **15** div $\mathbf{F} = 1; \iiint 1dV = \frac{\pi}{3}; \iiint 1\,dV = \frac{1}{6}$
17 div $(\frac{\mathbf{R}}{\rho^7}) = \frac{\text{div}\,\mathbf{R}}{\rho^7} + \mathbf{R}\cdot\text{grad}\frac{1}{\rho^7} = \frac{3}{\rho^7} - \frac{7}{\rho^8}\mathbf{R}\cdot\text{grad}\rho$
19 Two spheres, $\mathbf{n}$ radial out, $\mathbf{n}$ radial in, $\mathbf{n} = \mathbf{k}$ on top, $\mathbf{n} = -\mathbf{k}$ on bottom, $\mathbf{n} = \frac{x\mathbf{i}+y\mathbf{j}}{\sqrt{x^2+y^2}}$ on side;
$\mathbf{n} = -\mathbf{i}, -\mathbf{j}, -\mathbf{k}, \mathbf{i}+2\mathbf{j}+3\mathbf{k}$ on 4 faces; $\mathbf{n} = \mathbf{k}$ on top, $\mathbf{n} = \frac{1}{\sqrt{2}}(\frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j} - \mathbf{k})$ on cone
21 V = cylinder, $\iiint$ div $\mathbf{F}\,dV = \iint(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y})dx\,dy$ (z integral = 1); $\iint \mathbf{F}\cdot\mathbf{n}dS =$
$\int Mdy - Ndx, z$ integral = 1 on side, $\mathbf{F}\cdot\mathbf{n} = 0$ top and bottom; Green's flux theorem.
23 div $\mathbf{F} = \frac{-3GM}{a^3} = -4\pi G$; at the center; $\mathbf{F} = 2\mathbf{R}$ inside, $\mathbf{F} = 2(\frac{a}{\rho})^3\mathbf{R}$ outside
25 div $\mathbf{u}_r = \frac{2}{\rho}, q = \frac{2\epsilon_0}{\rho}, \iint \mathbf{E}\cdot\mathbf{n}\,dS = \iint 1dS = 4\pi$ **27** F (div $\mathbf{F} = 0$); F; T($\mathbf{F}\cdot\mathbf{n} \le 1$); F
29 Plane circle; top half of sphere; div $\mathbf{F} = 0$

Section 15.6 Stokes' Theorem and the Curl of F (page 595)

1 curl $\mathbf{F} = \mathbf{i}+\mathbf{j}+\mathbf{k}$ **3** curl $\mathbf{F} = \mathbf{0}$ **5** curl $\mathbf{F} = \mathbf{0}$ **7** $f = \frac{1}{2}(x+y+z)^2$
9 curl $x^m\mathbf{i} = \mathbf{0}$; $x^n\mathbf{j}$ has zero curl if $n = 0$ **11** curl $\mathbf{F} = 2y\mathbf{i}$; $\mathbf{n} = \mathbf{j}$ on circle so $\iint \mathbf{F}\cdot\mathbf{n}dS = 0$
13 curl $\mathbf{F} = 2\mathbf{i}+2\mathbf{j}, \mathbf{n} = \mathbf{i}, \iint$ curl $\mathbf{F}\cdot\mathbf{n}dS = \iint 2\,dS = 2\pi$

15 Both integrals equal $\int \mathbf{F} \cdot d\mathbf{R}$; Divergence Theorem, V = region between S and T, always div curl $\mathbf{F} = 0$

17 Always div curl $\mathbf{F} = 0$ **19** $f = xz + y$ **21** $f = e^{x-z}$ **23** $\mathbf{F} = y\mathbf{k}$

25 curl $\mathbf{F} = (a_3b_2 - a_2b_3)\mathbf{i} + (a_1b_3 - a_3b_1)\mathbf{j} + (a_2b_1 - a_1b_2)\mathbf{k}$ **27** curl $\mathbf{F} = 2\omega\mathbf{k}$; curl $\mathbf{F} \cdot \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}} = 2\omega/\sqrt{3}$

29 $\mathbf{F} = x(a_3z + a_2y)\mathbf{i} + y(a_1x + a_3z)\mathbf{j} + z(a_1x + a_2y)\mathbf{k}$

