# Final Exam 17.881/882

1. Pleasant Street is ordinarily a pleasant place, but recently a terrible stench of unknown origin has invaded the neighborhood. To solve this problem, the street set up a Sanitary Council (SC) to find the source of the foul smell and eliminate it. This gives Mr. W an idea. Mr. W is generally a good citizen, but he dislikes his next door neighbor, Mr. H, and decides to use this opportunity to force Mr. H out of the neighborhood. Moreover, Mr. W suspects that Mr. H is in fact the source of the smell. He has a problem, however, the F bloc. This is a group of citizens that like Mr. H, and also have a veto on the SC. Hence, Mr. W faces a decision. He may bring a proposal before the SC accusing Mr. H of being the origin of the stench (P), or he may leave Mr. H alone (L). If Mr. W makes a proposal, then the SC must meet and either accept (A) or reject (R) it. Finally, whatever the decision made by the SC, the F bloc can either veto the proposal (V) or not veto it (NV). If the SC rejects the proposal, then a veto obviously does not do anything substantial, but the F bloc still enjoys its show of power. On the other hand, the F bloc is not so obnoxious that it wants to use their veto if the SC actually accepts the proposal. Hence the extensive form of the game is the following:
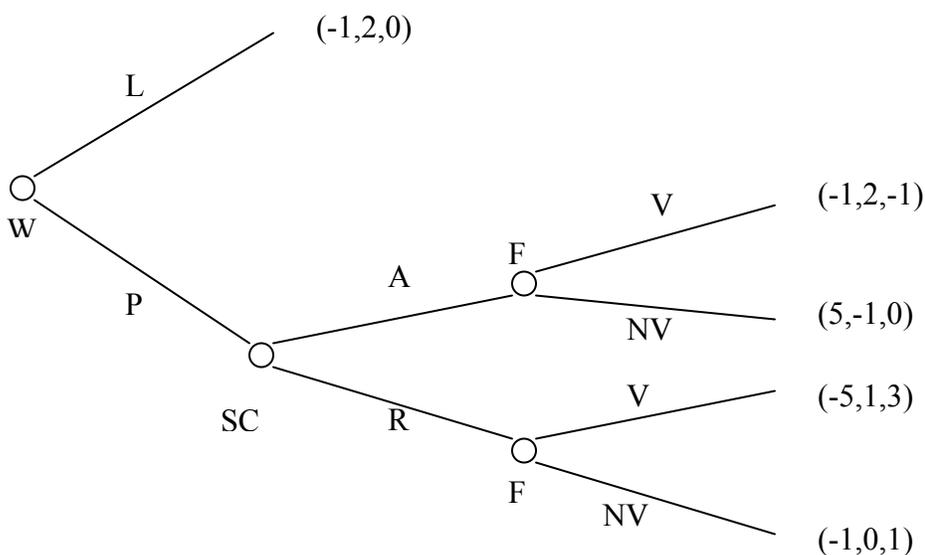
(a) How many strategies do W, SC, and F have?

(b) Set up the Normal form of this game and find the Nash equilibria. (Hint: set up two matrices for SC and F and have W choose between them with his strategy.)

(c) Find the subgame perfect Nash equilibria of this game.

Now assume that the F bloc must decide if they will veto *simultaneously* with the decision of the SC. How many strategies does each player have now? Find the subgame perfect Nash equilibria of this new game. Remember to state the equilibria formally.

2. Even though he is defeated by the veto, Mr. W is not the kind of guy who gives up easily. He talks to another neighbor, Mr. B, who also lives next door to Mr. H and dislikes Mr. H. They decide to take matters into their own hands. They will throw garbage into Mr. H's backyard every night until he gets disgusted and leaves. There is a problem with the plan however. If only one of them throws garbage, then it is easy to identify where it is coming from, in which case Mr. H can bring the culprit before the SC and demand that he pay a large fine. Mr. W and Mr. B can only avoid this if they both throw garbage. In that case Mr. H will find his backyard covered with rubbish and accuse all of his adjacent neighbors, and Mr. W and Mr. B will only pay $1/4^{th}$ of the fine each. Assume that the utility value of "garbage in the rascal's backyard" is 3, and does not vary whether is one or two loads. The total cost of the fine is 4. If no garbage is thrown (and hence no fine is imposed) they are left with 0.

   (a) Find the normal form of this game for each night, where the strategies are Throw (T) or Not Throw (NT). What kind of game does this look like? Find the Nash equilibria.

   (b) Suppose Mr. W. and Mr. B attempt to implement their plan for one week (seven nights). Can they use the fact that they have seven nights to coordinate and achieve a better outcome for themselves?

   (c) Now imagine Mr. W. and Mr. B attempt to implement their plan forever. They discount future payoffs by δ. Imagine each one uses the following trigger stategy: "Throw garbage as long as the other has thrown garbage up to tonight." What is the minimum δ required for this to be an equilibrium?

3. Mr. W and Mr. B are unable to trust each other, but Mr. W has a new plan. He suspects that Mr. H. is operating an illegal fish market from his basement, and this is the source of the Pleasant Street stench. If he can obtain photographs of the illegal fish market and bring them before the SC, then it will surely force Mr. H to leave. Realizing that Mr. W must use a large amount of water to store and clean the fish, he decides to monitor Mr. H's water use before deciding whether to break into Mr. H's house to take photographs. There is one problem with the plan, however. Mr. W knows that Mr. H enjoys taking long baths, which also use a large amount of water. So he might not be able to determine whether Mr. H's high water use is due to baths or fish. And, if Mr. W breaks into Mr. H's house only to take photographs of Mr. H in the bath, then he will not only be embarrassed but also in big trouble for breaking and entering. Mr. H knows of Mr. W's suspicions, and must decide how much water to use.

Set this game up as a signaling game, as follows. First, nature chooses Mr. H's type: fish market (F) or bath (B). Then Mr. H decides on his water use (the signal): use water (W) or do not use water (NW). Finally, ignorant of the type but observing the use of water, Mr. W decides whether to take photographs (G) or stay home (S).

Payoffs are as follows: If Mr. H operates a fish market using water gives him utility *H*, while using water for a bath (when he does not have fish) gives him utility *L*. If Mr. H is photographed while bathing, then his utility is reduced by *P*. If he is caught operating a fish market, then photographs reduce his utility by *p*. Not using water yields a utility of 0, but if he is operating the fish market, then photographs still reduce his utility by *p* (if he does not use water, then clearly he cannot be photographed in the bath).

Mr. W gets utility *A* if he obtains photographs of a fish market, and he gets –*a* if he trespasses but does not get photographs of a fish market. If he stays at home, he gets 0.

Finally, Mr. W's prior belief that Mr. H is operating a fish market is ½.

   (a) Suppose *a*=10 and *A*=5. Construct a perfect Bayesian equilibrium where Mr. W never goes to take photographs. Be formal and explicit and check all the elements and requirements for the equilibrium to exist.

   (b) Now, additionally, assume *H*=10, *L*=4, p=2 and P=5. Construct a separating equilibrium. Again, be formal and remember to check all necessary requirements.