

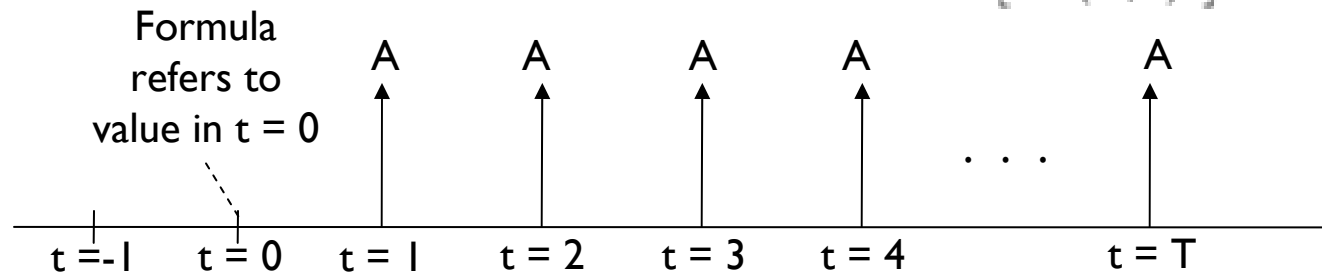
15.401 Recitation

Extra Session: Final Exam Review

General Advice

- Read carefully
- Show your work! Answers only give you partial credit
- Write down the formulas you use
- Draw timelines for cash flows
- Make sure you apply the annuity/ perpetuity formulas correctly – Example:

$$PV (\text{Annuity}) = A \times \frac{1}{r} \left[1 - \frac{1}{(1+r)^T} \right]$$



- State your assumptions
- Leave plenty of decimal places for interest rates (e.g., 1.2345%)



Sample Final Solutions

- ❑ Q1 – True or False
- ❑ Q2 – Present Value
- ❑ Q3 – Fixed Income
- ❑ Q4 – CAPM
- ❑ Q5 – Common Stock
- ❑ Q6 – Forwards & Futures
- ❑ Q7 – Capital Budgeting
- ❑ Q8 – Portfolio Theory / CAPM
- ❑ Q9 – Portfolio Theory
- ❑ Q10 – Options

Q1 – True or False

- a) The duration of a bond maturing at date T is always less than the duration of a zero-coupon bond maturing on the same date.

Advice:

- Look for the formulas of how duration is calculated
- Remember that the duration of zero-coupon bond is directly determined by its maturity

Q1 – True or False

a) The duration of a bond maturing at date T is always less than the duration of a zero-coupon bond maturing on the same date.

TRUE. Bonds with coupons always have a maturity that is lower than its maturity date. They accelerate cash flows received from the investment reducing the duration. On the other hand, zero-coupon bonds that, for example, mature in 3 years will necessarily have a maturity of 3 years. This is determined by the formula:

$$D = \sum_{t=1}^T \frac{PV(CF_t)}{B} \times t = \frac{1}{B} \sum_{t=1}^T \frac{CF_t}{(1+y)^t} \times t.$$

- Zero-Coupons have a duration of “T” since there is only one CF: $B = \frac{CF_T}{(1+y)^T}$
- Coupon Bonds maturing in “T” have a duration of less than “T” since there are cash flows occurring at T-1, T-2, etc.

Q1 – True or False

b) The market price of a share of stock equals the discounted value of the stream of future earnings per share

Advice:

- Look for relevant formulas → DCF
- Read carefully. For example:
 - Earnings \neq Dividends
 - Revenues \neq Profits
 - Face Value \neq Present Value

Q1 – True or False

b) The market price of a share of stock equals the discounted value of the stream of future earnings per share

Answer:

FALSE. The market value of a share of stock equals the discounted value of the stream of future dividends per share. It “works” with earnings if and only if the plowback (b) ratio = 0, making the payout (p) ratio = 1 and thus EPS = Dividends. Mathematically:

$$P_0 = \frac{D_1}{r - g}$$

Where D_1 = Dividends, not EPS

Q1 – True or False

c) Growth stocks must have a plowback ratio > 1

Advice:

- Clearly lay out the definition of a growth stock

Q1 – True or False

c) Growth stocks must have a plowback ratio > 1

Answer:

FALSE. Recall that Stocks of companies that have access to growth opportunities are considered growth stocks, Growth opportunities are investment opportunities that earn expected returns higher than the required rate of return on capital. By this definition, a growth stock has:
 $ROE > r$

The plowback ratio does not need to be greater than 1

Q1 – True or False

d) If a commercial airline wants to hedge its risk against oil prices, it should go short in oil futures

Advice:

- Think of examples and counterexamples
- Remember class and definitions!

Q1 – True or False

d) If a commercial airline wants to hedge its risk against oil prices, it should go short in oil futures

Answer:

FALSE. One can argue that airlines are short fuel cost. So to hedge that they have to go **long** oil future. On the other hand, they can pass down the cost of higher fuel prices to their clients so it is equally likely that they don't need to hedge 100% of their exposure (recall they will be in a disadvantage if they hedge and their competition did not hedge and prices fall as they have done recently).

In general, entering into a futures contract does not provide a hedge in itself. This is why not all airlines hedge all their gas purchases. Actually, the range vary widely from 0% of gas bought on futures to 100% of gas bought on futures, with notorious examples such as Southwest Airlines.

Q1 – True or False

e) The value of an American call option is always equal to the value of a European call option

Advice:

- Read carefully, and seek to understand the difference between American and European options:
 - European → Can only be exercised at expiration
 - American → Can be exercised on or before expiration
- Use your logic: Would you pay the same? Is there an effect due to time value of money?

Q1 – True or False

e) The value of an American call option is always equal to the value of a European call option

Answer:

FALSE. American Call options can be worth more than European options as they can be exercised before expiration. Therefore, the person who exercises the option could benefit from having the money early as well from not taking the risk of a reduction in the price of the underlying asset. Ultimately, this is related to the issue of dividend payment but we will not get into that here

Q1 – True or False

f) Holding everything else constant, the price of a European call option is increasing with increasing risk free interest rate.

Advice:

- Look for formulas or other content in your “cheat sheet” that may help you to answer the question. For example:

Put-Call Parity:

$$C + \frac{K}{(1+r)^T} = P + S$$

Increase in...	Value of call	Value of put
Strike price (K)	Decrease	Increase
Price of underlying asset (S)	Increase	Decrease
Volatility of the underlying asset (σ)	Increase	Increase
Maturity (T)	Increase	Increase
Interest rate (r)	Increase	Decrease

Q1 – True or False

f) Holding everything else constant, the price of a European call option is increasing with increasing risk free interest rate.

Answer:

- TRUE. According to the call-put parity, if everything else stays constant, when the risk-free rate increases the value of a call will necessarily have to increase:

$$C \uparrow + \frac{K}{(1+r \uparrow)^T} = P \downarrow + S$$

Q1 – True or False

g) Investors do not get rewarded for bearing idiosyncratic risk.

