15.401 Finance Theory

MIT Sloan MBA Program

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Lectures 18–20: Capital Budgeting

Critical Concepts

- NPV Rule
- Cash Flow Computations
- Discount Rates
- Discount Rates Over Time
- Project Interactions
- Alternatives to the NPV Rule
- The Practice of Capital Budgeting
- Key Points

Readings

- Brealey, Myers, and Allen Chapter 5–6, 9, 22
- Graham and Harvey (2001)

NPV Rule

Objective: Increase Firm's Current Market Value

- Implication: take projects with positive NPV
- Project has cashflows of:

$$\{CF_0, CF_1, \cdots, CF_T\}$$

- Its current market value is

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \cdots + \frac{CF_T}{(1+r_T)^T}$$

- This is the addition to the firm's market value by the project (recall value additivity).

NPV Rule

Investment Criteria:

- For a single project, take it if and only if its NPV is positive
- For many *independent* projects, take all those with positive NPV
- For *mutually exclusive* projects, take the one with positive and highest NPV

To Compute the NPV of a Project, We Need To Consider:

- Cash flows
- Discount rates
- Strategic options

Cash Flow Calculations

Main Points:

1. Use cash flows, not accounting earnings
2. Use after-tax cashflows
3. Use cash flows attributable to the project (compare firm value with and without the project):
   - Use incremental cash flows
   - Forget sunk costs: bygones are bygones
   - Include investment in working capital as capital expenditure
   - Include opportunity costs of using existing equipment, facilities, etc.
   - Correct for biases from fighting for resources inside firm

Cash Flow Calculations

Consider Project Cashflows:

$$
\begin{aligned}
CF &= [\text{Project Cash Inflows}] - [\text{Project Cash Outflows}] \\
&= [\text{Operating Revenues}] \\
&\quad - [\text{Operating Expenses without depreciation}] \\
&\quad - [\text{Capital Expenditures}] \\
&\quad - [\text{Income Taxes}]
\end{aligned}
$$

- Defining operating profit by

$$
\begin{aligned}
\text{Operating Profit} &= \text{Operating Revenues} \\
&\quad - \text{Operating Expenses w/o Depreciation}
\end{aligned}
$$

Cash Flow Calculations

- The income taxes are

$$\begin{aligned}[\text{Project Income Taxes}] &= [\text{Tax Rate}][\text{Operating Profit}] \\ &\quad - [\text{Tax Rate}][\text{Depreciation}]\end{aligned}$$

- Note that accounting depreciation does affect cash flows because it reduces the company's tax bill.
- Let τ denote the ``effective'' tax rate. Then

$$\begin{aligned}CF &= (1-\tau)[\text{Operating Profits}] - [\text{Capital Expenditures}] \\ &\quad + (\tau)[\text{Depreciation}]\end{aligned}$$

Cash Flow Calculations

Example: Capital Expenditure and Accounting Earnings vs. Cash Flows

A machine purchased for $1,000,000 with a life of 10 years generates annual revenues of $300,000 and operating expenses of $100,000. Assume that machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%. What is its NPV?

$$
\begin{aligned}
\text{Accounting Earnings} &= \$3{,}00\text{K} \; - \; \$100\text{K} \; - \; \$100K \\
&= \$100\text{K}
\end{aligned}
$$

$$
\begin{aligned}
\text{After-Tax Cashflow} &= (1-0.4) \times (\$3{,}00\text{K} - \$100\text{K}) \; + \\
&\qquad 0.4 \times \$100K \\
&= \$160\text{K}
\end{aligned}
$$

Cash Flow Calculations

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	- 1,000,000
1	300,000 - 100,000 - 100,000 = 100,000	(1-0.4)(100,000) = 60,000	(1-0.4) (300,000-100,00) + 40,000 = 160,000
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

- The accounting earnings do *not* accurately reflect the actual timing of cash flows

Discount Rates

So Far, We Have Shown That:

- A project's discount rate (i.e., required rate of return) is the expected rate of return demanded by investors for the project
- The discount rate(s) in general depend on the timing and risk of the cashflow(s)
- The discount rate is usually different for different projects
- Therefore, it is in general incorrect to use a company-wide "cost of capital" to discount cash flows of all projects

What Is The Required Rate of Return on a Project?

