Production Planning

Basic Inventory Model
Workforce Scheduling

Enhance Modeling Skills
Dynamic Models

Dynamic Inventory Model

- Modeling Time
- Modeling Inventory
- Unusual Network Example

Singapore Electric Generator

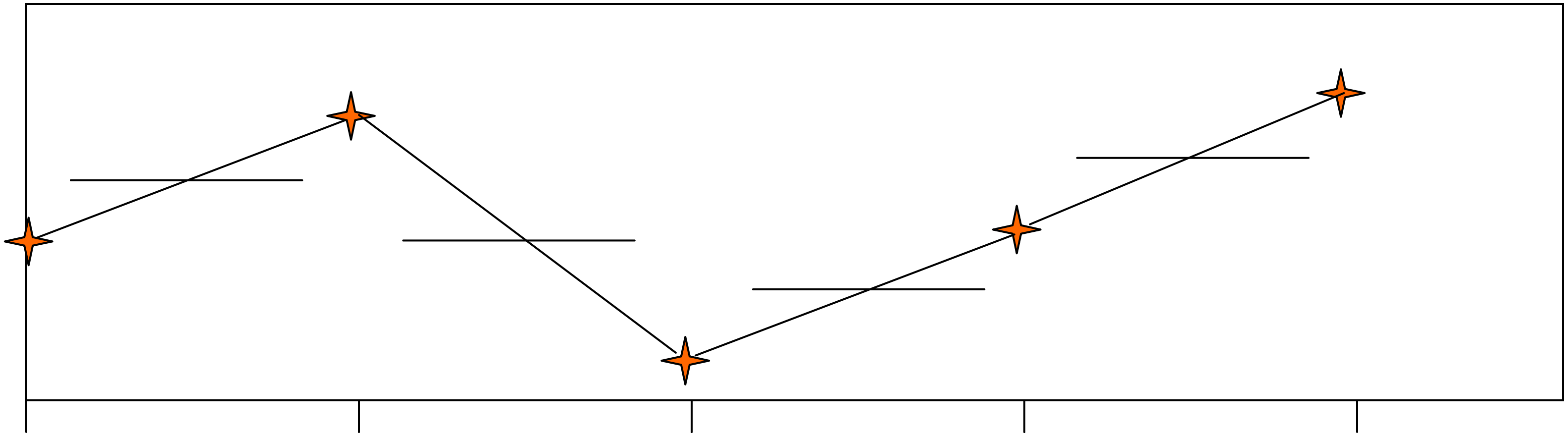
Singapore Electric Generator Production					
Unit Costs	Jan	Feb	Mar	Apr.	May
Production	$ 28.00	$ 27.00	$ 27.80	$ 29.00	
Inventory	$ 0.30	$ 0.30	$ 0.30	$ 0.30	
Production Qty	0	0	0	0	
Production Limits	60	62	64	66	
Beginning Inventory	15	-43	-79	-113	
Delivery Reqmts	58	36	34	59	Minimum
Ending Inventory	(43)	(79)	(113)	(172)	7
Production Cost	$ -	$ -	$ -	$ -	
Inventory Cost	$ (4.20)	$ (18.30)	$ (28.80)	$ (42.75)	Total
Total Cost	$ (4.20)	$ (18.30)	$ (28.80)	$ (42.75)	$ (94.05)

Inventory

- Balancing Your Checkbook
  - Previous Balance + Income - Expenses = New Balance
- Modeling Dynamic Inventory
  - Starting Inv. + Production - Shipments = Ending Inv.