Advice:

- Carefully read the language – there is an important difference between “idiosyncratic” risk (also called non-systematic risk) and “systematic” risk
- If you do not know the answer for this one, study a lot from here until tomorrow!!!

Q1 – True or False

g) Investors do not get rewarded for bearing idiosyncratic risk.

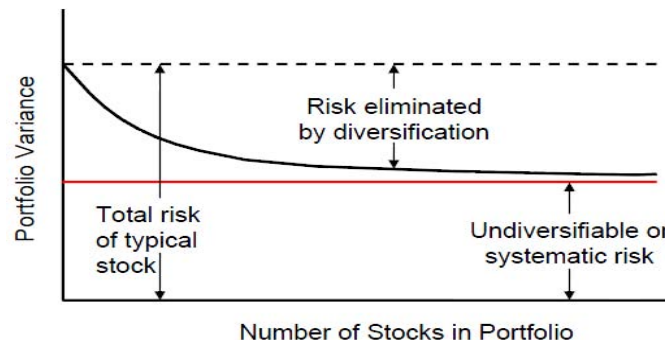
- TRUE. Investors only get rewarded for bearing the systemic risk, which is directly tied to the β of a stock.

Notice that:

$$\overbrace{\text{Var}[\tilde{r}_i]}^{\text{total risk}} = \overbrace{\beta_{iM}^2 \text{Var}[\tilde{r}_M]}^{\text{systematic risk}} + \overbrace{\text{Var}[\tilde{\varepsilon}_i]}^{\text{non-systematic risk}}$$

$$\bar{r}_i - r_{FF} = \beta_{iM}(\bar{r}_M - r_{FF}) \quad (\text{CAPM})$$

According to portfolio theory/CAPM, the idiosyncratic risk can be easily diversified and therefore it is not expected that you will get compensated for this:



Q1 – True or False

h) CAPM implies that all risky assets must have a positive risk premium.

Advice:

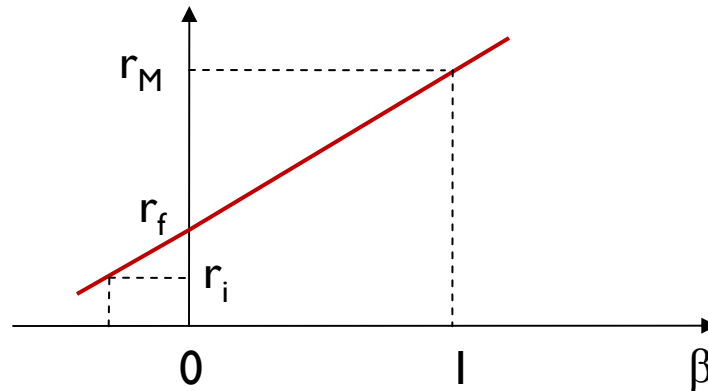
- Use the direct cue that the question is giving:
This can be explained though CAPM!

Q1 – True or False

h) CAPM implies that all risky assets must have a positive risk premium.

Answer:

- FALSE. A risky asset whose return is negatively correlated with the market has a negative beta and negative risk premium. Follow the line:



$$\text{Risk Premium} = r_i - r_f$$

Q2 – Present Value

- The annual membership fee at your health club is \$750 per year and is expected to increase at 5% per year. A life membership is \$7,500 and the discount rate is 12%. In order to justify taking the life membership, what would your minimum life expectancy need to be?

Advice:

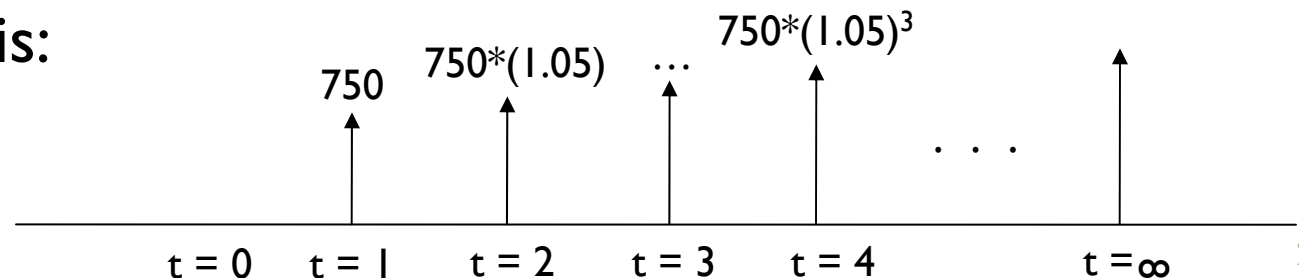
- State your assumptions
- Draw a timeline

Q2 – Present Value

- The annual membership fee at your health club is \$750 per year and is expected to increase at 5% per year. A life membership is \$7,500 and the discount rate is 12%. In order to justify taking the life membership, what would your minimum life expectancy need to be?

Answer:

- Assumption: For the finance option, the first \$750 fee membership fee payment is made after one year (that is in $t=1$, not $t=0$). Moreover, it is \$750 in $t=1$ (not $750*1.05 = \$787.5$ in $t=1$)
- This is:



Q2 – Present Value

- The annual membership fee at your health club is \$750 per year and is expected to increase at 5% per year. A life membership is \$7,500 and the discount rate is 12%. In order to justify taking the life membership, what would your minimum life expectancy need to be?

Answer:

Using the Growing Annuity formula we must solve:

$$7,500 = \frac{A}{r-g} \times \left[1 - \left(\frac{1+g}{1+r} \right)^T \right]$$

Where $A=\$750$; $g=0.05$; $r=0.12$

Solving for “T”:

$$T = \frac{\text{Ln} \left[1 - \left(\frac{7,500 \times 0.07}{750} \right) \right]}{\text{Ln} \left(\frac{1.05}{1.12} \right)}$$

This results in $T=18.65 \approx 19$ years

Q3 – Fixed Income

- The current prices of three US Treasury Bonds are as follows:

Maturity	Coupon Rate	Price
1	0%	\$97.474
2	5%	\$99.593
3	6%	\$100.148

- Assume that coupons paid yearly and all bonds have a PAR value of \$100.
- Advice: Leave at least 4 decimals for interest rates

Q3 – Fixed Income

a) What are the 1-, 2- and 3-year spot rates?

