RECITATION NOTES #3

Review of Production and Cost Concepts

Thursday - September 23, 2004

OUTLINE OF TODAY'S RECITATION

1. **The Production function:** brief review of production function and isoquants

2. **Economic Cost and User Cost of Capital:** definitions

3. **Cost concepts:** Types of costs and how to calculate them

4. **Economies of scale and scope:** definition and terminology warnings

5. **Learning curve effects:** definition and examples

6. **Numeric Examples:** applying these concepts in practice

1. THE PRODUCTION FUNCTION
1.1 Definition
1.2 Production with one variable input
1.3 Production with two variable input

1.1 Definition
In the production process, firms turn inputs, which are also called factors of production, into outputs. We can divide inputs into the broad categories of *labor, materials and capital*.

The relationship between the inputs to the production process and the resulting output is described by a production function. *The production function indicates the maximum output Q that a firm will produce for every specified combination of inputs.* Assuming for simplicity that there are two inputs, labor L and capital K, the production function can be written as:

$Q = f(K, L)$

Example: L measures the number of workers and K the amount of machinery employed by a firm to produce widgets. The production function gives the number Q of widgets that can be produced for any given combination of the inputs.

Every production function refers to a given technology. As technology advances, the production function will change to reflect the higher level of output that can be obtained with the same

inputs. Production functions describe what is technically feasible when the firm operates efficiently. This means that inputs will not be used if they decrease output.

The production function also refers to a specific time horizon. In the *short run*, for instance, some factors of production cannot be changed (e.g. the amount of capital/equipment): these factors are called *fixed inputs*. Only the *variable inputs* appear in the production function. In the *long run* all the inputs are considered variable.

1.2 Production with one variable input

Let's consider the case in which capital is fixed, but labor is variable. In this situation the production function can be written as:

$$Q = f(L)$$

The total product Q will increase by increasing the labor input. At a certain point, however, increasing labor becomes counter productive and the production function reaches its maximum. It does not make sense to add labor above the level that results in the maximum output.

*The **average product of labor** is defined as the output per unit of labor input:*

$$AP_L = Q / L$$

Graphically, the slope of the line drawn from the origin to the corresponding point on the total product curve gives the average product of labor (see P&R p.183 for graphs.) In the example, the average product of labor is increasing up to a certain level of labor input, at which point it reaches a maximum, then it starts decreasing.

*The **marginal product of labor** is defined as the incremental output per incremental unit of labor input:*

$$MP_L = \Delta Q / \Delta L$$

Graphically, the slope of the total product curve at that point gives the marginal product of labor.

When the marginal product is greater than the average product, the average product is increasing. This is because the last unit of labor contributes more to output than the previous units, therefore the average goes up. Similarly, *when the marginal product is less than the average product, the average product is decreasing.*

When the AP_L reaches its maximum, the marginal product of labor must equal the average product of labor, otherwise AP_L would either be increasing (if $MP_L > AP_L$) or decreasing (if $MP_L < AP_L$):

$$MP_L = AP_L \qquad \text{when } AP_L \text{ is maximum}$$

When the total product reaches its maximum, $MP_L = 0$ because adding an additional unit of input does not result in any increase in output.

The *law of diminishing returns* holds for most production processes. It states that *as the use of an input increases (with other inputs fixed), a point will eventually be reached at which the resulting addition to output decreases.* For instance, when labor input is low (and capital is fixed), small increments in labor input add substantially to output as workers are allowed to develop specialized tasks. Eventually, however, it will be more and more difficult to improve output by adding workers while holding the amount of equipment fixed. The extra workers become less effective and the marginal product of labor falls. Diminishing returns refers to changes in quantity of output and not to quality.

1.2 Production with two variable inputs
Let's go back to the general production function with two inputs, labor and capital:

$$Q = F(K, L)$$

This function can be represented graphically using isoquants. *An isoquant is a curve that shows all the possible combinations of inputs that yields the same output.* (see P&R p. 192 for graphs)

The law of diminishing returns still applies. Diminishing returns are observed by holding one variable fixed and looking at the marginal product of the other.

