15.093 Optimization Methods

Lecture 19: Line Searches
and Newton's Method

1 Last Lecture

Slide 1

- *Necessary Conditions for Optimality*
  (identifies candidates)
  x^* local min $\Rightarrow \nabla f(x^*) = 0$, $\nabla^2 f(x^*)$ PSD
- *Sufficient Conditions for Optimality*
  $\nabla f(x^*) = 0$ $\nabla^2 f(x)$ psd $x \in B_\epsilon(x^*) \Rightarrow x^*$ loc. min.
- *Characterizations of Convexity*
  a. f convex $\Leftrightarrow f(y) \geq f(x) + \nabla f(x)'(y - x)$ $\forall x, y$
  b. f convex $\Leftrightarrow \nabla^2 f(x)$ PSD $\forall x$
- f convex then global min $\Leftrightarrow \nabla f(x^*) = 0$

2 Steepest descent

2.1 The algorithm

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Step 0 Given $\boldsymbol{x}^0$, set $k := 0$.

Step 1 $\boldsymbol{d}^k := -\nabla f(\boldsymbol{x}^k)$. If $||\boldsymbol{d}^k|| \leq \epsilon$, then stop.

Step 2 Solve $\min_\lambda h(\lambda) := f(\boldsymbol{x}^k + \lambda \boldsymbol{d}^k)$ for the step-length λ^k, perhaps chosen by an exact or inexact line-search.

Step 3 Set $\boldsymbol{x}^{k+1} \leftarrow \boldsymbol{x}^k + \lambda^k \boldsymbol{d}^k$, $k \leftarrow k + 1$.
Go to **Step 1**.

3 Outline

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1. Bisection Method - Armijo's Rule
2. Motivation for Newton's method
3. Newton's method
4. Quadratic rate of convergence
5. Modification for global convergence

4 Choices of step sizes

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- $Min_\lambda f(x^k + \lambda d^k)$
- Limited Minimization: $Min_{\lambda \in [0,s]} f(x^k + \lambda d^k)$
- Constant stepsize $\lambda^k = s$ constant

- Diminishing stepsize: $\lambda^k \to 0,\ \sum_k \lambda^k = \infty$
- Armijo Rule

4.1 Bisection Line- Search Algorithm

4.1.1 Convex functions

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$$\bar{\lambda} := \arg\min_{\lambda} h(\lambda) := \arg\min_{\lambda} f(\bar{\boldsymbol{x}} + \lambda \bar{\boldsymbol{d}})$$

If $f(\boldsymbol{x})$ is convex, $h(\lambda)$ is convex.

Find $\bar{\lambda}$ for which $h'(\lambda) = 0$

$$h'(\lambda) = \nabla f(\bar{\boldsymbol{x}} + \lambda \bar{\boldsymbol{d}})' \, \bar{\boldsymbol{d}}$$

4.1.2 Algorithm

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Step 0. Set $k = 0$. Set $\lambda_L := 0$ and $\lambda_U := \hat{\lambda}$.

Step k. Set $\tilde{\lambda} = \dfrac{\lambda_U + \lambda_L}{2}$ and compute $h'(\tilde{\lambda})$.
If $h'(\tilde{\lambda}) > 0$, re-set $\lambda_U := \tilde{\lambda}$. Set $k \leftarrow k + 1$.
If $h'(\tilde{\lambda}) < 0$, re-set $\lambda_L := \tilde{\lambda}$. Set $k \leftarrow k + 1$.
If $h'(\tilde{\lambda}) = 0$, stop.

4.1.3 Analysis

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- At the kth iteration of the bisection algorithm, the current interval $[\lambda_L, \lambda_U]$ must contain a point $\bar{\lambda}$: $h'(\bar{\lambda}) = 0$
- At the k^{th} iteration of the bisection algorithm, the length of the current interval $[\lambda_L, \lambda_U]$ is

$$\text{length} = \left(\frac{1}{2}\right)^k (\hat{\lambda}).$$

- A value of λ such that $|\lambda - \bar{\lambda}| \leq \epsilon$ can be found in at most

$$\left\lceil \log_2 \left(\frac{\hat{\lambda}}{\epsilon}\right) \right\rceil$$

steps of the bisection algorithm.