This question is always answered using:

$$B = \frac{C_1}{1+r_1} + \dots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T + P}{(1+r_T)^T}$$

Specifically,

$$B_1 = \$97.474 = \frac{\$100}{(1+r_1)^1} \Rightarrow r_1 = 2.5915\%$$

$$B_2 = \$99.593 = \frac{\$5}{(1+0.025915)^1} + \frac{\$105}{(1+r_2)^2} \Rightarrow r_2 = 5.2872\%$$

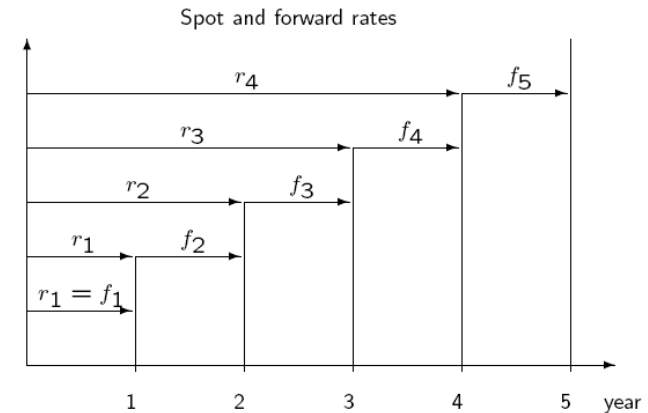
$$B_3 = \$100.148 = \frac{\$6}{(1+0.025915)^1} + \frac{\$6}{(1+0.052872)^2} + \frac{\$106}{(1+r_3)^3} \Rightarrow r_3 = 6.0447\%$$

The spot-rates are: $r_1=2.59\%$; $r_2=5.29\%$; $r_3=6.04\%$

Q3 – Fixed Income

b) What are the year 1 to 2 and year 1 to 3 forward rates?

Advice: Solve using generic formulas:



$$(1 + r_1) \times (1 + f_{1-2}) = (1 + r_2)^2 \Rightarrow f_{1-2} = \frac{(1 + r_2)^2}{(1 + r_1)} - 1 = \frac{(1.052872)^2}{1.025915} - 1$$

$$f_{1-2} = 8.0537\%$$

$$(1 + r_1) \times (1 + f_{1-3})^2 = (1 + r_3)^3 \Rightarrow f_{1-3} = \left[\frac{(1 + r_3)^3}{(1 + r_1)} \right]^{1/2} - 1 = \left[\frac{(1.060447)^3}{1.025915} \right]^{1/2} - 1$$

$$f_{1-3} = 7.8147\%$$

Q3 – Fixed Income

b) What are the year 1 to 2 and year 1 to 3 forward rates?

In the same way, we can also calculate the forward rate from year 2 to 3:

$$(1+r_2)^2 \times (1+f_{2-3}) = (1+r_3)^3 \Rightarrow f_{2-3} = \left[\frac{(1+r_3)^3}{(1+r_2)^2} \right] - 1 = \left[\frac{(1.060447)^3}{(1.052872)^2} \right] - 1$$
$$f_{2-3} = 7.5763\%$$

Q3 – Fixed Income

c) What is the price of a three-year bond with a 8% annual coupon?

Advice: Keep in mind the most basic Fixed Income formulas

- With spot rates: $B = \frac{C_1}{1+r_1} + \dots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T+P}{(1+r_T)^T}$.

- With YTM: $B = \sum_{t=1}^T \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}$.

In this case, we use the first option because we know the spot rates:

$$B = \frac{8}{(1.025915)^1} + \frac{8}{(1.052872)^2} + \frac{108}{(1.060447)^3} = \$105.5788$$

Q4 – CAPM

- Consider three stocks: Q, R and S:

	Beta	STD (annual)	Forecast for Nov 2009	
			Dividend	Stock Price
Q	0.45	35%	\$0.50	\$45
R	1.45	40%	0	\$75
S	-0.20	40%	\$1.00	\$20

- Use a risk-free rate of 2.0% and an expected market return of 9.5%. The market's standard deviation is 18%. Assume that the next dividend will be paid after one year, at $t = 1$.

Q4 – CAPM

a) According to CAPM, what is the expected rate of return of each stock:

Use basic CAPM formula:

$$\bar{r}_i = r_f + \beta \times (\bar{r}_M - r_f)$$

$$\bar{r}_Q = 2\% + 0.45 \times (9.5\% - 2\%) \Rightarrow \bar{r}_Q = 5.375\%$$

$$\bar{r}_R = 2\% + 1.45 \times (9.5\% - 2\%) \Rightarrow \bar{r}_R = 12.875\%$$

$$\bar{r}_S = 2\% + (-0.20) \times (9.5\% - 2\%) \Rightarrow \bar{r}_S = 0.5\%$$

Answer: $r_Q = 5.375\%$; $r_R = 12.875\%$; $r_S = 0.5\%$

Be aware of the difference between expected return and expected risk premium

Q4 – CAPM

b) What should today's price be for each stock, assuming CAPM is correct:

Note that:

- The expected return rate is the appropriate discount rate for each stock
- You should clarify your assumption of whether the price if quoted Nov 2009 price is before or after the dividend payment – Seems logical to assume that quoted price if immediately after the dividend payment

$$Q = \frac{0.5}{1.05375} + \frac{45}{1.05375} \Rightarrow Q = 43.1791 \approx \$43.18$$

$$R = \frac{75}{1.12875} \Rightarrow R = 66.4452 \approx \$66.45$$

$$S = \frac{1}{1.005} + \frac{20}{1.005} \Rightarrow S = 20.8955 \approx \$20.90$$

Q5 – Common Stock

- Company T's current return on equity (ROE) is 16%. It pays out one-quarter of earnings as cash dividends (payout ratio = .25). Current book value per share is \$35. The company has 5 million shares outstanding.

- Assume that ROE and payout ratio stay
- constant for the next four years. After that, competition forces ROE down to 10% and the company increases the payout ratio to 60%. The company does not plan to issue or retire shares. The cost of capital is 9.5%

Q5 – Common Stock

a) What is stock T worth?

- Advice:

- Do a year by year table until steady state is reached

- Answer:

	Year	1	2	3	4	5
Data Given	BB BVPS	\$35.00	\$39.20	\$43.90	\$49.17	\$55.07
	Investment	\$ 4.20	\$ 4.70	\$ 5.27	\$ 5.90	\$ 2.20
<u>Very useful:</u>	EB BVPS	\$39.20	\$43.90	\$49.17	\$55.07	\$57.28
	EPS	\$ 5.60	\$ 6.27	\$ 7.02	\$ 7.87	\$ 5.51
	DPS	\$ 1.40	\$ 1.57	\$ 1.76	\$ 1.97	\$ 3.30
$g = ROE \cdot b$	PV(DPS)	\$ 1.28	\$ 1.31	\$ 1.34	\$ 1.37	
	PV(TV)	\$41.79				
$p = 1 - b$	Total share price	\$47.08				
$EPS = BVPS \cdot ROE$	ROE	16%	16%	16%	16%	10%
	g	12%	12%	12%	12%	4%
	Payout Ratio	25%	25%	25%	25%	60%
$DPS = EPS \cdot p$	Plowback Ratio	75%	75%	75%	75%	40%
	Cost of Capital	9.50%				

Q5 – Common Stock

b) How much of stock T's value is attributable to growth opportunities (PVGO)?