Average Balances

- Assuming Smooth Cash Flows



⌘ Averages (Starting + Ending)/2

Challenge

- Formulate a Solver Model

Singapore Electric Generator

Singapore Electric Generator Production

Unit Costs	Jan	Feb	Mar	Apr.	May
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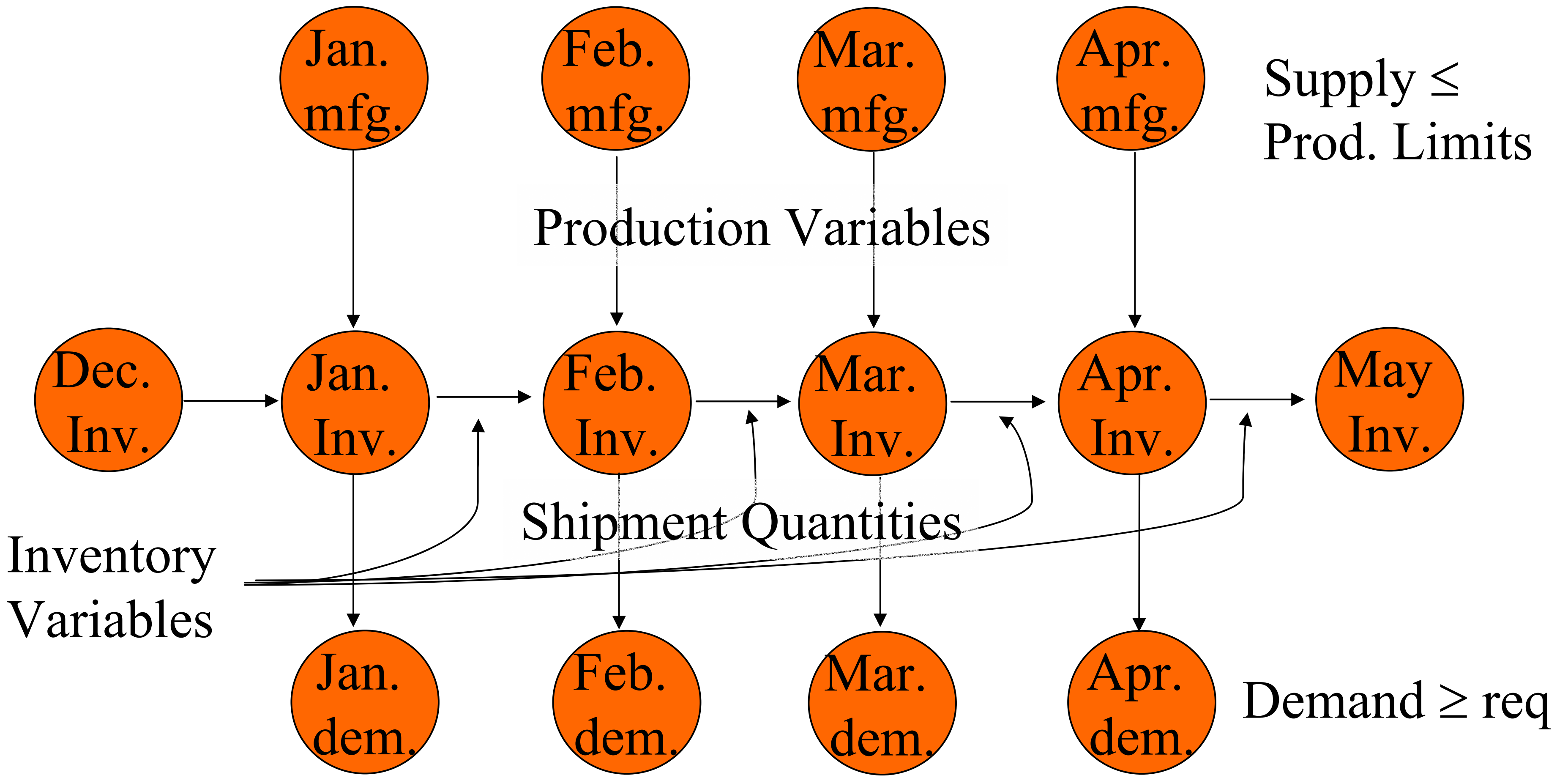
A Network Formulation



A Network Formulation

Singapore Electric Generator Production

Unit Costs	Dec	Jan	Feb	Mar	Apr.	May
Production		$ 28.00	$ 27.00	$ 27.80	$ 29.00	
Inventory		$ 0.30	$ 0.30	$ 0.30	$ 0.30	
Production Qty		0	0	0	0	
Production Limits		60	62	64	66	
Delivery Reqmts		58	36	34	59	
Calc. Ending Inv.		-43	(36)	(34)	(59)	Minimum
Ending Inventory	15	-	-	-	-	7
Production Cost		$ -	$ -	$ -	$ -	
Inventory Cost		$ 2.25	$ -	$ -	$ -	Total
Total Cost		$ 2.25	$ -	$ -	$ -	$ 2.25

Another View

s.t. InitialBalance:
Production['Jan'] - EndingInv['Jan'] = 43

s.t. MonthlyBalances['Feb']:
Production['Feb'] + EndingInv['Jan'] - EndingInv['Feb'] = 36

s.t. MonthlyBalances['Mar']:
Production['Mar'] + EndingInv['Feb'] - EndingInv['Mar'] = 34

s.t. MonthlyBalances['Apr']:
Production['Apr'] + EndingInv['Mar'] - EndingInv['Apr'] = 59

s.t. FinalBalance:
EndingInv['Apr'] >= 7

Scheduling Postal Workers

- Each postal worker works for 5 consecutive days, followed by 2 days off, repeated weekly.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Demand	17	13	15	19	14	16	11