31 curl $\mathbf{F} = -2\mathbf{k}$, $\iint -2\mathbf{k} \cdot \mathbf{R}dS = \int_0^{2\pi}\int_0^{\pi/2} -2\cos\phi(\sin\phi\, d\phi\, d\theta) = -2\pi$; $\int y\, dx - x\, dy =$
$\int_0^{2\pi}(-\sin^2 t - \cos^2 t)dt = -2\pi$

33 curl $\mathbf{F} = 2\mathbf{a}$, $2\iint(a_1x + a_2y + a_3z)dS = 0 + 0 + 2a_3\int_0^{2\pi}\int_0^{\pi/2}\cos\phi\sin\phi\, d\phi\, d\theta = 2\pi a_3$

35 curl $\mathbf{F} = -\mathbf{i}$, $\mathbf{n} = \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$, $\iint \mathbf{F} \cdot \mathbf{n}dS = -\frac{1}{\sqrt{3}}\pi r^2$

37 $g = \frac{y^2}{2} - \frac{x^3}{3}$ = stream function; zero divergence

39 div $\mathbf{F}$ = div $(\mathbf{V} + \mathbf{W})$ = div $\mathbf{V}$ so y = div $\mathbf{V}$ so $\mathbf{V} = \frac{y^2}{2}\mathbf{j}$ (has zero curl). Then $\mathbf{W} = \mathbf{F} - \mathbf{V} = xy\mathbf{i} - \frac{y^2}{2}\mathbf{j}$

41 curl (curl $\mathbf{F}$) = curl $(-2y\mathbf{k}) = -2\mathbf{i}$; grad (div $\mathbf{F}$) = grad $2x = 2\mathbf{i}$; $\mathbf{F}_{xx} + \mathbf{F}_{yy} + \mathbf{F}_{zz} = 4\mathbf{i}$

43 curl $\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = \mathbf{a}\sin t$ so $\mathbf{E} = \frac{1}{2}(\mathbf{a} \times \mathbf{R})\sin t$

45 $\mathbf{n} = \mathbf{j}$ so $\int M dx + P dz = \iint(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x})dx\, dz$ **47** $M_y^* = M_y + M_z f_y + P_y f_x + P_z f_y f_x + P f_{xy}$

49 $\int \mathbf{F} \cdot d\mathbf{R} = \iint$ curl $\mathbf{F} \cdot \mathbf{n}\, dS$; $\iint \mathbf{F} \cdot \mathbf{n}dS = \iiint$ div $\mathbf{F}\, dV$

CHAPTER 16 MATHEMATICS AFTER CALCULUS

Section 16.1 Linear Algebra (page 602)

1 All vectors $c\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **3** Only $x = 0$ **5** Plane of vectors with $x_1 + x_2 + x_3 = 0$

7 $x_p = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $A(x_p + x_0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **9** $A(x_p + x_0) = b + 0 = b$; another solution

11 $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}; b = \begin{bmatrix} c \\ c \\ c \end{bmatrix}$

13 $CC^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}; C^{\mathrm{T}}C = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$; (2 by 3) (2 by 3) is impossible

15 Any two are independent **17** C and F have independent columns

19 det $F = 3$ **21** $F^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

23 $\det(F - \lambda I) = \det\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 1 = 3 - 4\lambda + \lambda^2 = 0$ if $\lambda = 1$ or $\lambda = 3$;

$F\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ -1 \end{bmatrix}, F\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

25 $y = e^t\begin{bmatrix} 1 \\ -1 \end{bmatrix}, y = e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, y = \frac{e^t}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{e^{3t}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

27 $\det\begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^3 - 3(1-\lambda) + 2 = \lambda^3 - 3\lambda^2 = 0$ if $\lambda = 3$ or $\lambda = 0$ (repeated)

29 $\det\begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix} = \lambda^2 - 5\lambda = 0$ if $\lambda = 0$ or $\lambda = 5$; $A\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0\begin{bmatrix} 2 \\ -1 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

31 $H = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ **33** F if $b \neq 0$; T; T; F ($e^{\lambda t}$ is not a vector); T

Section 16.2 Differential Equations (page 610)

1 $3Be^{3t} - Be^{3t} = 8e^{3t}$ gives $B = 4 : y = 4e^{3t}$ **3** $y = 3 - 2t + t^2$ **5** $Ae^t + 4e^{3t} = 7$ at $t = 0$ if $A = 3$