Advice:

- Check that your answer makes sense!

Answer:

Year	1	2	3	4	5
BB BVPS	\$35.00	\$39.20	\$43.90	\$49.17	\$55.07
Investment	\$ 4.20	\$ 4.70	\$ 5.27	\$ 5.90	\$ 2.20
EB BVPS	\$39.20	\$43.90	\$49.17	\$55.07	\$57.28
EPS	\$ 5.60	\$ 6.27	\$ 7.02	\$ 7.87	\$ 5.51
DPS	\$ 1.40	\$ 1.57	\$ 1.76	\$ 1.97	\$ 3.30
PV(DPS)	\$ 1.28	\$ 1.31	\$ 1.34	\$ 1.37	
PV(TV)	\$41.79				
Total share price	\$47.08				
ROE	16%	16%	16%	16%	10%
g	12%	12%	12%	12%	4%
Payout Ratio	25%	25%	25%	25%	60%
Plowback Ratio	75%	75%	75%	75%	40%
Cost of Capital	9.50%				

PVGO $\$ (11.87)$ } $PVGO = Price - \frac{EPS}{r}$

If we calculate PVGO as Price – EPS/r, we obtain a negative PVGO!

How come if ROE > cost of capital at any time?

Q5 – Common Stock

- ROE on current assets decreases over time (16% → 10%), which means that, by taking current ROE, we overestimate the value of assets in place.
- Recall that Stock Price Can Be Decomposed Into Two Components
 - Present value of earnings under steady state (usually no growth)
 - Present value of growth opportunities
- The first component is equal to $\$35 * 10\% / 9.5\% = \36.84
- “Real” PVGO = $\$47.08 - \$36.84 = \underline{\$10.24}$

Final Answers:

- a) Price = \$47.08
- b) PVGO = \$10.24

Q6 – Forwards & Futures

- Spot and futures prices for Gold and the S&P in September 2007 are given below.

	07-September	07-December	08-June
COMEX Gold (\$/oz)	\$693	\$706.42	\$726.7
CME S&P 500	\$1453.55	\$1472.4	\$1493.7

Table 1: Gold and S&P 500 Prices on September 7, 2007

- a) Use prices for gold to calculate the effective annualized interest rate for Dec 2007 and June 2008. Assume that the convenience yield for gold is zero.

Advice:

- Read carefully: Note the dates!
- Remember how to “annualize” monthly effective rates

Q6 – Forwards & Futures

- a) Use prices for gold to calculate the effective annualized interest rate for Dec 2007 and June 2008. Assume that the convenience yield for gold is zero.

Answer:

The price of futures contract is related to the spot price by the following equation (spot-future parity):

$$F = S_0(1 + r)^T$$

For 7-Dec contract we have:

$$\$706.42 = \$693 (1 + r_{3\text{-months}})^{1/4} \rightarrow r_{3\text{-months}} = 7.97\%$$

Similarly, for the 8-Jun contract we have:

$$\$726.7 = \$693 (1 + r_{9\text{-months}})^{3/4} \rightarrow r_{9\text{-months}} = 6.54\%$$

Q6 – Forwards & Futures

b) Suppose you are the owner of a gold mine and would like to fix the revenue generated by your future production. Explain how the future market enables such hedges.

Answer:

You are naturally “long gold”. So, enter into a contract to sell your Dec 2007 production at the price of \$706.42/oz and your June 2008 production at \$726.70/oz. You can lock-in now the price at which you sell your future production!

Q6 – Forwards & Futures

c) Calculate the convenience yield on the S&P index between September 07 and December 07:

Assumption:

- $r_f = 7.97\%$ (annual risk-free interest rate calculated in part “a”, which related to the same period)

Answer:

$$F = S_0 \times (1 + r_f - d)^{1/4}$$

$$1,472.40 = 1,453.55 \times (1 + 0.0797 - d)^{1/4}$$

$$d = 1 + 0.0797 - \left(\frac{1,472.40}{1,453.55} \right)^4 = 0.02681$$

$$d = 2.68\%$$

The convenience yield is 2.68%

Q7 – Capital Budgeting

- You have developed the technology to use gold to produce high capacity fiber optic switches. The technology has cost \$ 5 million to develop. You need \$50 million of initial capital investment to start production. Sales of the switch sales will be \$20 million per year for the next 5 years and then drop to zero. The main cost of production is gold. Each year, you need 20,000 ounces of gold. Gold is currently selling for \$250 per ounce. Your supplier thinks that the gold price will appreciated at 5% per year for the next 5 years. The cost of capital is 10% for the fiber-optics business. The tax rate is 35%. The capital investment can be depreciated linearly over the next 5 years.

a) Calculate the after-tax cash flows of the project

Advice:

- Use a year-by-year table

Q7 – Capital Budgeting

Answer: Figures expressed in \$MM:

		Yr 0	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
CAPEX:	A	-50					
Revenues:	B		20	20	20	20	20
COGS:	C		-5	-5.25	-5.51	-5.79	-6.08
Depreciation:	D		-10	-10	-10	-10	-10
Net Income:	$E=B+C+D$		5	4.75	4.49	4.21	3.92
Tax:	$F=-0.35*E$		-1.75	-1.66	-1.57	-1.47	-1.38
TOTAL CF:	$A+B+C+F$	-50	13.25	13.09	12.92	12.74	12.55

Note:

- Sunk cost of \$5 MM doesn't matter!
- For Finance, we only care about After-Tax Cash Flow

Q7 – Capital Budgeting

b) Should you take the project?

Answer: Discount the factors at the appropriate discount rate:

	Yr 0	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
Non-Discounted CF:	-50	13.25	13.09	12.92	12.74	12.55
Discount Factor	1.10^0	1.10^1	1.10^2	1.10^3	1.10^4	1.10^5
Discounted CF	-50	12.05	10.82	9.70	8.70	7.79

NPV = Sum of Discounted CFs = \$-0.94 MM

Answer: NPV < 0 → Do not take the project!

Q8 – Portfolio Theory/CAPM

- It is November, 2007. The following variance-covariance matrix, for the market (S&P 500) and stocks T and U, is based on monthly data from November 2002 to October 2007. Assume T and U are included in the S&P 500. The betas for T and U are $T = 0.727$ and $U = 0.75$.

	<i>S&P500</i>	T	U
<i>S&P500</i>	0.0256	0.0186	0.0192
T	0.0186	0.1225	0.0262
U	0.0192	0.0262	0.0900

- Average monthly risk premiums from 2002 to 2007 were:
 - S&P500 : 1.0%
 - T : 0.6%
 - U : 1.1%
- Assume the CAPM is correct, and that the S&P is a good proxy for the market. Assume that that the expected future market risk premium is 0.6% and the risk-free interest rate is 0.3% per month.

