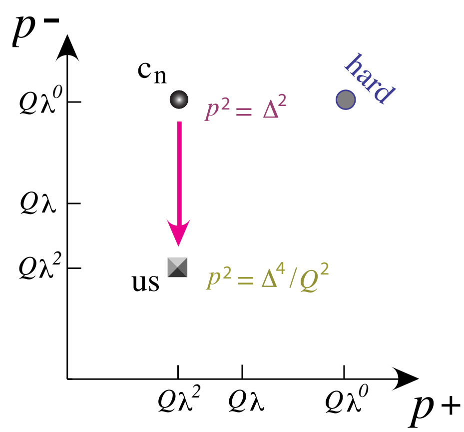


Figure 8: $\mathrm{SCET_I}$ zero-bin from one collinear direction scaling into the ultrasoft region.

there are ultrasoft subtractions for the collinear modes, but no collinear subtractions for the ultrasoft modes.

It also should be remarked that depending on the choice of infrared regulators, the subtraction terms very often give scaleless integrations of combined dimension $d-4$ in dimensional regularization. These then just yield terms proportional to $(1/\epsilon^j_{\rm UV} - 1/\epsilon^j_{\rm IR})$, which are only important to properly interpret whether factors of $1/\epsilon$ from the naive collinear loop integration that used Eq. (4.60) are UV poles that require a counterterm, or are IR poles that correspond with physical IR singularities in QCD. In particular this is often the case for the simplest measurements with an offshellness IR regulator for collinear external lines. More complicated measurements (such as those depending on a jet algorithm) or other choices of IR regulators (like a gluon mass or a cutoff) will lead to zero-bin subtractions that are not scaleless.

We will return to this discussion when carrying out explicit examples of collinear loops in section 7.

5 Symmetries of SCET

In quantum field theory Lagrangians are often built up from symmetries and dimensional analysis. So far our leading order SCET Lagrangians were derived directly from QCD at tree level. To go further, and determine whether loops can change the form of the Lagrangians (through Wilson coefficients or additional operators) we need to exploit symmetries and power counting. In this section, we will introduce the SCET gauge symmetries and reparameterization invariance (RPI) as a way to constrain SCET operators. We will find that the gauge symmetry formalism is a simple restatement of the standard QCD picture except with two separate gauge fields. RPI is a manifestation of the Lorentz symmetry which was broken by the choice of light-cone coordinates, and which acts independently in each collinear sector. We will also examine the spin symmetries of the SCET Lagrangian, although here we will find that there are no surprises beyond what we know from QCD.

5.1 Spin Symmetry

To examine the spin symmetry of $\mathcal{L}^{(0)}_{n\xi}$ it is convenient to write the Lagrangian in a two component form. From Eq. (3.11) we can write

$$\xi_n = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_n \\ \sigma^3 \varphi_n \end{pmatrix} , \tag{5.1}$$

where φ_n is a two-component field, dim φ_n = dim $\xi_n = 3/2$, and $\varphi_n \sim \lambda$. With this two-component field the SCET Lagrangian is

$$\mathcal{L} = \varphi_n^\dagger \left[in \cdot D + iD^\mu_{n\perp} \frac{1}{i\bar{n} \cdot D} iD^\nu_{n\perp} (g^\perp_{\mu\nu} + i\epsilon^\perp_{\mu\nu}\sigma_3) \right] \varphi_n \,. \tag{5.2}$$

Due to the σ_3 the spin symmetry is not an $SU(2)$, but rather just the $U(1)$ helicity symmetry corresponding to spin along the direction of motion n of the collinear fields. The relevant generator is

$$S_z = i\epsilon^{\mu\nu}_\perp [\gamma^\mu, \gamma^\nu] \to h = \sigma_3. \tag{5.3}$$

We can relate this symmetry to the chiral symmetry by noting that under chiral symmetry ξ_n transforms as

$$\xi_n \to \gamma^5 \xi_n = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma^3 \phi_n \\ \phi_n \end{pmatrix} \qquad \text{so} \quad \varphi_n \to \sigma_3 \varphi_n \,. \tag{5.4}$$

This $U(1)_A$ axial-symmetry is broken by fermion masses and non-perturbative instanton effects. Just like in QCD it is a useful symmetry for determining the structure of perturbation theory results. This implies that in SCET it is useful for determining the basis of operators we obtain when integrating out hard particles, and for relating Wilson coefficients.

