Problem Set 10 Solution

17.881/882

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1 Gibbons 3.2 (p.169)

1.1 Strategy Spaces

Firm 1 has two types or two information sets and must pick an action for each type. Firm 2 has only one type and can only pick one action.

The strategy spaces are: For Firm 1: $\{q_1(a_H), q_1(a_L)\}\epsilon\mathbf{R}^+ X\mathbf{R}^+$. For Firm 2: $q_2\epsilon\mathbf{R}^+$

1.2 Bayesian Nash Equilibrium

Let $q_1^*(a_H)$ and $q_2^*(a_H)$ denote firm 1's quantity choices as a function of a_j. Also let q_2^* denote firm 2's quantity choice.

If demand is high, firm 1 will choose $q_1^*(a_H)$ to solve

$$\max_{q_1}[a_H - q_1 - q_2^* - c]q_1$$

Similary, if demand is low, firm 1 will choose $q_1^*(a_L)$ to solve

$$\max_{q_1}[a_L - q_1 - q_2^* - c]q_1$$

Finally, firm 2 knows that $a_j = a_H$ with probability θ and should anticipate that firm 1's quantity choice will be $q_1^*(a_H)$ or $q_1^*(a_L)$ depending on a_j. Thus firm 2 chooses q_2^* to solve

$$\max_{q_2}\theta[a_H - q_1^*(a_H) - q_2 - c]q_2 + (1-\theta)[a_L - q_1^*(a_L) - q_2 - c]q_2$$

The first-order conditions for these three optimization problems are the following:

$$q_1^*(a_H) = \frac{a_H - c - q_2^*}{2} \quad (1)$$

$$q_1^*(a_L) = \frac{a_L - c - q_2^*}{2} \quad (2)$$

$$q_2^* = \frac{\theta[a_H - q_1^*(a_H) - c] + (1-\theta)[a_L - q_1^*(a_L) - c]}{2} \quad (3)$$

3 can be rewritten, using 1 and 2, as:

$$\begin{aligned} q_2^* &= \frac{[\theta a_H + (1-\theta)a_L - c]1/2 - q_2^*/2}{2} \quad (4) \\ \implies q_2^* &= \frac{\theta a_H + (1-\theta)a_L - c}{3} \end{aligned}$$

Using 4, 1 and 2 can be rewritten, respectively, as:

$$q_1^*(a_H) = \frac{a_H - c}{3} + (1-\theta)\frac{a_H - a_L}{6} \quad (5)$$

$$q_1^*(a_L) = \frac{a_L - c}{3} - \theta\frac{a_H - a_L}{6} \quad (6)$$

Thus, assuming that parameters are such that $q_2^*, q_1^*(a_H), q_1^*(a_L)$ as given by 4, 5, 6 are all positive, then these equations characterize the Bayesian Nash Equilibrium of this game.