The Binomial Distribution

Summer 2003

Internet Bubble

"Several industry experts believe that 30% of internet companies will run out of cash in 6 months and that these companies will find it very hard to secure additional funding" <u>Association for Computing Machinery</u>, August, 2001

- Pick 10 companies at random. How many would we expect to run out of cash in 6 months?
- What is p(4 run out of cash in 6 months)?
- What is p(2 run out…)?
- What is p(all 10 run out…)?

Binomial Distribution

Derived from theory, not from experience

- An experiment consists of n *"trials"*

- Each trial results in : *yes* or *no* *("binomial" means "2 names" or "2 labels")*

- Trials are independent of each other

- Each trial has same probability: success p, failure 1-p

        r.v. X = # successes in n trials

Random variable X has binomial distribution with parameters n and p
"Binomial(n,p)"

An Example "Experiment"

Select, randomly, one team of 3 students from a large pool of students composed of 70% US, and 30% International.
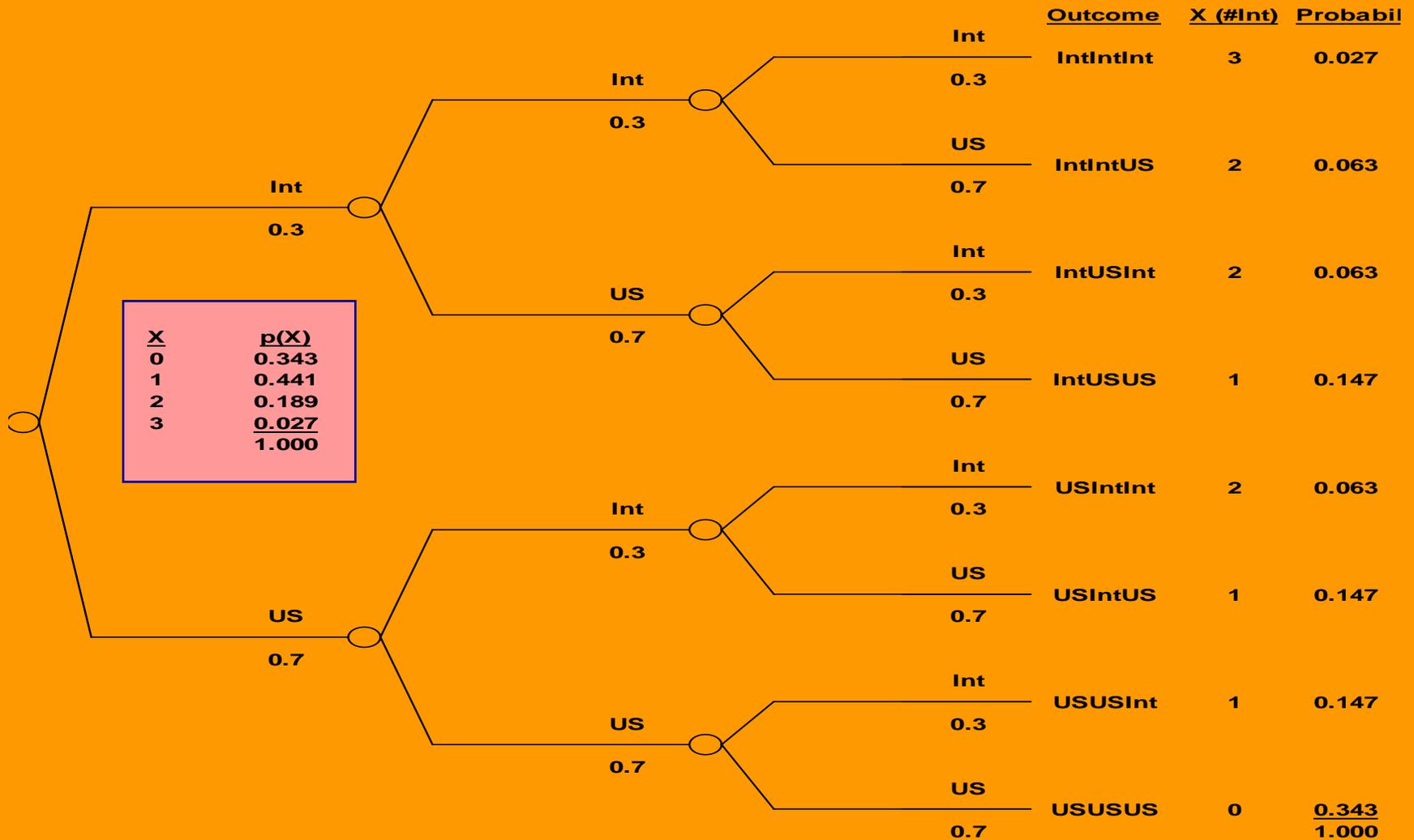
r.v. X = number of international students on the three student team selected randomly.

- n = 3 trials
- define: "yes": picking an international student
                       "no": picking a US student
- P(yes) = p = 0.30;   P(no) = 1 - p = 0.70

> X is a Binomial (n=3 , p=0.30)

(compare to experiment 2 in which we flipped three coins with p(H) = 0.3 and looked at the total # of heads)

Probability tree and probability distribution for r.v. X (total # Int students)

Outcome	X (#Int)	Probabil
IntIntInt	3	0.027
IntIntUS	2	0.063
IntUSInt	2	0.063
IntUSUS	1	0.147
USIntInt	2	0.063
USIntUS	1	0.147
USUSInt	1	0.147
USUSUS	0	0.343
		1.000

Int
0.3

Int
0.3

US
0.7

Int
0.3

Int
0.3

US
0.7

US
0.7

Int
0.3

Int
0.3

US
0.7

US
0.7

US
0.3

US
0.7

X	p(X)
0	0.343
1	0.441
2	0.189
3	0.027
	1.000

Binomial Probability Distribution Formula

Let X be Binomial(n, p):

The probability of having x "yes" events (also called "successes" in n trials is

$$P(X=x) = \frac{n!}{x!(n-x)!} \ p^x \ (1-p)^{n-x}$$

(where x! = x(x-1)(x-2)...1, and 0! = 1)

Check in Example 1: n=3 , p=0.3

$$P(X=1) = \frac{3!}{1!/(3-1)!} \ (0.3) \ (0.7)^2 = 0.441$$

Internet Bubble Again

- What is the p(4 of 10 run out of cash)?