5.2 Gauge Symmetry

The standard gauge transformation in QCD is

$$U(x) = \exp[i\alpha^A(x) T^A] \,. \tag{5.5}$$

When we go to SCET we need to have gauge transformations which do not inject large momenta into our EFT fields, that is, the transformations must leave us withing our effective field theory. For example, if we used a gauge transformation where α^A satisfied

$$i\partial_\mu \alpha^A \sim Q\alpha^A \tag{5.6}$$

then $\xi_n' = U(x)\xi_n$ would no longer have $p^2 \le Q^2\lambda^2$ and would not be described by SCET. There are two acceptable SCET gauge transformations which are defined by their momentum scale. They are

$$\text{collinear} \quad U_n(x): \quad i\partial^\mu U_n(x) \sim Q(\lambda^2, 1, \lambda) U_n(x) \tag{5.7}$$

$$\text{ultrasoft} \quad U_u(x): \quad i\partial^\mu U_u(x) \sim Q(\lambda^2, \lambda^2, \lambda^2) U_u(x). \tag{5.8}$$

There is also a global color transformation which for convenience we group together with the U_u. To avoid double counting, in the collinear transformation we fix $U_n(n \cdot x = -\infty) = 1$. We can implement a collinear gauge transformation on the collinear fields ξ_{n,p_l} via a Fourier transform. Since $\psi(x) \to U(x)\psi(x)$ is equivalent to $\tilde{\psi}(p) \to \int dq\, \tilde{U}(p-q)\tilde{\psi}(q)$, the transformation involves a convolution in label momenta. To understand how the collinear gauge field transforms under a collinear gauge transformation, we need to recall that there is a background usoft gauge field A^μ_{us}. Consequently we must take $\partial_\mu \to \mathcal{D}^{us}_\mu$ so that A^μ_n transforms as a quantum field in an A^μ_{us} background. Therefore the collinear gauge transformations are

$$\begin{aligned} \xi_{n,p}(x) &\to (U_n)_{p-q}(x)\,\xi_{n,q}(x)\,, \\ A^\mu_{n,p}(x) &\to U_{n,p-q}(x)\Big(gA^\mu_{n,q-q'}(x) + \delta_{q,q'}\,i\mathcal{D}^\mu_{us}\Big)U^\dagger_{n,q'}(x)\,, \end{aligned} \tag{5.9}$$

where we sum over repeated momentum label indices. It is convenient to setup a matrix notation for these convolutions by defining

$$(\hat{U}_n)_{p_\ell,q_\ell} \equiv (U_n)_{p_\ell - q_\ell}\,, \tag{5.10}$$

where the LHS is the (p_ℓ, q_ℓ) element of a matrix in momentum space, and the RHS is a number (both are of course also matrices in color). Then Eq. (5.9) with a sum over repeated indices becomes $\xi_{n,p_\ell} \to (\hat{U}_n)_{p_\ell,q_\ell}\xi_{n,q_\ell}$. And if we suppress indices then we have $\xi_n \to (\hat{U}_n)\xi_n$.

Finally the ultrasoft fields do not transform under a collinear gauge transformation, since the resulting field would have a large momentum and hence no longer be ultrasoft. Essentially this means that by definition our collinear gauge transformations do not turn ultrasoft gluons into collinear gluons.