4.1.4 Example

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$$\begin{aligned} h(\lambda) &= (x_1 + \lambda d_1) - 0.6(x_2 + \lambda d_2) + 4(x_3 + \lambda d_3) + \\ &\quad +0.25(x_4 + \lambda d_4) - \sum_{i=1}^{4} \log(x_i + \lambda d_i) - \\ &\quad - \log\left((5 - \sum_{i=1}^{4} x_i - \lambda \sum_{i=1}^{4} d_i\right) \end{aligned}$$

where $(x_1, x_2, x_3, x_4) = (1, 1, 1, 1)'$ and $(d_1, d_2, d_3, d_4) = (-1, 0.6, -4, -0.25)'$. SLIDE 9

$$\begin{aligned} h(\lambda) &= 4.65 - 17.4225\lambda - \log(1 - \lambda) - \log(1 + 0.6\lambda) - \\ &\quad - \log(1 - 4\lambda) - \log(1 - 0.25\lambda) - \log(1 + 4.65\lambda) \end{aligned}$$

and

$$\begin{aligned} h'(\lambda) &= -17.4225 + \frac{1}{1 - \lambda} - \frac{0.6}{1 + 0.6\lambda} + \frac{4}{1 - 4\lambda} + \\ &\quad + \frac{0.25}{1 - 0.25\lambda} - \frac{4.65}{1 + 4.65\lambda} \end{aligned}$$

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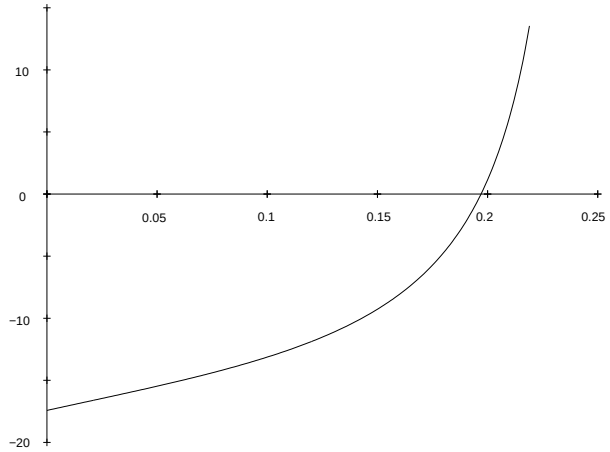

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k	λ_l^k	λ_u^k	$h'(\tilde{\lambda})$
1	0.0000000	1.0000000	NaN
2	0.0000000	0.5000000	NaN
3	0.0000000	0.2500000	−11.520429338348
4	0.1250000	0.2500000	−2.952901763683
5	0.1875000	0.2500000	13.286386294218
6	0.1875000	0.2187500	2.502969022220
7	0.1875000	0.2031250	−0.605144021505
8	0.1953125	0.2031250	0.831883373151
9	0.1953125	0.1992188	0.087369215988
10	0.1953125	0.1972656	−0.265032213496

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k	λ_l^k	λ_u^k	$h'(\bar{\lambda})$
20	0.1970253	0.1970272	−0.000184301091
30	0.1970268	0.1970268	−0.000000146531
40	0.1970268	0.1970268	0.000000000189
41	0.1970268	0.1970268	0.000000000023
42	0.1970268	0.1970268	−0.000000000059
43	0.1970268	0.1970268	−0.000000000018
44	0.1970268	0.1970268	0.000000000003
45	0.1970268	0.1970268	−0.000000000008
46	0.1970268	0.1970268	−0.000000000002
47	0.1970268	0.1970268	0.000000000000

4.2 Armijo Rule

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Bisection is accurate but may be expensive in practice
Need cheap method guaranteeing sufficient accuracy
Inexact line search method. Requires two parameters: $\epsilon \in (0,1)$, $\sigma > 1$.
$\bar{h}(\lambda) = h(0) + \lambda\epsilon h'(0)$
$\bar{\lambda}$ acceptable by Armijo's rule if:
• $h(\bar{\lambda}) \leq \bar{h}(\bar{\lambda})$
• $h(\sigma\bar{\lambda}) \geq \bar{h}(\sigma\bar{\lambda})$ (prevents the step size be small)
(i.e. $f(x^k + \bar{\lambda}d^k) \leq f(x^k) + \bar{\lambda}\epsilon\nabla f(x^k)'d^k$)

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We get a range of acceptable stepsizes.
Step 0: Set $k = 0$, $\lambda^0 = \bar{\lambda} > 0$
Step k: If $h(\lambda^k) \leq \bar{h}(\lambda^k)$, choose λ^k stop. If $h(\lambda^k) > \bar{h}(\lambda^k)$ let $\lambda^{k+1} = \frac{1}{\sigma}\lambda^k$
(for example, $\sigma = 2$)

5 Newton's method

5.1 History

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Steepest Descent is simple but slow
Newton's method complex but fast
Origins not clear

Raphson became member of the Royal Society in 1691 for his book "Analysis Aequationum Universalis" with Newton method.
Raphson published it 50 years before Newton.