Q8 – Portfolio Theory/CAPM

Advice:

- Ask yourself whether the variance-covariance matrix includes the weights (“w”) or not. In this case, it doesn’t
 - Also note that **Risk Premiums** are given here
- a) What were the alphas for stocks T and U over the last 60 months?

Answer:

Using the CAPM regression formula:

$$\alpha_i = (\bar{r}_i - r_F) - \beta_{estimated}(\bar{r}_M - r_F)$$

Therefore:

$$\alpha_T = 0.6 - 0.727 \times 1 = -0.127\%$$

$$\alpha_U = 1.1 - 0.75 \times 1 = 0.35\%$$

Q8 – Portfolio Theory/CAPM

b) What are the expected future rates of return for T and U?

Answer:

Using the CAPM regression formula:

$$\bar{r}_i = r_f + \beta \times (\bar{r}_M - r_f)$$

$$\bar{r}_T = 0.3\% + 0.727 \times (1\%) \Rightarrow \bar{r}_T = 1.027\%$$

$$\bar{r}_U = 0.3\% + 0.750 \times (1\%) \Rightarrow \bar{r}_U = 1.050\%$$

Note:

- We used the Market risk premium 1% based on the return from 2002 to 2007. You could have also use the projected return of 0.6% given above, depending on which one you have more confidence in. Be clear on the exam and you will get full credit

Q8 – Portfolio Theory/CAPM

- c) What are the optimal portfolio weights for the S&P 500, T and U? Explain qualitatively

Answer:

According to the Theory in Finance I, all rationale investors should always invest in a point along the Capital Market Line (CML.) That is, in a combination of the risk-free assets and the market portfolio.

In this case, the S&P 500 is the proxy we have for the market portfolio, and the question does not give the option of investing in the risk-free asset.

Therefore, invest all the money in the S&P 500. The point is that S&P 500 already includes some weight of T&U, just in the correct proportion that a rational investor would want to have !

Q9 – Portfolio Theory

- Expected returns and standard deviations of three risky assets are as follows:

	Expected Return	Standard Deviation	Correlations		
			A	B	C
A	11%	30%	1.0	.3	.15
B	14.5%	45%	.3	1.0	.45
C	9%	30%	.15	.45	1.0

Advice:

- Ask yourself what exactly is expressed in the matrix. In this case it is just the correlations! – not the variances-covariances

Q9 – Portfolio Theory

- a) Calculate the expected return and standard deviation of a portfolio of stocks A, B and C. Assume an equal investment in each stock.

Answer: Using the two basic portfolio theory formulas:

$$\bar{r}_P = (w_A \times r_A) + (w_B \times r_B) + (w_C \times r_C)$$
$$\sigma_P^2 = (w_A^2 \times \sigma_A^2) + (w_B^2 \times \sigma_B^2) + (w_C^2 \times \sigma_C^2) + (2 \times w_A \times w_B \times \sigma_{AB}) + (2 \times w_A \times w_C \times \sigma_{AC}) + (2 \times w_B \times w_C \times \sigma_{BC})$$

We will also use... $\sigma_{xy} = \rho_{xy} \times \sigma_x \times \sigma_y$

... To get:

$$\sigma_P^2 = (w_A^2 \times \sigma_A^2) + (w_B^2 \times \sigma_B^2) + (w_C^2 \times \sigma_C^2) + (2 \times w_A \times w_B \times \rho_{AB} \times \sigma_A \times \sigma_B) + (2 \times w_A \times w_C \times \rho_{AC} \times \sigma_A \times \sigma_C) + (2 \times w_B \times w_C \times \rho_{BC} \times \sigma_B \times \sigma_C)$$

Q9 – Portfolio Theory

Substituting in these equations we obtain:

$$\bar{r}_p = \left(\frac{1}{3} \times 11\%\right) + \left(\frac{1}{3} \times 14.5\%\right) + \left(\frac{1}{3} \times 9\%\right) \Rightarrow \bar{r}_p = 11.5\%$$

$$\begin{aligned} \sigma_p^2 &= \left(\frac{1^2}{3} \times 0.30^2\right) + \left(\frac{1^2}{3} \times 0.45^2\right) + \left(\frac{1^2}{3} \times 0.30^2\right) + \\ &+ \left(2 \times \frac{1}{3} \times \frac{1}{3} \times 0.30 \times 0.30 \times 0.45\right) + \left(2 \times \frac{1}{3} \times \frac{1}{3} \times 0.15 \times 0.30 \times 0.30\right) + \\ &+ \left(2 \times \frac{1}{3} \times \frac{1}{3} \times 0.45 \times 0.45 \times 0.30\right) \\ &\Rightarrow \sigma_p^2 = 0.068017 \end{aligned}$$

$$\sigma_p = \sqrt{0.068017} \Rightarrow \sigma_p = 0.2608 = 26.08\%$$

Answer: $E[r_p] = 11.5\%$; $\sigma_p = 26.08\%$

Q9 – Portfolio Theory

b) Compute the Sharpe ratio of a portfolio that has 30% in A, 30% in B and 40% in C:

Assumption: $r_f = 4\%$

Answer: Using the same procedure as in part “a” we obtain:

$$\bar{r}_p = 11.25\%$$

$$\sigma_p = 25.66\%$$

Therefore the Sharpe Ratio is:

$$SR = \frac{\bar{r}_p - r_f}{\sigma_p} = \frac{11.25\% - 4\%}{25.66\%}$$

$$SR = 0.282$$

Q9 – Portfolio Theory

c) Assume a portfolio of asset B and C. Determine the weight in asset B, such that the total portfolio risk is minimized:

Answer:

Let $(w, 1-w)$ be the weights for (B, C), then

$$\arg \min_w \left[w^2 \cdot 0.45^2 + (1-w)^2 \cdot 0.30^2 + 2w(1-w)(0.45)(0.45)(0.30) \right]$$

