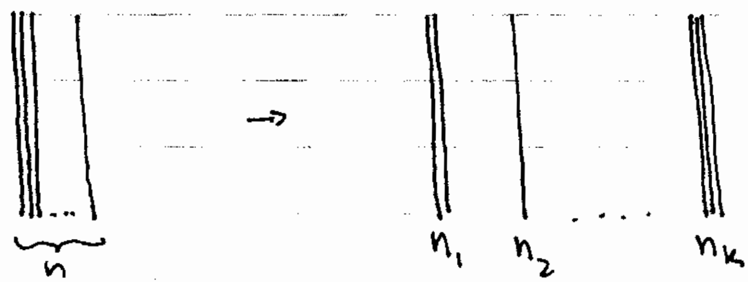
8.871

Solutions to problem set #6

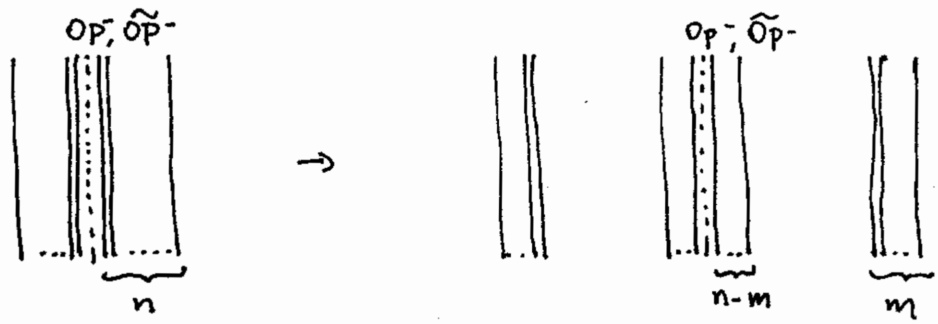
1.

a)



$$U(n) \rightarrow \prod_{i=1}^{k} U(n_i), \quad \sum_{i=1}^{k} n_i = n, \quad k \leq n$$

b)



Op^- : $$SO(2n) \rightarrow SO(2(n-m)) \times U(m) \rightarrow SO(2(n-m)) \times \prod_{i=1}^{k} U(m_i)$$

$$\sum_{i=1}^{k} m_i = m, \quad k \leq m$$

$\widetilde{Op}^+$: $$SO(2n+1) \rightarrow SO(2(n-m)+1) \times U(m) \rightarrow SO(2(n-m)+1) \times \prod_{i=1}^{k} U(m_i)$$

$$\sum_{i=1}^{k} m_i = m, \quad k \leq m$$

c) Same picture as in (b) but with an Op^+ orientifold

$$Sp(n) \to Sp(n-m) \times U(m) \to Sp(n-m) \times \prod_{i=1}^{k} U(m_i)$$

$$\sum_{i=1}^{k} m_i = m, \qquad k \leq m$$

d) Here we can move the branes around, give a vev to $\frac{1}{g_s} = e^{-\Phi}$ or both.

Moving the branes around while keeping $\frac{1}{g_s} = 0$ gives:

$$E_n \to E_{n-m} \times \prod_{i=1}^{k} U(m_i) \qquad \sum_{i=1}^{k} m_i = m, \quad k \leq m$$

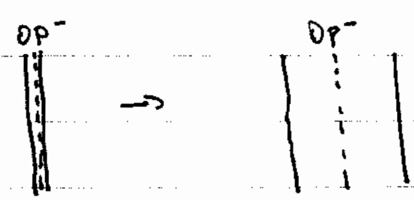
Making $\frac{1}{g} > 0$ gives:

$$E_n \to D_{n-m-1} \times U(1) \times \prod_{i=1}^{k} U(m_i) \qquad \sum_{i=1}^{k} m_i = m, \; k \leq m.$$

$B_1 = A_1$

$$Sp(1) \to U(1)$$
$$SU(2) \to U(1)$$

$D_1 = U(1)$



$$SO(2) \to U(1)$$

$$U(1) \to U(1)$$

$D_2 = A_1 \times A_1$

$$SO(4) \to U(2) = U(1) \times SU(2) \to U(1) \times U(1)$$

$$SU(2) \times SU(2) \to U(1) \times SU(2) \to U(1) \times U(1)$$

$D_3 = A_3$

$$\begin{array}{llll} SO(6) \to & SO(4) \times U(1) & = SU(2) \times SU(2) \times U(1) = & SU(2) \times U(2) \\ & \downarrow & & \\ & SO(2) \times U(2) & = U(1) \times U(2) \to U(1)^3 \to U(3) & \end{array}$$

$$\begin{array}{llll} SU(4) \to & U(1) \times SU(3) = U(3) & \to U(2) \times U(1) & \\ & \downarrow & & \\ & SU(2) \times U(2) & \to SU(2) \times U(1)^2 & \to U(1)^3 \end{array}$$