- Simple case: single discount rate can be used for all cashflows of a project (the term structure of discount rates is flat)
- General case (the term structure of discount rates is not flat)

Discount Rates

Use CAPM to Estimate Cost of Capital

- Project's required rate of return is determined by the **project beta**:

$$\bar{R}_{\text{project}} = R_f + \beta_{\text{project}} \left(\bar{R}_m - R_f \right)$$

- What matters is the project beta, not the company beta!
- What if project beta is unknown?
  - Find comparable "pure-play" company and use its beta
  - Find comparable historical project and use its cashflows to estimate beta
  - Use intuition and empirical judgment to guesstimate beta

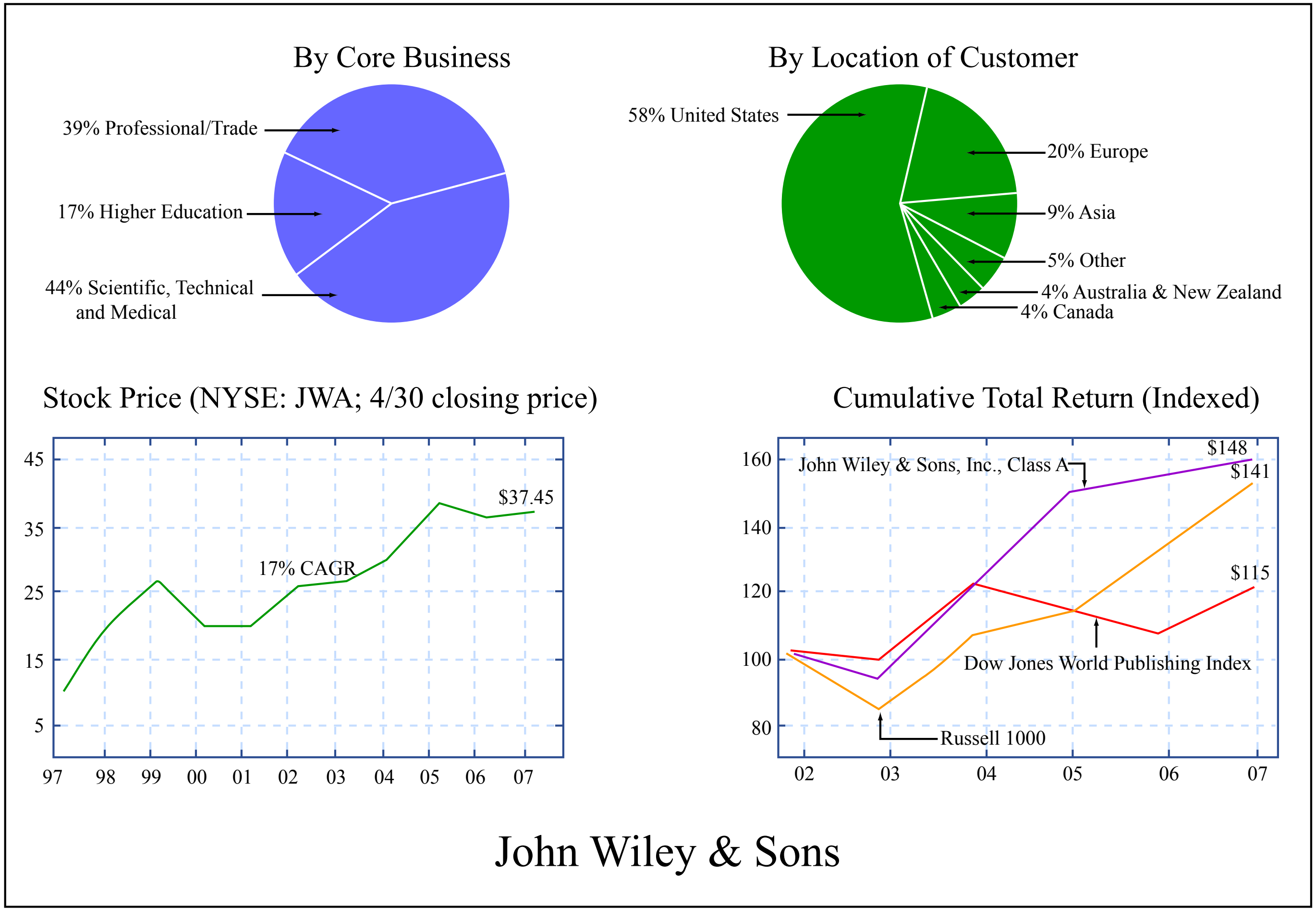
Discount Rates

Example:

Bloomberg, a provider of financial data and analytics, is considering entering the publishing business (Bloomberg Press), and must evaluate the NPV of the estimated cashflows from this business. What cost of capital should it use for these NPV calculations?

- Bloomberg should not use its own beta to discount Bloomberg Press cashflows
- Bloomberg should use the beta of a publishing company (e.g., John Wiley & Sons)
- What about using McGraw-Hill's beta?

Discount Rates



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Discount Rates

Example (cont):

- Beta of JW/A (from http://finance.yahoo.com): 1.29
- Riskfree rate: 5%
- Market risk premium: 6%

$$\bar{R}_{\text{project}} = R_f + \beta_{\text{project}}(\bar{R}_m - R_f)$$
$$= 0.05 + 1.29 \times 0.06 = 12.7\%$$

- Use judgment in interpreting and adjusting this estimate
- Estimates are always wrong!
- How good is the approximation?

Discount Rates Over Time

Discount Rates Over Different Horizons Are, In General, Different

- The term structure of discount rates arises from two sources:

1. Term structure of interest rates: The discount rates in absence of risk can be different for sure cashflows at different dates
2. Term structure of risk premia:
   a) The risk of cashflows at different dates is different
   b) The price of risk is different for different dates