First-order condition:

$$2w \cdot 0.45^2 - 2 \cdot 0.30^2 + 2w \cdot 0.30^2 + 2 \cdot 0.06075 - 4w \cdot 0.06075 = 0$$

$$w^* = 0.171$$

The minimum variance portfolio is:

$$\underline{w_B = 17.1\%} ; \underline{w_C = 82.9\%}$$

Q10 – Options

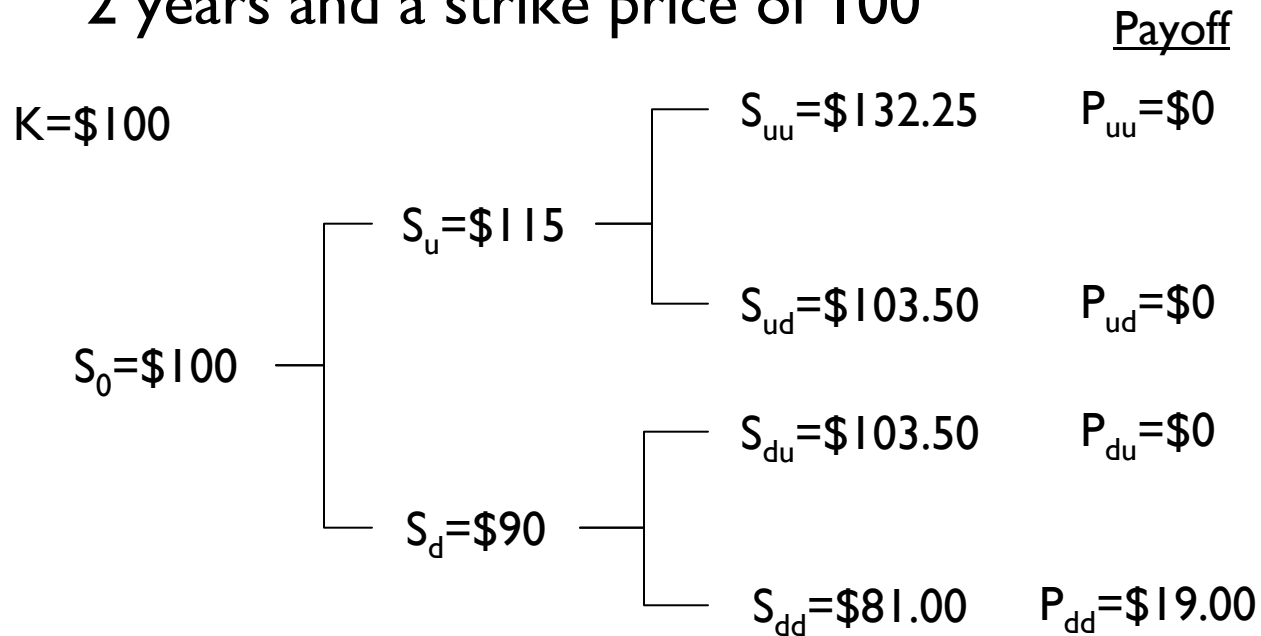
- You are asked to price options on KYC stock. KYC's stock price can go up by 15 percent every year, or down by 10 percent. Both outcomes are equally likely. KYC does not pay dividend. The risk free rate is 5% (EAR), and the current stock price of KYC is \$100.
- a) Price a European put option on KYC with maturity of 2 years and a strike price of 100

Advice:

- Be aware the given probabilities of going up or down (in this case equal probabilities) are irrelevant
- Solve by whatever method is easier for you – e.g. risk-neutral probability or the direct replication approach.
- If you solve with the binomial model (like we will), draw the tree!

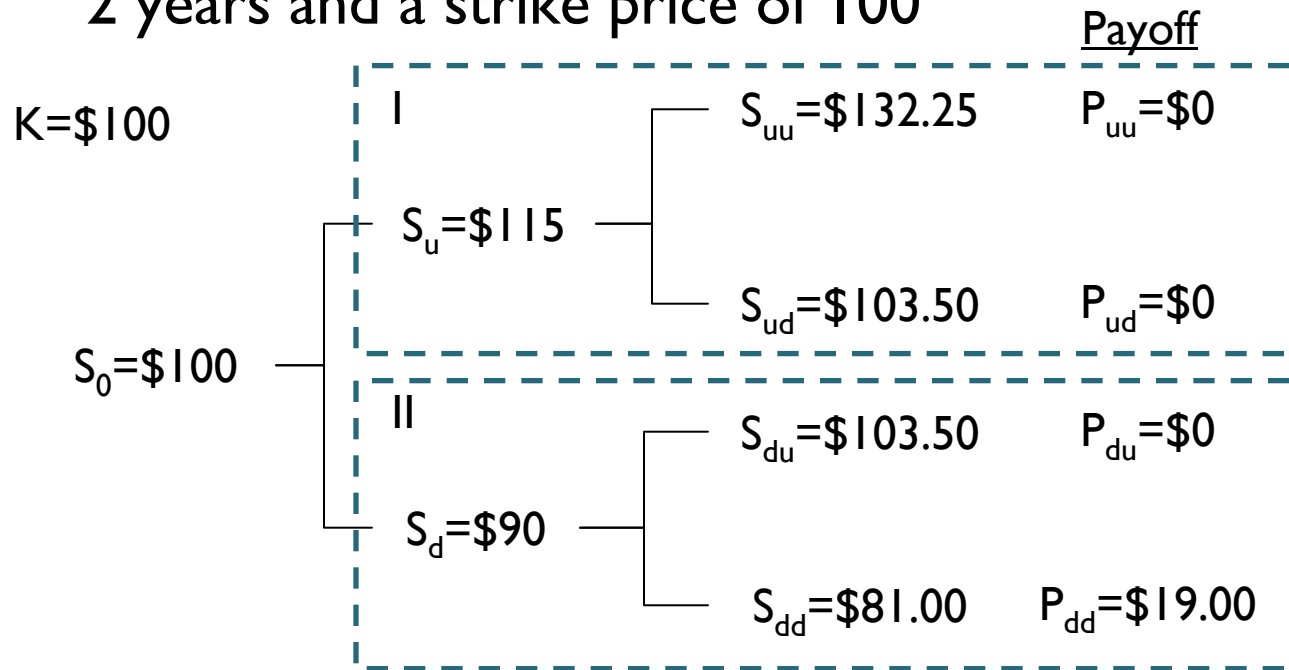
Q10 – Options

- a) Price a European put option on KYC with maturity of 2 years and a strike price of 100



Q10 – Options

- a) Price a European put option on KYC with maturity of 2 years and a strike price of 100