- $P(X=x) = \dfrac{n!}{x!(n-x)!} p^x(1-p)^{n-x}$

- $P(X=4) = \dfrac{10!}{4!6!} .3^4 (.7^6) = .20$

- $P(X=10) = ?$

Excel Calculates Binomial Probabilities

Use the function wizard icon or the Insert_Function Command to choose the Statistical function BINOMDIST. This function calculates either the individual binomial probability P(X=Value) or the cumulative binomial probability P(X<=Value).

Mean, Variance and SD for Binomial Probability Distribution (r.v.)

Let X be Binomial(n, p):

The probability of having x successes in n trials is:

$$P(X=x)= \frac{n!}{x!(n-x)!}\ p^x (1-p)^{n-x}$$

(where x! = x(x-1)(x-2)...1, and 0! = 1)

$E(X) = np = 3 * 0.3 = 0.9$

$VAR(X) = np(1-p) = 3 * 0.3 * 0.7 = 0.63$

$SD(X) = \sqrt{n\ p\ (1-p)} = 0.794$

Calculations shown for Binomial (n=3, p=0.3)

Note: this is equivalent to counting success = 1 and failure = 0

Another example:

A laser production facility is known to have a 75% yield; that is, 75% of the lasers manufactured by the facility pass the quality test. Suppose that today the facility is scheduled to produce 15 lasers.

   a) What is the expected number of lasers to pass the test (the mean)?
   b) What is the variance?
   c) What is the standard deviation?
   d) What is the probability that at least 14 lasers will pass the quality test?

Laser Production

- What is the mean?

  E(X) = np = 15*.75 = 11.25 lasers

- What is the variance?

  Var(X) = np(1-p) = 15*.75*.25 = 2.812

- What is the standard deviation?

  SD(X) = sqrt (Variance) = 1.677 lasers

- What is the p of 14 or more passing?

  P(14 or more) = p(14) + p(15) = .067 + .013 = .08

Acquiring A Company Decision

- You represent Company A (the Acquirer), which is considering acquiring Company T (the Target) by means of a cash offer for 100% of Company T shares. You are unsure what price to offer because the value of Company T depends on the outcome of a major oil exploration project. If the project fails, Company T will be worth nothing, but if it succeeds, shares of Company T could be worth up to $100. All share values between $0 and $100 are considered equally likely. By all estimates, whatever the value of Company T under current management, it will be worth 50% more under the management of Company A. The Board of Directors wants your recommendation today for how much to offer per share for Company T. You do not know the outcome of the exploration project, however, you expect Company T to delay their decision about accepting your offer until after they know the results of the project. Company T will accept any offer above their true value and reject any offer below. There will be no negotiation or opportunity to make a second offer. What do you recommend as a take-it-or-leave-it price for Company T? $ _____ per share

Setting Up the Problem

- What is our goal? To maximize expected profit.
- What are the options? Make an offer of somewhere between $0 and $100/share.

  If you have trouble setting up the continuous problem, try making it discrete, e.g., what if I offered $20, $40, $60, $80?

- What are the outcomes? Profit = Revenue – Costs. Revenue = 1.5 * Book Value. Costs = Offer. So Profit = 1.5 BV – O.
- What is E(BV)? And what are the probabilities?

Probabilities

- We are told there is a uniform distribution between $0 and $100. Therefore, is the expected value = (0 + 100)/2 = $50/share?

- No. The seller will not accept any offer less than the book value. Therefore, if my offer is accepted, the book value must be uniform between $0 and Offer.

  E(BV) is therefore (0 + Offer)/2 = .5 Offer !!

- And, Profit = 1.5 BV − Offer

  = 1.5*.5 Offer − Offer = -.25 Offer

Uniform Distributions

$0   Offer = $60 $100

Conditional Probabilities

■ The key to the Acquiring A Company decision is to realize that the seller is making a decision conditional on receiving an <u>acceptable</u> offer

■ If you offer $60/share, the offer will be accepted only if the value is <= $60/share

■ Your expected book value is therefore conditional on your offer! If you offer $60/share, the expected book value is not $50/share, but rather $30/share (all values between $0 and $60 equally likely)

■ Your E(profit) is always <0, so don't bid !

Your Answers

- 37% answered $50/share, which is the EMV of T; ignores the conditional situation
- 19% bid between $51-$75 which tries to give each side some profit
- 9% gave the correct answer ($0, or don't make a bid)
- Even $1/share bid will lose an expected 25 cents per share (times millions of shares)

Next Lecture

Question: Given the choice to invest $1 million in these three firms, which one(s) would you choose?

Asset	Expected Return (% per year)	Risk (% per year)	Correlation		
			Exxon	Boeing	GM
Exxon	11.2%	13.9%	-----	0.41	0.13
Boeing	9.1	16.5	0.41	----	-0.22
GM	12.1	15.8	0.13	-0.22	----

More information would help

Question: How do we describe the relationship between two random variables?   Covariance - Correlation