- Use the discount rate that corresponds to the risk at the time the cashflow is generated

Discount Rates Over Time

Example:

A firm is investing in an oil exploration project:

- Drilling takes place over the coming year
- At the end of the first year:
  - with probability 1/3, it finds 3 million barrels of oil
  - with probability 2/3, it finds nothing
- Conditional on successful exploration, 3 million barrels of oil will be produced by the end of the second year (There is no more oil after that)
- Expected after-tax profit per barrel is $20
- The riskfree rate is 5%
- Industry discount rate of oil production is 20%
- The exploration risk is non-systematic (beta of 0.0)
- What is the NPV of the project?

Discount Rates Over Time

Example (cont):

- Potential value of the cash flows after drilling (at t=1):

$$CF_1 = \begin{cases} 0 & \text{with prob } 2/3 \\ \frac{20\times 3}{1.20} = \$50\text{MM} & \text{with prob } 1/3 \end{cases}$$

- The value of the project at date 0:

$$\text{PV} = \frac{\text{E}[CF_1]}{1+R_1} = \frac{\text{E}[CF_1]}{1.05}$$

$$= \frac{0 \times (2/3) + \$50 \times (1/3)}{1.05} = \$15.9\text{MM}$$

Project Interactions

Deciding Among A Set of Projects

- If projects are independent, apply NPV rule to each project
- If projects are dependent (e.g., mutually exclusive—accepting one rules out the others), we have to compare their NPVs

Optimal Timing of Projects

- Reject project
- Accept project now
- Accept project later
- Sometimes waiting can increase NPV!

Project Interactions

Example:

Potential demand for your product is projected to increase over time. If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity cost of capital is 10%. Denoting by FPV the project's NPV at the time of introduction, we have:

Year to Start	FPV	% Change in FPV	NPV
1	100	–	91
2	120	20	99
3	138	15	104
4	149	8	102

Before year 4, the return to waiting is larger that the opportunity cost of capital, 10%. As long as the growth rate of FPV remains below 10% after year 4, it is best to wait and introduce at the end of year 3.