$$I \begin{cases} 132.25a + 1.05b = 0 \\ 103.50a + 1.05b = 0 \end{cases}$$

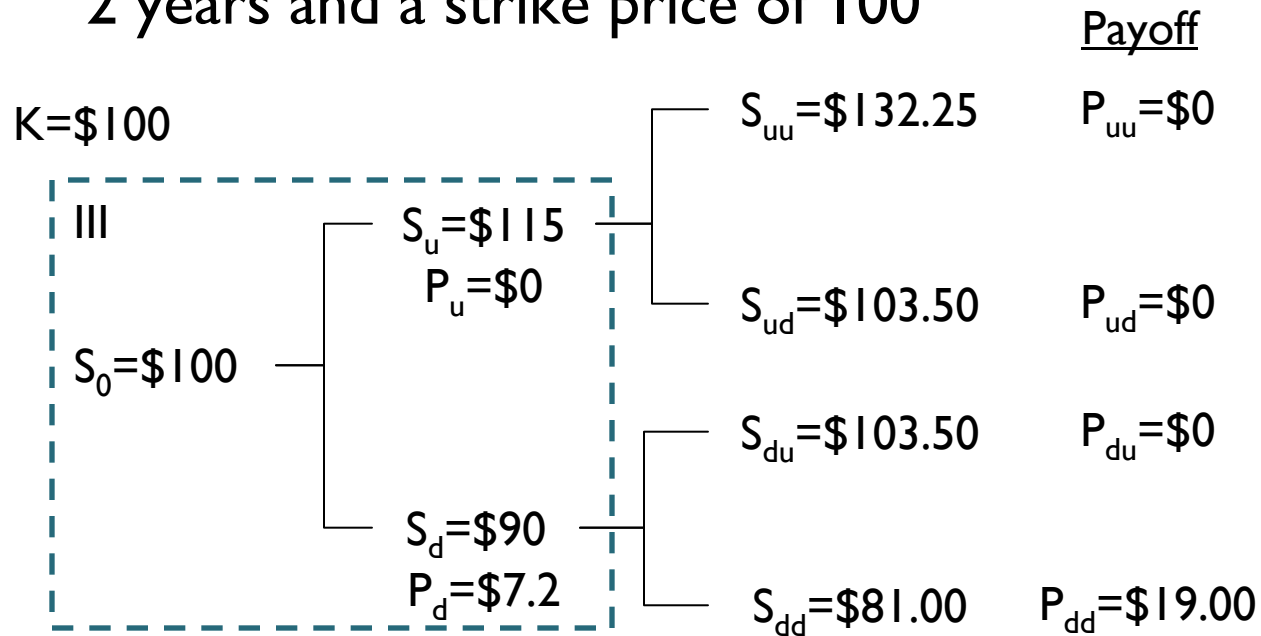
$$\left. \begin{matrix} a = 0 \\ b = 0 \end{matrix} \right\} P_u = 115a + b = 0$$

$$II \begin{cases} 103.50a + 1.05b = 0 \\ 81.00a + 1.05b = 19 \end{cases}$$

$$\left. \begin{matrix} a = -0.84 \\ b = 83.2 \end{matrix} \right\} P_d = 90a + b = 7.2$$

Q10 – Options

- a) Price a European put option on KYC with maturity of 2 years and a strike price of 100



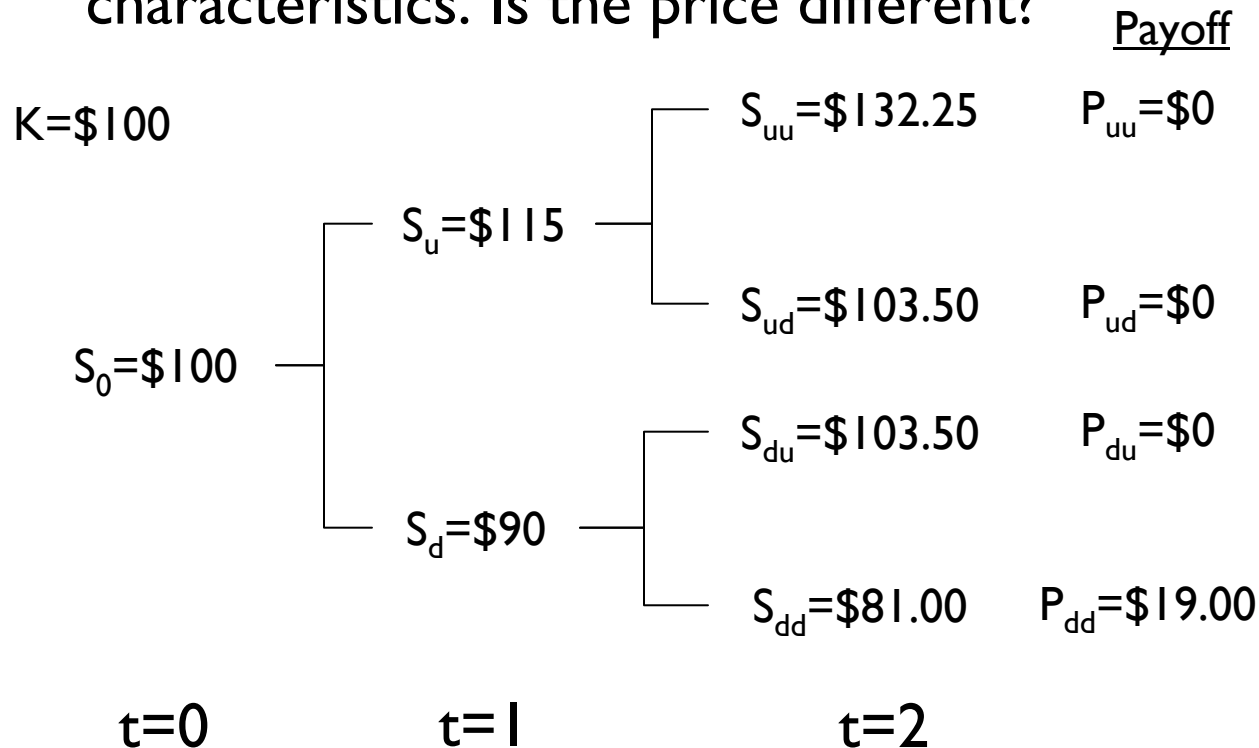
$$\text{III } \begin{cases} 115a + 1.05b = 0 \\ 90a + 1.05b = 7.2 \end{cases}$$

$$\left. \begin{matrix} a = -0.29 \\ b = 31.76 \end{matrix} \right\} P_d = 100a + b = \$2.76$$

