Practice for Final Exam
15.063 Communicating with Data
Summer 2003

- *Closed book, but you can use the Formula Sheet.*
- *Show all your work in the exam.*
- *Calculators are allowed.*
- *You will have 3 hours from the time we start.*

Problem 1 (20 points. a, b: 7 points each. c: 6 points)

Gill Bates has an opportunity to purchase a lot of electronic components for resell from a local manufacturer. He can purchase the entire lot and resale it at a profit of $10,000.00, but only if the components in the lot are of good quality. If the components in the lot are defective, Robert will lose $3,000.00 in the transaction.
In the past, four out of five lots from this manufacturer have been of good quality (one out of five has been defective).
Robert has an opportunity to hedge his bet by having the lot inspected before making the purchase decision: For $200, he can have the lot inspected. He believes that if the lot is good, the inspector will, with a probability of 0.15, declare the lot defective. Further, if the lot is defective, the inspector will, with a probability of 0.1 declare the lot good.

Let G represent the event that the lot is good;
Let D represent the event that the lot is defective;
Let Q represent the event that the inspector declares the lot to be of good quality;
Let N represent the event that the inspector declares the lot to be of poor quality.

(a) Fill out the Conditional Probability Table.

(b) Find the following probabilities: P(Q), P(N), P(D|N), P(G|N), P(D|Q), P(G|Q).

(c) Build (but do not solve) the decision tree. Be sure to clearly mark each node (whether it is a decision node or event node), the decision or event at each branch, the probability for each event branch, and the payout associated with each leaf (i.e., the outcome at the end of each path in the tree)

Problem 2 (20 points. a, b: 7 points each. c: 6 points)

The Human Resources department at *MOIRTC Technologies* is planning to interview 7 software engineers for the 3 job openings that emerged last week. According to job market survey results, the salary asked by software engineers with 3 years' working experiences is uniformly distributed between $65,000 and $85,000. The 7 interviewees all have 3 years' working experience and the salaries they will ask are independent and in line with the probability distribution indicated by the survey. Suppose *MOIRTC* decided to set the starting salary for the 3 openings at $70,000 each.

(a) What is the probability that all 7 of the interviewees ask for a salary that is less than or equal to $70,000?

(b) Compute the probability that there are at least 3 among the 7 interviewees (i.e., 3 or more interviewees) whose salary expectation is less than or equal to $70,000.

(c) There are 300 software engineers with 3 years' working experiences on the job market right now. What is the probability that at least 30 of them (i.e., 30 or more) ask for a salary that is less than or equal to $70,000? (Hint: use an appropriate approximation, and explain why you may do so.)

Problem 3 **(20 points. a, b: 7 points each. c: 6 points)**

The following data represent the number of years of education for each employee in a random sample of 15 employees in a large corporation

Employee ID Number	*Years of Education*
01	6
02	7
03	7
04	8
05	12
06	12
07	12
08	13
09	13
10	13
11	14
12	15
13	16
14	18
15	21

(a) What is your estimate of the mean ($\bar{x}$) and standard deviation (s) of the number of years of education for the employees in this corporation?

(b) Construct a 95 % confidence interval for the true mean.

(c) What sample size would be needed to obtain a 95% confidence interval whose tolerance level is plus or minus half a year?

Problem 4 (20 points. a, b, c: 4 points each d: 8 points)

CWD Engineering employs a large number of engineers. Mary wants to analyze the extent to which the salaries of these engineers are related to their education and experience. She has obtained data for a sample of 20 engineers: (1) their yearly salary; (2) their undergraduate GPA (grade point average); and (3) their number of years of experience.

Engineer Number	Yearly Salary ($)	Undergrad. GPA	Years of Experience	Engineer Number	Yearly Salary ($)	Undergrad. GPA	Years of Experience
1	118,683	4.00	14	11	50,960	2.47	6
2	116,420	3.80	17	12	106,694	3.79	11
3	56,021	2.82	4	13	95,850	2.81	8
4	71,274	2.85	6	14	80,346	2.67	10
5	99,204	3.68	13	15	67,010	2.17	7
6	91,695	3.26	13	16	47,545	2.12	4
7	79,564	2.48	14	17	108,438	3.48	11
8	100,771	4.00	16	18	85,605	3.82	5
9	62,091	2.93	2	19	71,877	2.68	8
10	96,634	3.42	12	20	88,541	2.60	12

Mary used a multiple regression model to predict yearly salary on the basis of these data. The resulting computer output is shown below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.91534
R Square	0.83784
Adjusted R Square	0.81876
Standard Error	9,038.06
Observations	20

ANOVA

	df	*SS*	*MS*	*F*
Regression	2	7,174,865,080	3,587,432,540	43.917032
Residual	17	1,388,672,010	81,686,589	
Total	19	8,563,537,091		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	4,012.80	10,691.55	0.37532	0.71206
Undergraduate GPA	18,188.59	4,016.87	4.52804	0.00030
Years of Experience	2,538.87	573.40	4.42773	0.00037

(a) Write a complete equation for the simple linear regression model that incorporates the estimated coefficients provided by this computer output. Make sure to define *in words* all the variables used in this equation, and the *units* in which each is expressed.

(b) Use the regression model in (a) to predict the annual yearly salary of engineer Joe Smith, who has a 3.50 undergraduate GPA and 10 years of experience.

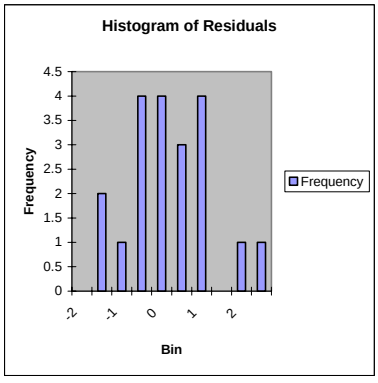
(c) According to the regression model in (a) above, the *actual* yearly salary of engineer Joe Smith is *most* likely to differ from the predicted value found in question (b) above by: (choose the BEST answer. Hint: consider the SD of the error term in your regression model)

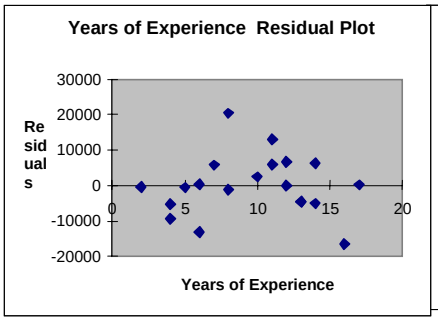
(cA) about ±$1.00	(cB) about ±$10.00	(cC) about ±$100.00
(cD) about ±$1,000.00	(cE) about ±$10,000.00	(cF) about ±$100,000.00

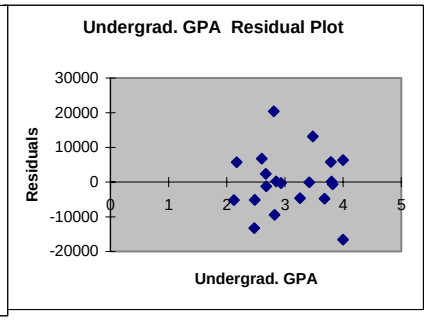
The correlation matrix of the data in this problem is:

	Yearly Salary ($)	*Undergrad. GPA*	*Years of Experience*
Yearly Salary ($)	1		
Undergrad. GPA	0.806741	1	
Years of Experience	0.801412	0.543407	1

Some more output of the regression is:







(d) In general, does the model satisfy the assumptions that a linear regression makes? Are both variables in the model statistically significant predictors? Do you feel comfortable using this regression model?