Alternatives to NPV

In Practice, Other Investment Rules Are Also Used

1. **Payback Period**
2. **Internal Rate of Return (IRR)**
3. **Profitability Index (PI)**

- Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.
- These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.
- The bottom line is: **The NPV rule dominates these alternatives**

Payback Period

Payback Period is the minimum *k* such that

$$CF_1 + CF_2 + \cdots + CF_k \geq -CF_0 = I_0$$

- In words, *k* is the minimum length of time such that the sum of cash flows from a project is positive

Decision Criterion Using Payback Period

- For independent projects: Accept if *k* is less than or equal to some fixed threshold *t**
- For mutually exclusive projects: Among all the projects having $k \leq t^*$, accept the one that has the minimum payback period

Payback Period

Shortcomings

- Ignores time-value of money
- Ignores cashflows after *k*

Discounted Payback:

$$\frac{CF_1}{1+R_1} + \cdots + \frac{CF_k}{(1+R_k)^k} \geq -CF_0 = I_0$$

- Still ignores cashflows after *k*

⇒ **Use NPV!**

Profitability Index

Profitability Index (PI) is the ratio of the present value of *future* cash flows and the initial cost of a project:

$$\text{PI} \equiv \frac{\text{PV}}{-CF_0} = \frac{\text{PV}}{I_0}$$

Decision Criterion Using PI

- For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule)
- For mutually exclusive projects: Among the projects with PI greater than one, accept the one with the highest PI.

PI Gives The Same Answer As NPV When

1. There is only one cash outflow, which is at time 0
2. Only one project is under consideration

Shortcomings:

- PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects

	CF_0	CF_1	IRR	NPV at 10%	PI at 10%
Project 1	−1,000	2,000	100%	818.18	1.82
Project 2	−2,000	3,600	80%	1,272.73	1.64
Project 2−1	−1,000	1,600	60%	454.55	1.45

$\Rightarrow$ **Use NPV!**

Internal Rate of Return (IRR)

IRR Defined By Solution To:

$$I_0 = \frac{CF_1}{(1+\mathrm{IRR})} + \frac{CF_2}{(1+\mathrm{IRR})^2} + \cdots + \frac{CF_t}{(1+\mathrm{IRR})^t}$$

Decision Criterion Using IRR

- For independent projects: Accept a project if its IRR is greater than some fixed IRR*, the threshold rate
- For mutually exclusive projects: Among the projects having IRR's greater than IRR*, accept one with the highest IRR

Internal Rate of Return (IRR)

IRR Rule Leads To The Same Decisions As NPV If

1. There is only one cash outflow, which occurs at time 0
2. Only one project is under consideration
3. The opportunity cost of capital is the same for all periods
4. The threshold rate is set equal to opportunity cost of capital

Shortcomings:

- Non-existent or multiple IRRs in certain cases
- Incorrect rankings for loans and other projects with negative cashflows in future periods
- Ignore scale

⇒ Use NPV!

Internal Rate of Return (IRR)

Example:

Incorrect ranking for loan-type cashflow sequences

	CF_0	CF_1
Project 1	-100	120
Project 2	100	-120

- The IRR of both projects is 20%
- If actual opportunity cost is 10%, IRR says to accept both projects
- However,
  - Project 1 has a positive NPV only if $R < 20\%$
  - Project 2 has a positive NPV only if $R > 20\%$
- Should take project 1 and reject project 2

Internal Rate of Return (IRR)

Example:

Non-existence of IRR

	CF_0	CF_1	CF_2
Project 1	-105	250	-150
Project 2	105	-250	150

- No IRR exists for these two projects

Internal Rate of Return (IRR)

