15.093 - Recitation 5

Michael Frankovich and Shubham Gupta

October 9, 2009

1 BT Exercise 5.5

Solution

The tableau is:

	0	0	$\bar{c}_3$	0	$\bar{c}_5$
1	0	1	-1	0	β
2	0	0	2	1	γ
3	1	0	4	0	δ

a) The necessary and sufficient conditions for optimality are $\bar{c}_3 \geq 0$ and $\bar{c}_5 \geq 0$.

b) Continuing the simplex method, with x_3 the entering variable, x_1 will leave the basis. In the new tableau, the optimal bfs is obtained;

	0	0	0	0	$\bar{c}_5$
7/4	1/4	1	0	0	$\beta + \delta/4$
1/2	-1/2	0	0	1	$\gamma - \delta/2$
3/4	1/4	0	1	0	$\delta/4$

c) If $\bar{c}_3 \geq 0$ and $\bar{c}_5 \geq 0$, then the current solution is optimal. Now consider the case when $\bar{c}_3 < 0$ or $\bar{c}_5 < 0$. Note any feasible solution must satisfy $Ax = b$, $x \geq 0$ and so $B^{-1}Ax = B^{-1}b$ for any basis B. Hence we read the followign three equations from the tableau:

$$x_2 - x_3 + \beta x_5 = 1 \quad (1)$$
$$2x_3 + x_4 + \gamma x_5 = 2 \quad (2)$$
$$x_1 + 4x_3 + \delta x_5 = 3 \quad (3)$$

Eqn (2) tells us $x_3,\ x_4$ and x_5 are bounded, then eqns (1) and (3) tell us x_2 and x_1, respectively, are bounded. So the polyehdron is bounded and so has an optimal cost, since it is nonempty.

d) The current basis is optimal. B^{-1} is the last three columns of the tableau. Why? We need to ensure primal feasibility is maintained. We require $B^{-1}(b + \epsilon e_1) = B^{-1}b + \epsilon B^{-1}e_1 = (1, 2, 3)' + \epsilon(-1, 2, 4)' \geq 0$, which occurs iff $-3/4 \leq \epsilon \leq 1$.

e) Note that x_1 is the third basic variable. So we have then that the new $\hat{c}_B = c_B + \epsilon e_3$. Feasibility is not affected. The optimality condition is $\hat{c} - \hat{c}'_B B^{-1}A = c' + \epsilon e'_1 - c'_B B^{-1}A - \epsilon e'_3 B^{-1}A = \bar{c}' + \epsilon e'_1 - \epsilon(1, 0, 4, 0, \delta) = \bar{c}' - \epsilon(0, 0, 4, 0, \delta) \geq 0$. So we require

$$
\begin{aligned}
&\epsilon \leq \bar{c}_3/4, \\
&\epsilon \leq \bar{c}_5/\delta, \quad \delta > 0, \\
&\epsilon \geq \bar{c}_5/\delta, \quad \delta < 0.
\end{aligned}
$$

2 Dantzig-Wolfe Decomposition

See Bertsimas and Tsitsklis, chapter 6.

15.093J / 6.255J Optimization Methods
Fall 2009