$B_2 = C_2$

$$\begin{array}{ll} Sp(2) \to & Sp(1) \times U(1) = SU(2) \times U(1) \\ \downarrow & \\ U(2) & \end{array}$$

$$\begin{array}{ll} SO(5) \to & SO(3) \times U(1) = SU(2) \times U(1) \\ \downarrow & \\ U(2) & \end{array}$$

$\underline{E_1 = A_1}$

$$E_1 \xrightarrow{\frac{1}{g_s} > 0} U(1)$$

$$SU(2) \longrightarrow U(1)$$

$\underline{E_2 = A_1 \times U(1)}$

$$E_2 \xrightarrow{\frac{1}{g_s} > 0} SO(2) \times U(1) = U(1)^2$$

$$SU(2) \times U(1) \rightarrow U(1) \times U(1)$$

$\underline{E_3 = A_1 \times A_2}$

$E_3 \longrightarrow E_2 \times U(1) = SU(2) \times U(1) \times U(1)$ → and descending

$\frac{1}{g_s} > 0 \downarrow$

$SO(4) \times U(1) = SU(2) \times SU(2) \times U(1)$ → and descending

$SU(3) \times SU(2) \longrightarrow U(1) \times U(1) \times SU(2)$ → and descending

$\downarrow$

$SU(2) \times SU(2) \times U(1)$ → and descending

$E_4 = A_4$

$$
\begin{tikzcd}
E_4 \arrow[r] \arrow[d] & E_3 \times U(1) = SU(2)\times SU(3)\times U(1) = SU(2)\times U(3) \arrow[r] & \cdots \\
SO(6)\times U(1) = SU(4)\times U(1) \arrow[r] & \cdots &
\end{tikzcd}
$$

$$
\begin{tikzcd}
SU(5) \arrow[r] \arrow[d] & SU(2)\times U(3) \arrow[r] & \cdots \\
SU(4)\times U(1) \arrow[r] & \cdots &
\end{tikzcd}
$$

$E_5 = SO(10)$

$$
\begin{tikzcd}
 & & \vdots & & \vdots \\
E_5 \arrow[r] \arrow[d] & E_4 \times U(1) = & U(5) \arrow[u] \arrow[r] & E_3 \times U(2) = & SU(3)\times SU(2)\times U(2) \arrow[u] \\
SO(8)\times U(1) \arrow[r] & \cdots & & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & SO(4)\times U(3) = SU(2)\times SU(2)\times U(3) \arrow[r] & \cdots \\
SO(10) \arrow[ur] \arrow[r] \arrow[d] & U(5) \arrow[r] & \cdots \\
SO(8)\times U(1) \arrow[r] & \cdots &
\end{tikzcd}
$$

2.

a), b), c)

The little group in d spacetime dimensions is $SO(d-2)$. V_d consists of a vector ($d-2$ d.o.f.), $10-d$ scalars and 8 fermionic d.o.f. . Using $G_d \to G_{d-1} + V_{d-1}$ to work our way down from 10d we find the following content for the gravity multiplet:

10d	$g_{\mu\nu}$	$B_{\mu\nu}$	ϕ	ψ_μ	λ		
	35	28	1	56	$8'$		$64b + 64f$
9d	$g_{\mu\nu}$	$B_{\mu\nu}$	C_μ	ϕ	ψ_μ	λ	
	27	21	7	1	48	8	$56b + 56f$
8d	$g_{\mu\nu}$	$B_{\mu\nu}$	$2C_\mu$	ϕ	$\psi_{\mu\pm}$	$\lambda_\pm$	
	20	15	$2\cdot 6$	1	$20+20'$	$4+4'$	$48b + 48f$
7d	$g_{\mu\nu}$	$B_{\mu\nu}$	$3C_\mu$	ϕ	$2\psi_\mu$	2λ	
	14	10	$3\cdot 5$	1	$2\cdot 16$	$2\cdot 4$	$40b + 40f$
6d	$g_{\mu\nu}$	$B^+_{\mu\nu} + B^-_\mu$	$4C_\mu$	ϕ	$2\psi_{\mu\pm}$	$2\lambda_\pm$	
	$(3,3)$	$(3,1)+(1,3)$	$4\cdot(2,2)$	$(1,1)$	$2(3,2)+2(2,3)$	$2(2,1)+2(1,2)$	

$SO(4) \simeq SU(2) \times SU(2)$ $\quad$ $32\,b + 32\,f$

5d	$g_{\mu\nu}$	$B_{\mu\nu}$	$5C_\mu$	ϕ	$4\psi_\mu$	4λ	
	5	3	$5\cdot 3$	1	$4\cdot 4$	$4\cdot 2$	$24b + 24f$