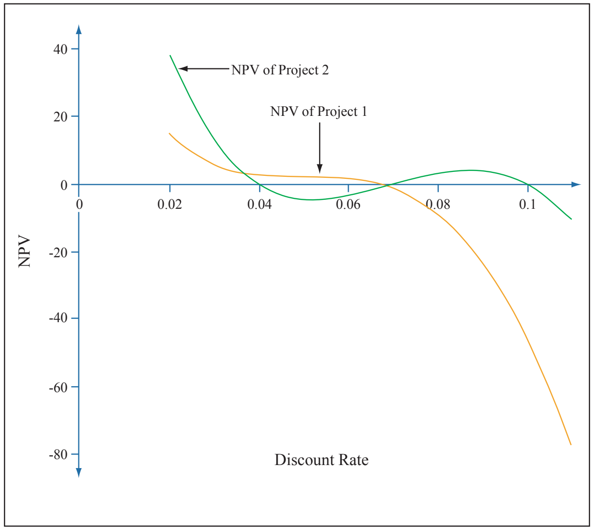
Example:

Multiple IRR's

	CF_0	CF_1	CF_2	CF_3
Project 1	−500,000	1,575,000	−1,653,750	578,815
Project 2	−500,000	1,605,000	−1,716,900	612,040

$$\text{IRR}_1 = 7\%$$

$$\text{IRR}_2 = \begin{cases} 4\% \\ 7\% \\ 10\% \end{cases}$$



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Internal Rate of Return (IRR)

Example:

Incorrect project ranking using IRR for mutually exclusive projects:

a) Projects of different scales:

	CF_0	CF_1	IRR	NPV at 10%
Project 1	−10,000	20,000	100%	8,181.82
Project 2	−20,000	36,000	80%	12,727.27

- One workaround to this problem is to use **incremental** cashflows:
  - See if lower investment (project 1) is a good idea
  - See if incremental investment (project 2) is a good idea

	CF_0	CF_1	IRR	NPV at 10%
Project 1	−10,000	20,000	100%	8,181.82
Project 2	−20,000	36,000	80%	12,727.27
Project 2−1	−10,000	16,000	60%	4,545.45

Internal Rate of Return (IRR)

Example (cont):

b) Projects with different time patterns of cash flows:

CF_t	0	1	2	3	4	5	etc.	IRR	NPV at 10%
Project 1	−90	60	50	40	0	0	⋯	33%	35.92
Project 2	−90	18	18	18	18	18	⋯	20%	90.00
Project 2−1	0	−42	−32	−22	18	18	⋯	15.6%	54.08

⇒ **Use NPV!**

Comparison of Methods Used by Large U.S. and Multinational Firms

	Large U.S. Firms	Multi-Nationals	
	Percentage Using Each Method	Use as Primary Method	Use as Secondary Method
Payback Period	80.3%	5.0%	37.6%
IRR	65.5	65.3	14.6
NPV	67.6	16.5	30.0
Other	—	2.5	3.2

The Practice of Capital Budgeting

Historical Comparison of Primary Use of Various Techniques

	1959	1964	1970	1975	1977	1979	1981
Payback Period	34%	24%	12%	15%	9%	10%	5.0%
IRR	19	38	57	37	54	60	65.3
NPV	—	—	—	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8

Source: S. Ross, R. Westerfield, and B. Jordon, *Essentials of Corporate Finance*, Irwin,1996.

Other Issues in Capital Budgeting

- Competitive response
  - CF forecasts should take into account responses of competitors.
- Capital rationing
- Sources of positive-NPV projects
  - Short-run competitive advantage (right place at right time)
  - Long-run competitive advantage (patent, technology, economies of scale, etc.)
  - Noise

Key Points

- Use the NPV rule for capital budgeting decisions: take all projects with positive NPV, or take highest-NPV project if mutually exclusive
- Consider project interactions separately
- Use after-tax cashflows for NPV calculations, not accounting earnings
- Use the CAPM to estimate cost of capital with project beta
- Be careful about risks that change over time or across different stages
- Be wary of alternative to NPV:
  - Payback rule, discounted payback rule
  - Profitability index
  - Internal rate of return

Additional References

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