15.057 Systems Optimization
Spring 2002
Final Exam

Instructions

You have two hours to complete the exam from the time you first begin it. Watch your time. If you are having difficulty with a question, you might wish to pass over it and return to it later if you have enough time at the end of the exam.

The exam is open book: you may use any notes or text material. You are encouraged to use Excel to build your models.

The exam is to be done individually, without collaboration with anyone else.

Good luck!!!

Problem 1 Linear Forms (30 points)

Consider the following AMPL models. In each case:

a. Indicate hether or not the given model is a linear model as it is stated and
b. If it is not a linear model as stated either show how to formulate an equivalent linear model (with no approximations) or state that no linear formulation is possible.

Do not forget to answer both part a. and part b.

i. We are trying to forecast future demand from historical demand. Our model of demand includes an initial level and a rate of growth -- both of which we are trying to estimate.

```
param T;                  /* The number of past quarters of demand data */
param Demand{1..T};      /* The demand in quarter 1, 2, …, T */

var InitialLevel;        /* The  initial level of demand in quarter 0 for our forecast
                             This could be negative as we are using it to fit the rest of the data  */

var GrowthRate;           /* The change in demand from quarter to quarter.
                             This could be negative as we might be selling buggy whips */

var Estimate{1..T};    /* This variable is a convenience. It is simply the demand we estimate in
                            each quarter given the values of the InitialLevel and GrowthRate */

/* We measure the quality of the forecast based on how well it fits historical data as estimated by the
sum of the absolute differences between the forecast and actual demand in quarters 1, 2, …T */
minimize TotalError:
        sum{t = 1..T} (Estimate[t] - Demand[t])*(Estimate[t]-Demand[t]);

s.t. DefineEstimate {t in 1..T}:
        Estimate[t] = InitialLevel + GrowthRate*t;
```

Part a: Is this a linear model as it is given?

Part b. If it is not written as a linear model, can it be expressed as a linear model? If so, show how to express it as a linear model.

ii. After grading an exam, the Prof wants to assign weights or total points to the questions so that the average score in the class is between two given values (say 75 and 80, for example).

```
Set QUESTIONS;                          /* The set of questions */

Param AvgPctCorrect{QUESTIONS};     /* average fraction correct for each question */
Param MinAverage;                       /* minimum average score */
Param MaxAverage;                       /* maximum average score */


Var Points{QUESTIONS} >= 0; /* The points assigned to each question */
Var TooHigh >= 0;                 /* The extent to which the total points exceed 100 */
Var TooLow >= 0;                  /* The extent to which the total points falls short of 100 */

Minimize Error:
        TooHigh +  TooLow;

s.t. Nearly100TotalPoints:
        sum{q in QUESTIONS} Points[q] = 100+TooHigh - TooLow;

s.t. MeetMin:
        (sum{q in QUESTIONS} AvgPctCorrect[q]*Points[q])/(100+TooHigh - TooLow)
    >= MinAverage;

s.t. MeetMax:
        (sum{q in QUESTIONS} AvgPctCorrect[q]*Points[q])/(100+TooHigh - TooLow)
   <= MaxAverage;
```

Part a: Is this a linear model as it is given?

Part b. If it is not written as a linear model, can it be expressed as a linear model? If so, show how to express it as a linear model.

iii. The weight limit version of the service constraint for distribution center dc to pool p applies if both distribution center dc serves pool p and the trailers between distribution center dc and pool p reach the weight limit rather than their cubic capacity.

```
Set POOLS;
Set STORES;
Set DCS;
Set PRODUCTS;

Param Demand{PRODUCTS, STORES}:
Param CubicCapacity;
Param WeightLimit;
Param MaxFillTime{DCS, POOLS};
Param Weight{PRODUCTS};
Param Cube{PRODUCTS};

var WeighOut{DCS, POOLS} binary;
var UseEdge{DCS, POOLS} binary;

var Flow[PRODUCTS, DCS, POOLS} >= 0;
var Assign{POOLS, STORES} binary;

s.t. ImposeTrailerFillbyWeight{dc in DCS, pool in POOLS}:
        MaxFillTime[dc, pool]*sum{prd in PRODUCTS} Weight[prd]*Flow[prd, dc, pool]
>= WeightLimit*WeighOut[dc, pool]*UseEdge[dc, pool];
```

```
s.t. ImposeTrailerFillbyCube{dc in DCS, pool in POOLS}:
        MaxFillTime[dc, pool]*sum{prd in PRODUCTS} Cube[prd]*Flow[prd, dc, pool]
>= CubicCapacity*CubeOut[dc, pool]*UseEdge[dc, pool];

s.t. DefineUseEdge{prd in PRODUCTS, dc in DCS, pool in POOLS}:
        Flow[prd, dc, pool] <= (sum{store in STORES} Demand[prd, store])*UseEdge[dc, pool];

s.t. WeightOrCube{dc in DCS, pool in POOLS}:
        WeighOut[dc, pool] + CubeOut[dc, pool] = 1;
```

Part a: Is this an integer ***linear*** model as it is given?

Part b. If it is not written as an integer linear model, can it be expressed as an integer linear model? If so, show how to express it as an integer linear model.

Problem 2 Sensitivity Analysis (10 points)

Question 4-61 on page 198 of Moore et al.

Problem 3 Basic Facts (30 points)

Indicate whether each of the following questions is true or false.

a. In solving an integer programming problem, Solver and CPLEX employ genetic algorithms

b. In solving an integer programming problem, Solver and CPLEX employ the simplex method.

c. The set of optimal solutions to an integer program describes a convex set.

d. The function $f(x) = x^2$ (that's x*x) is a convex function

e. The function $f(x,y) = x^2 + y^2$ (that's x*x + y*y) a convex function

f. All polynomials are convex functions.

g. If a real valued function f is a convex function then the function $g(x) = -f(x)$ is a convex function

Problem 4 Relaxations and Restrictions (20 points)

Suppose we are solving an integer programming problem with a minimization objective. Sort the values in the list of items (a)-(d) from smallest to greatest (you can fill the blanks in the following line, but make sure that your response is in the solution you are sending back).

_____ ≤ _____ ≤ _____ ≤ _____

a. Optimal objective function value of the linear programming relaxation with integrality restrictions on some, but not all, variables.
b. Optimal objective function value of the integer program.
c. Optimal objective function value of the complete linear programming relaxation (no integrality restrictions).
d. Optimal objective function value of the integer program if we fix the values of some of the variables.

Problem 5 Integer Programming Formulation (10 points)

Formulate an integer linear model of the following situation. This is intended to be part of a larger, more complicated model so the fact that we can solve the problem easily is irrelevant -- we are only talking about a small piece of the problem.

There are N items numbered 1, 2, N. If we choose item i, we earn r[i]. If we select exactly one of the items, we must pay b. You may introduce other variables if you wish. You will certainly need to add an objective (minimize total cost) and constraints.

```
param N;  # the number of items.
param r{1..N}; # the revenue on each item
param b; # the cost incurred if we choose exactly one item

var x{1..N} binary; # x[i] is one if we  choose item i and 0 otherwise.
```