Isoquants are typically convex downward sloping curves. Intuitively, this happens because, if output is held constant, it takes less capital to replace one unit of labor when labor is abundant than when it is scarce.

The measure of increased output associated with proportional increases in all inputs is fundamental to the long-run nature of the firm's production process.
- If output more than doubles when inputs are doubled, there are *increasing returns to scale*. This might happen because the increased scale allows more specialization of both workers and equipment.
- If output doubles when inputs are doubled, there are *constant returns to scale*. In other words it is the same to have two identical plants or a bigger plant with twice the labor and the capital.
- If output less than doubles when all inputs double, there are *decreasing returns to scale*.

In general, above a certain size, all businesses show decreasing returns to scale because of the complexities of organizing and managing very large operations.

2. ECONOMIC COST
2.1 Difference between Economic Cost and Accounting Cost
2.2 User Cost of Capital

2.1 Difference between Economic Cost and Accounting Cost
There are differences in economic and accounting costs. Economic analyses are forward-looking and are concerned with *opportunity costs*, the costs associated with opportunities that are foregone by not putting the resources to the highest value. For example, the cost of an MBA is not only the two years of tuition but also the two years of salary foregone. Similarly, economic analyses do not care about *sunk costs*, an expenditure that has been made and cannot be recovered. In determining costs, *explicit costs* such as wages, raw materials and capital are the primary concern. Depreciation costs are different in that economic analyses include the actual wear and tear loss over time.

2.2 User Cost of Capital
The User Cost of Capital is defined as the opportunity cost of holding capital for a firm. It is defined as:

$$UCC_t = (r + \%dep'n)V_t$$

Where:
> Vt = effective value of the firm's capital
> r = weighted average cost of capital of the Firm
> **% dep'n** = the wear and tear depreciation the firm's capital has faced, calculated as the effective percentage difference of the capital's value over time
> (In a formula: **% dep'n** $= (V_t - V_{t+1})/V_t$)

The most important consideration in estimating, r, the costs of capital to the firm is *risk*. Although this topic is covered in much more detail in corporate finance classes, often a firm will use a weighted average cost of capital (WACC), which reflects both the opportunity cost of its debt and of its equity.

3. COST CONCEPTS
3.1 Total, fixed, variable and sunk costs
3.2 Average and Marginal costs
3.3 Production costs and optimal production level

3.1 Total, fixed, variable and sunk costs
The Total Cost function has two main components: Fixed Costs and Variable Costs.

3.1.1 Fixed Costs
All costs that do not vary with the quantity of output produced are fixed costs.
Example: if you produce cars and you have to pay a monthly rent of $30,000 for your production facilities, this is a fixed cost. Regardless of your production level (it could be from 0 to infinity) as long as you do not close the plant you would still have to pay this fixed amount of money.

[**NOTE**: these costs are recoverable if the firm shuts down!]

3.1.2 Variable Costs
All costs that vary with the quantity of output produced are considered variable costs. **Example**: if you produce cars and you have to pay $250 per car for paint and finishing materials, then your total cost for painting and finishing material will depend on the amount of cars you will decide to produce. It will be 0 if you do not produce any car and it will be a multiple of $250 if you decide to produce many cars.

3.1.3 Sunk Costs
All costs/expenditures that cannot be recovered if the firm decides to interrupt its activities. They can also include costs yet to be paid which can not be avoided
Example: Past advertising expenditures cannot be recovered if you shut down your company.