4d	$g_{\mu\nu}$	$B_{\mu\nu}$	$6C_\mu$	ϕ	$4\psi_\mu$	$4\cdot\lambda$	
	2	1	$6\cdot 2$	1	$4\cdot 2$	$4\cdot 2$	16b + 16f
	(± 2)		(± 1)		$(\pm \frac{3}{2})$	$(\pm \frac{1}{2})$	(helicities)

3d	$g_{\mu\nu}$	$B_{\mu\nu}$	$7C_\mu$	ϕ	ψ_μ	$8\cdot\lambda$	8b + 8f
	0	0	$7\cdot 1$	1	0	$8\cdot 1$	

d), e)

The (0,2) supergravity multiplet in 6d is:

$g_{\mu\nu}$	$5\,B^-_{\mu\nu}$	$4\psi_\mu$	
(3,3)	5 (1,3)	4 (2,3)	24 b + 24 f

3.

Type IIB supergravity contains two $8'$s, two 56s and one $[5]_+$.

The contributions of these to the anomaly are:

$$\hat{I}_{8'} = -\hat{I}_8 = -\frac{tr(R^6)}{725760} - \frac{tr(R^4)tr(R^2)}{552960} - \frac{[tr(R^2)]^3}{1327104}$$

$$\hat{I}_{56} = -495\frac{tr(R^6)}{725760} + 225\frac{tr(R^4)tr(R^2)}{552960} - 63\frac{[tr(R^2)]^3}{1327104}$$

$$\hat{I}_{[5]_+} = 992\frac{tr(R^6)}{725760} - 448\,\frac{tr(R^4)tr(R^2)}{552960} + 128\,\frac{[tr(R^2)]^3}{1327104}$$

There are no gauge fields so only gravitational terms appear.

$$\hat{I}_{IIB} = 2\,\hat{I}_{8'} + 2\,\hat{I}_{56} + \hat{I}_{[5]_+} = 0 \;.$$

4.

a), b)

$\underline{E_8} \supset SO(14) \times U(1)$

$248 \to 91_0 + 14_2 + 14_{-2} + 1_0 + 64_1 + \overline{64}_{-1}$

$\underline{E_7} \supset SO(12) \times U(1)$

$56 \to 12_1 + 12_{-1} + 32_0$ fund

$133 \to 66_0 + 32'_1 + 32'_{-1} + 1_2 + 1_{-2} + 1_0$ adj.

$\underline{E_6} \supset SO(10) \times U(1)$

$27 \to 1_4 + 10_{-2} + 16_1$

$78 \to 1_0 + 45_0 + 16_{-3} + \overline{16}_3$

$\underline{E_5} = SO(10) \supset SO(8) \times U(1)$

$16 \to 8_{v(1)} + 8_{v(-1)}$

$45 \to 1_0 + 28_0 + 8_{s(2)} + 8_{s(-2)}$

$\underline{E_4} = SU(5) \supset SO(6) \times U(1) \simeq SU(4) \times U(1)$

$10 \to 4_3 + 6_{-2}$

$24 \to 1_0 + 15_0 + 4_{-5} + \bar{4}_5$

$\underline{E_3} = SU(2) \times SU(3) \supset SO(4) \times U(1) = SU(2) \times SU(2) \times U(1)$

$(3,2) \to (1,2)_{-2} + (2,2)_1$

$(8,1) + (1,3) \to (1,1)_0 + (3,1)_0 + (2,1)_3 + (2,1)_{-3} + (1,3)_0$

E_2, E_1 trivial.

5. The root system for E_n can be constructed in an 8 dimensional space, using maximal subgroups of E_n.

$E_8 \supset SO(16)$ $\qquad 248 \to 120 + 128$

$\pm e_i \pm e_j \ , \ i,j = 1 \ldots 8 \quad , \quad \frac{1}{2}(\pm e_1 \ldots \pm e_8)$ with $\prod(-) = +$

$E_7 \supset SO(12) \times SU(2)$ $\qquad 133 \to (1,3) + (32',2) + (66,1)$

$\pm e_i \pm e_j \ , \ i,j = 1 \ldots 6 \quad , \quad \pm(e_7 - e_8) \ , \ \pm \frac{1}{2}(e_7 - e_8) + \frac{1}{2}(\pm e_1 \ldots \pm e_6)$

with $\prod(-) = -$

$E_n \ , \quad 6 \geq n \geq 2 \qquad E_n \supset SO(2n-2) \times U(1)$

$\pm e_i \pm e_j \ , \ i,j = 1 \ldots n-1 \quad , \quad \pm \frac{1}{2}(e_8 - e_7 - \cdots - e_n \pm e_{n-1} \ldots \pm e_1)$

$\prod(-) = +$ with $+$ in front

$\prod(-) = -$ with $-$ in front.

Note that this is just one of many possible constructions.