Answer: The option is worth **\$2.76**

Q10 – Options

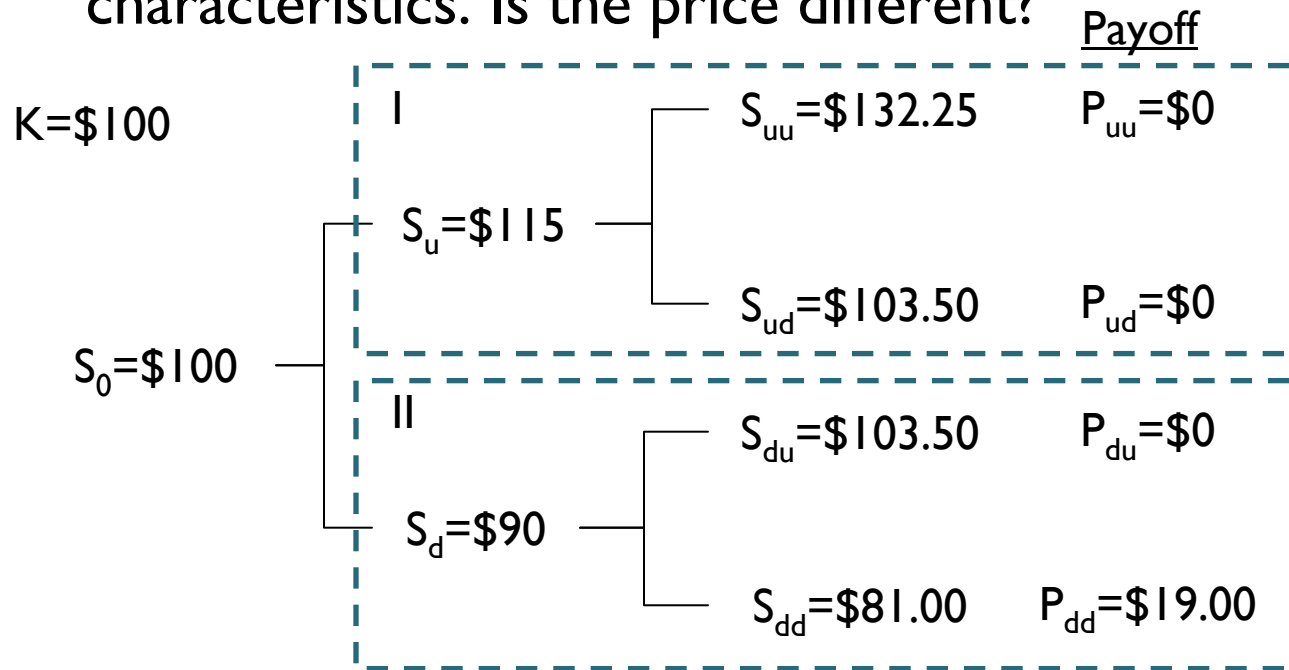
b) Price an American put option on KYC with the same characteristics. Is the price different?



Note: For every nodes before $t=2$ we will question whether it is better to keep the option or to exercise it

Q10 – Options

b) Price an American put option on KYC with the same characteristics. Is the price different?



$$I \quad \begin{cases} 132.25a + 1.05b = 0 \\ 103.50a + 1.05b = 0 \end{cases}$$

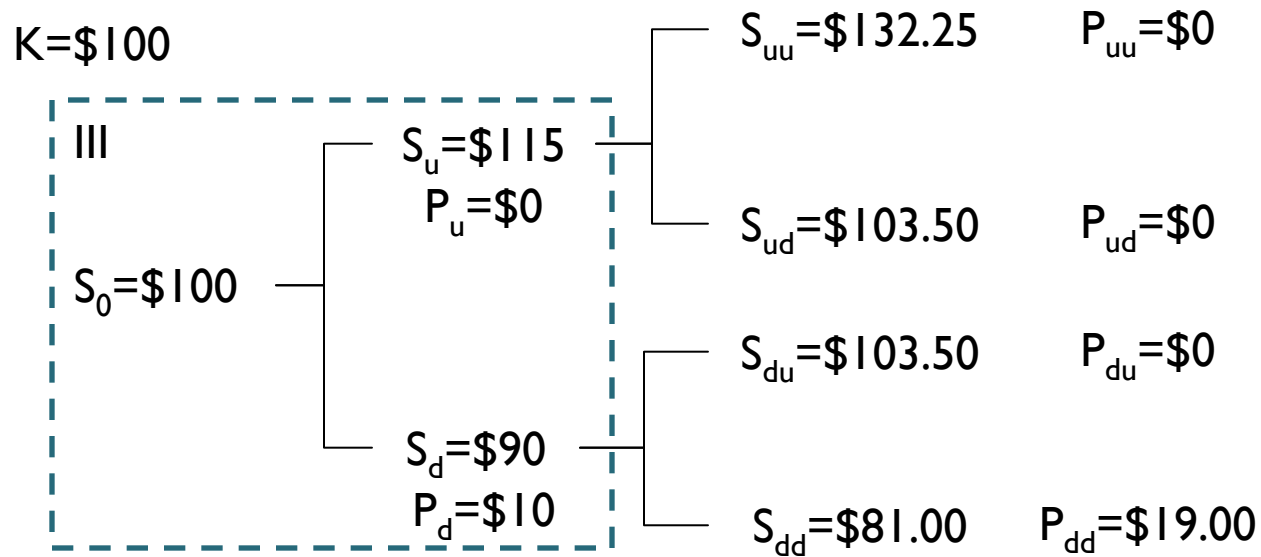
$$\left. \begin{array}{l} a = 0 \\ b = 0 \end{array} \right\} \begin{array}{l} P_u = 115a + b = 0 \\ \text{If we exercise, also } 0 \end{array}$$

$$II \quad \begin{cases} 103.50a + 1.05b = 0 \\ 81.00a + 1.05b = 19 \end{cases}$$

$$\left. \begin{array}{l} a = -0.84 \\ b = 83.2 \end{array} \right\} \begin{array}{l} P_d = 90a + b = 7.2, \text{ but now if we} \\ \text{exercise} = 10 > 7.2 \rightarrow P_d = \$10 \end{array}$$

Q10 – Options

b) Price an American put option on KYC with the same characteristics. Is the price different? Payoff



$$\text{III } \begin{cases} 115a + 1.05b = 0 \\ 90a + 1.05b = 10 \end{cases}$$

$$\begin{cases} a = -0.40 \\ b = 43.80 \end{cases} \left. \vphantom{\begin{cases} a \\ b \end{cases}} \right\} P_d = 100a + b = \$3.80, \text{ better than exercise it for } \$0$$

Answer: The option is now worth \$3.80... Because if it gets to S_d we would exercise it for \$10

Q10 – Options

c) Given the price of the put option that you calculated in a), specify the ranges of KYC share price at the option's maturity date for which you will be making a net profit

Advice:

- Remember to mention the time value of money

Answer:

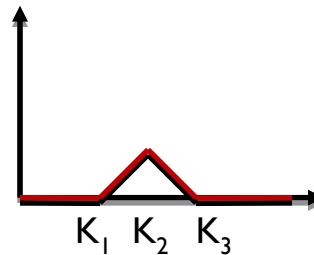
- At $t=2$, the option will be in the money for any price of S between \$0 and \$100 (European Put, $K=\$100$)
- However, in $t=0$ we paid \$2.76. Therefore, we would be making profits between \$0 and \$97.24 ($100-2.76$)
- ... But the initial \$2.76 after two years are worth:
 $2.76 \times (1.05)^2 = \$3.04$
- Answer: We will make a **net profit** in **[\$0**

Q10 – Options

d) Suppose you expect the price of KYC stock to have little variance in the future. How would you design a strategy (using options) to take advantage of this?

Answer:

- You think that the price of the asset will not change significantly from the expected value...
- ... Therefore, use a “butterfly” spread:





THANK YOU & GOOD LUCK!!!

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