- Minimize the number of postal workers (FTE's)

Challenge

- Formulate a Solver Model

Formulating the LP

Scheduling Postal Workers

Shift / Day	Mon - Fri	Tues - Sat	Wed - Sun	Thurs - Mon	Fri - Tues	Sat - Wed	Sun - Thurs	Demand
Mon	1			1	1	1	1	17
Tues	1	1			1	1	1	13
Wed	1	1	1			1	1	15
Thurs	1	1	1	1			1	19
Fri	1	1	1	1	1			14
Sat		1	1	1	1	1		16
Sun			1	1	1	1	1	11

Formulating as an LP

- The Objective
  - Total Workers Required
  - Minimize $I$5
- The decision variables
  - The number of workers assigned to each shift
  - $B$5:$H$5
- The Constraints
  - Enough workers each day
  - $I$6:$I$12 >= $J$6:$J$12

The linear program

$$
\begin{aligned}
\text{Minimize} \quad z = \; & MF + TS + WSu + ThM + FT + SW + SuTh \\
\text{subject to} \quad & MF + ThM + FT + SW + SuTh \geq 17 \\
& MF + TS + FT + SW + SuTh \geq 13 \\
& MF + TS + WSu + SW + SuTh \geq 15 \\
& MF + TS + WSu + ThM + SuTh \geq 19 \\
& MF + TS + WSu + ThM + FT \geq 14 \\
& TS + WSu + ThM + FT + SW \geq 16 \\
& WSu + ThM + FT + SW + SuTh \geq 11
\end{aligned}
$$

Non-negativity

The Decision Variable Decision

- Would it be possible to have the variables be the number of workers on each day?
- Conclusion: sometimes the decision variables incorporate constraints of the problem.
  - Hard to do this well, but worth keeping in mind
  - We will see more of this in integer programming.

Enhancement

- Some days we will have too many workers
- Excess
- Only concerned with the largest excess
- Minimize the largest Excess

Challenge

- Formulate a Solver Model

Formulating the LP

Scheduling Postal Workers

Shift / Day	Mon - Fri	Tues - Sat	Wed - Sun	Thurs - Mon	Fri - Tues	Sat - Wed	Sun - Thurs	Demand
Mon	1			1	1	1	1	17
Tues	1	1			1	1	1	13
Wed	1	1	1			1	1	15
Thurs	1	1	1	1			1	19
Fri	1	1	1	1	1			14
Sat		1	1	1	1	1		16
Sun			1	1	1	1	1	11

Minimize the Maximum

- Min Max{XS[Mon], XS[Tues], …}
- Min Z
- S.t. Z $\geq$ XS[Mon]
- S.t. Z $\geq$ XS[Tues]
- …
- **S.t. MF + ThM + FT + SW + SuTh – XS[Mon] = 17**
- **S.t. MF + TS + FT + SW + SuTh – XS[Tues] = 13**
- ….

Enhancement

- Ensure at least 30% of the workers have Sunday off
- Formulate a Solver Model

Formulating the LP

Scheduling Postal Workers

Shift / Day	Mon - Fri	Tues - Sat	Wed - Sun	Thurs - Mon	Fri - Tues	Sat - Wed	Sun - Thurs	Demand
Mon	1			1	1	1	1	17
Tues	1	1			1	1	1	13
Wed	1	1	1			1	1	15
Thurs	1	1	1	1			1	19
Fri	1	1	1	1	1			14
Sat		1	1	1	1	1		16
Sun			1	1	1	1	1	11

The linear program

Minimize $z = MF + TS + WSu + ThM + FT + SW + SuTh$

subject to

$$
\begin{aligned}
MF + ThM + FT + SW + SuTh &\geq 17\\
MF + TS + FT + SW + SuTh &\geq 13\\
MF + TS + WSu + SW + SuTh &\geq 15\\
MF + TS + WSu + ThM + SuTh &\geq 19\\
MF + TS + WSu + ThM + FT &\geq 14\\
TS + WSu + ThM + FT + SW &\geq 16\\
WSu + ThM + FT + SW + SuTh &\geq 11\\
.7(MF + TS) - 0.3*(WSu + ThM + FT + SW + SuTh) &\geq 0
\end{aligned}
$$

Non-negativity

Summary

- More LP Modeling
- LPs are more general than Networks
- Modeling Time
- Clever choices of decision variables