6 Newton's method

6.1 Motivation

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Consider

$$\min f(\boldsymbol{x})$$

- Taylor series expansion around $\bar{\boldsymbol{x}}$

$$f(\boldsymbol{x}) \approx g(\boldsymbol{x}) = f(\bar{\boldsymbol{x}}) + \nabla f(\bar{\boldsymbol{x}})'(x - \bar{x}) +$$

$$\frac{1}{2}(x - \bar{x})'\nabla^2 f(\bar{x})(x - \bar{x})$$

- Instead of $\min f(\boldsymbol{x})$, solve $\min g(\boldsymbol{x})$, i.e., $\nabla g(\boldsymbol{x}) = \mathbf{0}$
- $$\nabla f(\bar{\boldsymbol{x}}) + \nabla^2 f(\bar{\boldsymbol{x}})(\boldsymbol{x} - \bar{\boldsymbol{x}}) = \mathbf{0}$$

$$\Rightarrow \boldsymbol{x} - \bar{\boldsymbol{x}} = -(\nabla^2 f(\bar{\boldsymbol{x}}))^{-1}\nabla f(\bar{\boldsymbol{x}})$$

- The direction $\boldsymbol{d} = -(\nabla^2 f(\bar{\boldsymbol{x}}))^{-1}\nabla f(\bar{\boldsymbol{x}})$ is the Newton direction

6.2 The algorithm

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Step 0 Given $\boldsymbol{x}^0$, set $k := 0$.

Step 1 $\boldsymbol{d}^k := -(\nabla^2 f(\boldsymbol{x}^k))^{-1}\nabla f(\boldsymbol{x}^k)$.
If $||\boldsymbol{d}^k|| \leq \epsilon$, then stop.

Step 2 Set $\boldsymbol{x}^{k+1} \leftarrow \boldsymbol{x}^k + \boldsymbol{d}^k$, $k \leftarrow k + 1$.
Go to **Step 1**.

6.3 Comments

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- The method assumes that $\nabla^2 f(\boldsymbol{x}^k)$ is nonsingular at each iteration
- There is no guaranteee that $f(\boldsymbol{x}^{k+1}) \leq f(\boldsymbol{x}^k)$
- We can augment the algorithm with a line-search:

$$\min_{\lambda} f(\boldsymbol{x}^k + \lambda \boldsymbol{d}^k)$$

6.4 Properties

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Theorem If $\boldsymbol{H} = \nabla^2 f(\boldsymbol{x}^k)$ is PD, then $\boldsymbol{d}^k$ is a descent direction: $\nabla f(\boldsymbol{x}^k)'\boldsymbol{d}^k < 0$
Proof:

$$\nabla f(\boldsymbol{x}^k)'\boldsymbol{d}^k = -\nabla f(\boldsymbol{x}^k)'\boldsymbol{H}^{-1}\nabla f(\boldsymbol{x}^k) < 0$$

if $\boldsymbol{H}^{-1}$ is PD. But,

$$0 < \boldsymbol{v}'(\boldsymbol{H}^{-1})'\boldsymbol{H}\boldsymbol{H}^{-1}\boldsymbol{v} = \boldsymbol{v}'\boldsymbol{H}^{-1}\boldsymbol{v}$$

$$\Rightarrow \boldsymbol{H}^{-1} \text{ is PD}$$

6.5 Example 1

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$$f(x) = 7x - \log x, \qquad x^* = \frac{1}{7} = 0.14857143$$

$$f'(x) = 7 - \frac{1}{x}, \qquad f''(x) = \frac{1}{x^2}$$

$$\Rightarrow d = -(f''(x))^{-1} f'(x) = -\left(\frac{1}{x^2}\right)^{-1}\left(7 - \frac{1}{x}\right) = x - 7x^2$$

$$x^{k+1} = x^k + \left(x^k - 7(x^k)^2\right) = 2x^k - 7(x^k)^2$$

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k	x^k	x^k	x^k	x^k
0	1	0	0.01	0.1
1	-5	0	0.0193	0.13
2	-185	0	0.03599	0.1417
3	-239945	0	0.062917	0.14284777
4	-4E11	0	0.098124	0.142857142
5	-112E22	0	0.128849782	0.142857143
6		0	0.141483700	0.142857143
7		0	0.142843938	0.142857143
8		0	0.142857142	0.142857143