3.2 Average and Marginal costs
3.2.1 Average cost *is the total cost per unit of output.*

$$AC = TC/Q$$

3.2.2 Marginal cost *is the increase in cost resulting from producing one additional unit of output.*

$$MC = \Delta VC / \Delta Q = \Delta TC / \Delta Q$$

In the short run, capital will be fixed and labor will be variable so that the MC will be the unit cost of additional labor.

$$MC = w\Delta L / \Delta Q = w/ MP_L$$
w wages
MP_L marginal product of labor

3.3 Production Costs and optimal production level
The cost of output is the cost of the inputs used in the production. Let's call the cost of labor *w* (for wage) and the cost of capital *r* (for rent). The total cost C of producing any particular output using K units of capital and L units of labor is:

$$C = rK + wL$$

The same level of total cost can be determined by different mixes of labor and capital. *An isocost line includes all possible combinations of labor and capital that can be purchased for a given total cost.* The isocost line is a downward sloping straight line, crossing the labor axis at L = C/w and the capital axis at K = C/r. All isocost lines, for different total costs, are parallel with slope -w/r. (See P&R p. 218 for related graphs)

Isocost lines can be used to determine the optimal mix (in the sense of minimum cost) of labor and capital for the production of any given quantity Q of output. *The optimal combination of*

inputs is given by the point of tangency of the isoquant curve for quantity Q and the lowest isocost line intersecting the isoquant.

Rational firms will always try to produce output using the optimal mix of inputs to minimize costs. The total cost curve for such firms can then be obtained by graphing the minimum cost of production corresponding to different levels of output, determined as above. The total cost curve function showing the costs of different levels of production is therefore always the result of an optimization. In real life, firms seldom identify the optimal mix of labor and capital analytically. Rather, they continuously make trade-offs between labor and capital, iterating until they can no longer improve costs. This results in the optimal mix of inputs for the production of the desired output.

4. ECONOMIES OF SCALE AND SCOPE
4.1 Economies of Scale - Definition and example
4.2 Relationship between Economies of Scale, Marginal Costs and Average Costs
4.3 Short run vs Long Run Cost functions
4.4 Difference between Economies of Scale and Returns to Scale
4.5 Economies of Scope – Definition and Example

4.1 Definition and example
Economies of scale can be interpreted as the cost savings associated with the size of the business. If a firm experiences economies of scale, it means that it is sustaining unit cost savings at higher scales of production (in other words, for higher Q values, the lower the Average Costs).
Example: If a firm can produce 100 laptops at a cost of $2,000 each and 3000 laptops at a cost of $1,000 each, then it is experiencing economies of scale.

4.2 Relationship between Economies of Scale, Marginal Costs and Average Costs
In general, we can tell if a firm has economies of scale, diseconomies of scale or constant costs by observing the relationship between its Marginal Costs and its Average Costs.
- **Economies of Scale -** AC decreases with Q, so MC < AC
- **Constant Costs -** AC constant with Q, so MC = AC
- **Diseconomies of Scale -** AC increases with Q, so MC > AC

4.3 Difference between Economies of Scale and Return to scale
The term "Returns to Scale" refers to physical properties of production: if a firm doubles its inputs yields and obtains more than double units of output, it is experiencing positive returns to scale (or Increasing returns to scale). When we refer to Economies of scale, we are referring to a money amount.

4.4. Economy of Scope
Economies of scale should not be confused with Economies of scope. These are the cost savings that a firm experiences from producing multiple products using the same facilities/structures. These are also called "Joint production economies".

Example: A firm produces 200 laptops in plant A for $500,000 and 300 PDAs in plant B for $30,000. If the firm can produce all 200 laptops and 300 PDAs in plant A for $510,000, then it is experiencing economies of scope.

5. LEARNING CURVE EFFECTS
5.1 Definition
5.2 Types of learning curve functions

5.1 Definition
A production function is characterized by learning effects if the Average Cost of production diminishes the more experienced the workers become. Learning effects are represented in the cost equation by including a variable term for cumulative production.

5.2 Learning curve functions
A cost function where learning effects are relevant could be represented as follows:

$$AC = a - bN + cQ$$

Where N represents cumulative production in the past, and Q represents the Quantity of output to be produced this period. As this equation shows, the average cost of current and future production decreases as the cumulative past production (i.e., production experience) increases.