7 Add $y = Ae^{-t}$ because $y' + y = 0$; choose $A = -1$ so $-e^{-t} + 3 - 2t + t^2 = 2$ at $t = 0$

9 $y = \frac{e^{kt}-1}{k}$; $y = t$; by l'Hôpital $\lim_{k\to 0} \dfrac{e^{kt}-1}{k} = \lim_{k\to 0} \dfrac{te^{kt}}{1} = t$

11 Substitute $y = Ae^t + Bte^t + C\cos t + D\sin t$ in equation: $B = 1, C = \frac{1}{2}, D = -\frac{1}{2}$, any A

13 Particular solution $y = Ate^t + Be^t$; $y' = Ate^t + (A+B)e^t = c(Ate^t + Be^t) + te^t$ gives $A = cA + 1, A + B = cB, A = \frac{1}{1-c}, B = \frac{-1}{(1-c)^2}$

15 $\lambda^2 e^{\lambda t} + 6\lambda e^{\lambda t} + 5e^{\lambda t} = 0$ gives $\lambda^2 + 6\lambda + 5 = 0, (\lambda+5)(\lambda+1) = 0, \lambda = -1$ or -5 (both negative so decay); $y = Ae^{-t} + Be^{-5t}$

17 $(\lambda^2 + 2\lambda + 3)e^{\lambda t} = 0, \lambda = -1 \pm \sqrt{-2}$ has imaginary part and negative real part; $y = Ae^{(-1+\sqrt{2}i)t} + Be^{(-1-\sqrt{2}i)t}$; $y = Ce^{-t}\cos\sqrt{2}t + De^{-t}\sin\sqrt{2}t$

19 $d = 0$ no damping; $d = 1$ underdamping; $d = 2$ critical damping; $d = 3$ overdamping

21 $\lambda = -\frac{b}{2} \pm \frac{\sqrt{b^2-4c}}{2}$ is repeated when $b^2 = 4c$ and $\lambda = -\frac{b}{2}$; $(t\lambda^2 + 2\lambda)e^{\lambda t} + b(t\lambda + 1)e^{\lambda t} + cte^{\lambda t} = 0$ when $\lambda^2 + b\lambda + c = 0$ and $2\lambda + b = 0$

23 $-a\cos t - b\sin t - a\sin t + b\cos t + a\cos t + b\sin t = \cos t$ if $a = 0, b = 1, y = \sin t$

25 $y = A\cos 3t + B\cos 5t$; $y'' + 9y = -25B\cos 5t + 9B\cos 5t = \cos 5t$ gives $B = \frac{-1}{16}$; $y_0 = 0$ gives $A = \frac{1}{16}$

27 $y = A(\cos\omega t - \cos\omega_0 t)$, $y'' = -A\omega^2\cos\omega t + A\omega_0^2\cos\omega_0 t$, $y'' + \omega_0^2 y = \cos\omega t$ gives $A(-\omega^2 + \omega_0^2) = 1$; breaks down when $\omega^2 = \omega_0^2$

29 $y = Be^{5t}$; $25B + 3B = 1, B = \frac{1}{28}$ **31** $y = A + Bt = \frac{1}{2} + \frac{1}{2}t$

33 $y'' - 25y = e^{5t}$; $y'' + y = \sin t$; $y'' = 1 + t$; right side solves homogeneous equation so particular solution needs extra factor t

35 $e^t, e^{-t}, e^{it}, e^{-it}$ **37** $y = e^{-2t} + 2te^{-2t}$; $y(2\pi) = (1 + 4\pi)e^{-4\pi} \approx 0$

39 $y = (4e^{-rt} - r^2e^{-4t/r})/(4 - r^2) \to 1$ as $r \to 0$ **43** $h \leq 2; h \leq 2.8$

Section 16.3 Discrete Mathematics (page 615)

1 Two then two then last one; go around hexagon **3** Six (each deletes one edge)

5 Connected: there is a path between any two nodes; connecting each new node requires an edge

13 Edge lengths 1,2,4

15 No; 1,3,4 on left connect only to 2,3 on right; 1,3 on right connect only to 2 on left **17** 4

19 Yes **21** F (may loop); T **25** 16