6.6 Example 2

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$$f(x_1, x_2) = -\log(1 - x_1 - x_2) - \log x_1 - \log x_2$$

$$\nabla f(x_1, x_2) = \begin{bmatrix} \dfrac{1}{1 - x_1 - x_2} - \dfrac{1}{x_1} \\ \dfrac{1}{1 - x_1 - x_2} - \dfrac{1}{x_2} \end{bmatrix}$$

$$\nabla^2 f(x_1, x_2) = \begin{bmatrix} \left(\dfrac{1}{1 - x_1 - x_2}\right)^2 + \left(\dfrac{1}{x_1}\right)^2 & \left(\dfrac{1}{1 - x_1 - x_2}\right)^2 \\ \left(\dfrac{1}{1 - x_1 - x_2}\right)^2 & \left(\dfrac{1}{1 - x_1 - x_2}\right)^2 + \left(\dfrac{1}{x_1}\right)^2 \end{bmatrix}$$

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$$(x_1^*, x_2^*) = \left(\frac{1}{3}, \frac{1}{3}\right), \qquad f(x_1^*, x_2^*) = 3.295836867$$

k	x_1^k	x_2^k	$\|\|\boldsymbol{x}-\boldsymbol{x}^*\|\|$
0	0.85	0.05	0.58925565
1	0.7170068	0.09659864	0.45083106
2	0.5129752	0.17647971	0.23848325
3	0.35247858	0.27324878	0.06306103
4	0.33844902	0.32623807	0.00874717
5	0.33333772	0.33325933	7.4133E-05
6	0.33333334	0.33333333	1.1953E-08
7	0.33333333	0.33333333	1.5701E-16

7 Quadratic convergence

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- Recall

$$z = \lim_{n\to\infty} z_n, \qquad \limsup_{n\to\infty} \frac{|z_{n+1}-z|}{|z_n-z|^2} = \delta < \infty$$

- Example: $z_n = a^{2^n}$, $0 < a < 1$, $z = 0$

$$\frac{|z_{n+1}-z|}{|z_n-z|^2} = \frac{a^{2^{n+1}}}{a^{2^{n+1}}} = 1$$

7.1 Intuitive statement

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Theorem Suppose $f(\boldsymbol{x}) \in C^3$ (thrice cont. diff/ble), $\boldsymbol{x}^*$: $\nabla f(x^*) = 0$ and $\nabla^2 f(\boldsymbol{x}^*)$ is nonsingular. If Newton's method is started sufficiently close to $\boldsymbol{x}^*$, the sequence of iterates converges quadratically to $\boldsymbol{x}^*$.

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Lemma:
Suppose $f(\boldsymbol{x}) \in C^3$ on $\Re^n$ and x a given pt.
Then $\forall \epsilon > 0$, $\exists \beta > 0$: $||x-y|| \le \epsilon \Rightarrow$

$$||\nabla f(x) - \nabla f(y) - H(y)(x-y)|| \le \beta ||x-y||^2$$

7.2 Formal statement

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Theorem
Suppose f twice cont. diff/ble. Let $\boldsymbol{g}(\boldsymbol{x}) = \nabla f(\boldsymbol{x})$, $\boldsymbol{H}(\boldsymbol{x}) = \nabla^2 f(\boldsymbol{x})$, and $\boldsymbol{x}^* : \boldsymbol{g}(\boldsymbol{x}^*) = \boldsymbol{0}$. Suppose $f(\boldsymbol{x})$ is twice differentiable and its Hessian satisfies:

- $||\boldsymbol{H}(\boldsymbol{x}^*)|| \ge h$
- $||\boldsymbol{H}(\boldsymbol{x}) - \boldsymbol{H}(\boldsymbol{x}^*)|| \le L||\boldsymbol{x}-\boldsymbol{x}^*||$, $\forall \boldsymbol{x} : ||\boldsymbol{x}-\boldsymbol{x}^*|| \le \beta$