6. NUMERIC EXAMPLES
6.1 Example of types of costs
6.2 Example of production function
6.3 Example of Learning Effects

6.1 Example of types of costs

In the production of widgets, the fixed costs are $120. From the following production and total cost schedule, calculate the total variable cost, the average total, fixed and variable costs and the marginal cost.

Q	TC ($)	TFC ($)	TVC ($)	AC ($)	AFC ($)	AVC ($)	MC ($)
0	120						
1	180						
2	200						
3	210						
4	225						
5	260						
6	330						

What is the production level that minimizes average costs?

Answer:

FC = $ 120

Q	TC ($)	TFC ($)	TVC ($)	AC ($)	AFC ($)	AVC ($)	MC ($)
0	$ 120	$ 120	$ -	$ -	n.a	$ -	$ -
1	$ 180	$ 120	$ 60	$ 180	$ 120	$ 60	$ 60
2	$ 200	$ 120	$ 80	$ 100	$ 60	$ 40	$ 20
3	$ 210	$ 120	$ 90	$ 70	$ 40	$ 30	$ 10
4	$ 225	$ 120	$ 105	$ 56	$ 30	$ 26	$ 15
5	$ 260	$ 120	$ 140	$ 52	$ 24	$ 28	$ 35
6	$ 330	$ 120	$ 210	$ 55	$ 20	$ 35	$ 70

6.2 Example of production function

At another widget factory, labor costs $300/unit/day. Given the total quantity produced, determine the average and marginal productivity of labor, TVC, AVC and MC assuming that capital is fixed in the short run.

L	Q	AP_L	MP_L	TVC ($)	AVC ($)	MC ($)
1	100					
2	300					
3	700					
4	1000					
5	1200					
6	1300					
7	1350					

What amount of labor minimizes average variable costs?

Answer:

L	Q	APL	MPL	TVC ($)	AVC ($)	MC ($)
1	100	100	100	$ 300	$ 3.00	$3.00
2	300	150	200	$ 600	$ 2.00	$1.50
3	700	233	400	$ 900	$ 1.29	$0.75
4	1000	250	300	$ 1,200	$ 1.20	$1.00
5	1200	240	200	$ 1,500	$ 1.25	$1.50
6	1300	216	100	$ 1,800	$ 1.38	$3.00
7	1350	193	50	$ 2,100	$ 1.56	$6.00

6.3 Example of learning curve

A computer company's cost function, which relates its average cost of production AC to its cumulative output in thousands of computers N and its plant size in terms of thousands of computers produced per year Q, within the production range of 10,000 to 50,000 computers, is given by:

$$AC = 10 - 0.1N + 0.3Q$$

Q: Is there a learning curve effect?

A: Yes. The N term shows that there is a linear decrease in AC as the cumulative output increases.

Q: Are there economies or diseconomies of scale?

A: Diseconomies of scale. As the batch size increases, the average cost increases by 30%. If the coefficient on the Q term is greater than zero, then there are diseconomies of scale. If the coefficient is less than zero, there are economies to scale, and if the coefficient is equal to zero, there are constant costs.

During its existence, the firm has produced a total of 40,000 computers and is producing 10,000 computers this year. Next year it plans to increase its production to 12,000 computers. Will its average cost of production increase or decrease?

This year: $AC = 10 - 0.1*40 + 0.3*10$
              AC = $9 thousand

Next year: $AC = 10 - 0.1*50 + 0.3*12$
              AC = $8.6 thousand

Firms gain learning through optimizing manufacturing processes and procedures over a series of lots (or batches). Improving process workflows by setting up machines and tools between the production runs leads to the largest increases in learning.

In this example, the learning gained from the 10,000 units this year will decrease the average cost for next year. On the other hand, by increasing the production size in the next year (from 10,000 to 12,000), the firm will incur higher average costs due to diseconomies of scale.

These two forces work in opposite directions, with the net result of a lower total average cost. (The learning effect from the additional 10,000 units is stronger than the diseconomies of scale realized by increasing the batch size by 2,000 units.)