(Reminder: If $\boldsymbol{A}$ is a square matrix:

$$||\boldsymbol{A}|| = \max_{\boldsymbol{x}:||\boldsymbol{x}||=1} ||\boldsymbol{A}\boldsymbol{x}|| = \max\{|\lambda_1|, \ldots, |\lambda_n|\})$$

Suppose SLIDE 28

$$||\boldsymbol{x} - \boldsymbol{x}^*|| \leq \min\left(\beta, \frac{2h}{3L}\right) = \gamma.$$

Let $\bar{\boldsymbol{x}}$ be the next iterate in the Newton method. Then,

$$||\bar{\boldsymbol{x}} - \boldsymbol{x}^*|| \leq ||\boldsymbol{x} - \boldsymbol{x}^*||^2 \left(\frac{2h}{3L}\right) \quad \text{and}$$

$$||\bar{\boldsymbol{x}} - \boldsymbol{x}^*|| < ||\boldsymbol{x} - \boldsymbol{x}^*|| < \gamma.$$

7.3 Proof

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$\bar{\boldsymbol{x}} - \boldsymbol{x}^*$

$$\begin{aligned}
&= \boldsymbol{x} - \boldsymbol{H}(\boldsymbol{x})^{-1}\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{x}^* \\
&= \boldsymbol{x} - \boldsymbol{x}^* + \boldsymbol{H}(\boldsymbol{x})^{-1}\Big(\boldsymbol{g}(\boldsymbol{x}^*) - \boldsymbol{g}(\boldsymbol{x})\Big) \\
&= \boldsymbol{x} - \boldsymbol{x}^* + \boldsymbol{H}(\boldsymbol{x})^{-1} \\
&\quad \int_0^1 \boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x^*} - \boldsymbol{x}))(\boldsymbol{x^*} - \boldsymbol{x})dt \quad \text{(Lemma 1)} \\
&= \boldsymbol{H}(\boldsymbol{x})^{-1} \int_0^1 \left(\boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x^*} - \boldsymbol{x})) - \boldsymbol{H}(\boldsymbol{x})\right)(\boldsymbol{x^*} - \boldsymbol{x})dt
\end{aligned}$$

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$||\bar{\boldsymbol{x}} - \boldsymbol{x}^*||$

$$\begin{aligned}
&\leq ||\boldsymbol{H}(\boldsymbol{x})^{-1}|| \\
&\quad \int_0^1 \left|\left|\boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x^*} - \boldsymbol{x})) - \boldsymbol{H}(\boldsymbol{x})\right|\right| \cdot ||\boldsymbol{x} - \boldsymbol{x^*}||dt \\
&\leq ||\boldsymbol{H}(\boldsymbol{x})^{-1}|| \cdot ||\boldsymbol{x} - \boldsymbol{x^*}|| \int_0^1 L||\boldsymbol{x} - \boldsymbol{x^*}||\ t\ dt \\
&= \frac{1}{2}L||\boldsymbol{H}(\boldsymbol{x})^{-1}|| \cdot ||\boldsymbol{x} - \boldsymbol{x^*}||^2
\end{aligned}$$

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Now

$$\begin{aligned}
h &\leq ||\boldsymbol{H}(\boldsymbol{x}^*)|| \\
&= ||\boldsymbol{H}(\boldsymbol{x}) + \boldsymbol{H}(\boldsymbol{x}^*) - \boldsymbol{H}(\boldsymbol{x})|| \\
&\leq ||\boldsymbol{H}(\boldsymbol{x})|| + ||\boldsymbol{H}(\boldsymbol{x}^*) - \boldsymbol{H}(\boldsymbol{x})|| \\
&\leq ||\boldsymbol{H}(\boldsymbol{x})|| + L||\boldsymbol{x}^* - \boldsymbol{x}|| \\
&\Rightarrow ||\boldsymbol{H}(\boldsymbol{x})|| \geq h - L||\boldsymbol{x} - \boldsymbol{x}^*||
\end{aligned}$$

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$$\Rightarrow ||\boldsymbol{H}(\boldsymbol{x})^{-1}|| \leq \frac{1}{||\boldsymbol{H}(\boldsymbol{x})||} \leq \frac{1}{h - L||\boldsymbol{x} - \boldsymbol{x}^*||}$$

$$
\begin{aligned}
\Rightarrow ||\bar{\boldsymbol{x}} - \boldsymbol{x}^*|| &\leq ||\boldsymbol{x} - \boldsymbol{x}^*||^2 \frac{L}{2(h - L||\boldsymbol{x} - \boldsymbol{x}^*||)} \\
&\leq ||\boldsymbol{x} - \boldsymbol{x}^*||^2 \frac{L}{2(h - \frac{2}{3}h)} \\
&= \frac{3L}{2h} ||\boldsymbol{x} - \boldsymbol{x}^*||^2
\end{aligned}
$$

7.4 Lemma 1

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- Fix $\boldsymbol{w}$. Let $\phi(t) = \boldsymbol{g}(\boldsymbol{x} + t(\boldsymbol{x}^* - \boldsymbol{x}))'\boldsymbol{w}$
- $\phi(0) = \boldsymbol{g}(\boldsymbol{x})'\boldsymbol{w}, \quad \phi(1) = \boldsymbol{g}(\boldsymbol{x}^*)'\boldsymbol{w}$
- $\phi'(t) = \boldsymbol{w}'\boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x}^* - \boldsymbol{x}))(\boldsymbol{x}^* - \boldsymbol{x})$
- $\phi(1) = \phi(0) + \int_0^1 \phi'(t)dt \Rightarrow$

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$$
\forall \boldsymbol{w}: \quad \boldsymbol{w}'(\boldsymbol{g}(\boldsymbol{x}^*) - \boldsymbol{g}(\boldsymbol{x})) =
$$

$$
\boldsymbol{w}' \int_0^1 \boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x}^* - \boldsymbol{x}))(\boldsymbol{x}^* - \boldsymbol{x})dt
$$

$$
\Rightarrow \boldsymbol{g}(\boldsymbol{x}^*) - \boldsymbol{g}(\boldsymbol{x}) = \int_0^1 \boldsymbol{H}(\boldsymbol{x} + t(\boldsymbol{x}^* - \boldsymbol{x}))(\boldsymbol{x}^* - \boldsymbol{x})dt
$$

7.5 Critical comments

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- The iterates from Newton's method are equally attracted to local min and local max.
- We do not know $\beta,\ h,\ L$ in general.
- Note, however, that they are only used in the proof, not in the algorithm.
- We do not assume convexity, only that $H(\boldsymbol{x}^*)$ is nonsingular and not badly behaved near $\boldsymbol{x}^*$.

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7.6 Properties of Convergence

Proposition:

Let $r^k = ||x^k - x^*||$ and $C = \frac{3L}{2h}$. If $\|x^0 - x^*|| < \gamma$ then

$$
r^k \leq \frac{1}{C}(Cr^0)^{2^k}
$$

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Proposition:

If $||x^0 - x^*|| > \epsilon > 0 \Rightarrow ||x^k - x^*|| < \epsilon$

$$\forall k \geq \left\lceil \frac{\log(\frac{\log(\frac{1}{C\epsilon})}{\log(\frac{1}{Cr^0})}}{\log 2} \right\rceil$$

8 Solving systems of equations

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$$\boldsymbol{g}(\boldsymbol{x}) = \mathbf{0}$$

$$\boldsymbol{g}(\boldsymbol{x}^{t+1}) \approx \boldsymbol{g}(\boldsymbol{x}^t) + \nabla \boldsymbol{g}(\boldsymbol{x}^t)(\boldsymbol{x}^{t+1} - \boldsymbol{x}^t) \Rightarrow$$

$$\boldsymbol{x}^{t+1} = \boldsymbol{x}^t - (\nabla \boldsymbol{g}(\boldsymbol{x}^t))^{-1} \boldsymbol{g}(\boldsymbol{x}^t)$$

Application in optimization: $\boldsymbol{g}(\boldsymbol{x}) = \nabla f(\boldsymbol{x})$

9 Modifications for global convergence

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- Perform line search
- When Hessian is singular or near singular, use:

$$\boldsymbol{d}^k = -\left(\nabla^2 f(\boldsymbol{x}^k) + \boldsymbol{D}^k\right)^{-1} \nabla f(\boldsymbol{x}^k)$$

- $\boldsymbol{D}^k$ is a diagonal matrix:

$$\nabla^2 f(\boldsymbol{x}^k) + \boldsymbol{D}^k \text{ is PD}$$

10 Summary

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1. Line search methods:
   Bisection Method.
   Armijo's Rule.
2. The Newton's method
3. Quadratic rate of convergence
4. Modification for global convergence

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