Collinear Gauge Transformations : $U_n(x)$

Therefore our set of Collinear Gauge Transformations with the matrix notation for momentum space labels are

- $\xi_n(x) \to \hat{U}_n(x)\xi_n(x)$
- $A^\mu_n(x) \to \hat{U}_n(x)(A^\mu_n(x) + \frac{i}{g}\mathcal{D}^\mu_{us})\hat{U}^\dagger_n(x)$
- $q_{us}(x) \;\to\; q_{us}(x)$
- $A^\mu_{us}(x) \;\to\; A^\mu_{us}(x)$

When using the momentum label notation the condition $U_n(n\cdot x = -\infty) = 1$ becomes $(U_n)_{p_\ell \to 0} = \delta_{p_\ell,0}$ for the zero-bin $p_\ell = 0$ (the ultrasoft transformations do not modify large momenta, but the collinear transformations do).

For usoft gauge transformations, the field ξ_n and A^μ_n transform as quantum fields under a background gauge transformation, which is to say they transform as matter fields with the appropriate representation. The usoft fields have their usual gauge transformations from QCD.

Usoft Gauge Transformations : $U_u(x)$

Therefore for the Ultrasoft Gauge Transformations we have

- $\xi_n(x) \to U_{us}(x)\xi_n(x)$
- $A^\mu_n(x) \to U_{us}(x)A^\mu_n(x)U^\dagger_{us}(x)$
- $q_{us}(x) \;\to\; U_{us}(x)q_{us}(x)$
- $A^\mu_{us}(x) \;\to\; U_{us}(x)(A^\mu_{us}(x) + \frac{i}{g}\partial^\mu)U^\dagger_{us}(x)$

Since all of the fields transform, these ultrasoft gauge transformations connect fields in operators that are mixtures of collinear and ultrasoft fields. This differs from $U_n(x)$ which only connects collinear fields to each other.

It is important to note that the U_n and U_u gauge transformations are homogeneous in the power counting, so they do not change the order in λ for transformed operators. They are exact, there are no corrections to these transformations at higher orders in λ, and thus the power expansion will have gauge invariant operators at each order in λ.

The transformation of the fields yield transformations for objects that are built from the fields. An important case is the Wilson line W_n which is like the Fourier transform of $W(x,-\infty)$. In QCD a general Wilson line with the gauge field along a path will transform on each end as $W(x,y) \to U(x)W(x,y)U^\dagger(x)$. For the collinear gauge transformation we have fields in momentum space for labels, and position space representing residual momenta, and $U_n^\dagger(-\infty) = 1$, so the Wilson line transforms only on one side for collinear transformations. For ultrasoft transformations $W_n(x)$ is actually a local operator with all fields at x, and the product of multiple $\bar{n}\cdot A_n(x) \to U_{us}(x)\bar{n}\cdot A_n(x)U_{us}^\dagger(x)$ leads to one U_{us} and $U_{us}^\dagger$ on the left and right. Thus with the matrix notation

$$\begin{aligned} \text{collinear}: \qquad & W_n(x) \to \hat{U}_n(x)W_n(x)\,, \\ \text{ultrasoft}: \qquad & W_n(x) \to U_{us}(x)W_n(x)U_{us}^\dagger(x)\,. \end{aligned} \tag{5.11}$$

It is useful to consider the correspondence between the appearance of the Wilson line W_n in operators, and the collinear gauge symmetry. If we consider our example of the heavy-to-light current then without the Wilson line the operator $\bar{\xi}_n\Gamma h_v^{us}$ is not gauge invariant, transforming to $\bar{\xi}_n U_n^\dagger \Gamma h_v^{us}$. Here the ξ_n transforms because collinear gluons couple to ξ_n without taking it offshell, but h_v^{us} does not transform because this ultrasoft field can not interact with the collinear gluons while remaining near its mass shell. But recall that when the offshell collinear gluons are accounted for in matching onto the SCET operator that the $\bar{n}\cdot A_n \sim \lambda^0$ gluons generate a Wilson line W_n, so the complete result from tree level matching is

$$J_{\rm SCET} = \bar{\xi}_n W_n \Gamma h_v^{us}. \tag{5.12}$$

Now under a collinear gauge transformation $J_{\rm SCET} \to \bar{\xi}_n \hat{U}_n^\dagger \hat{U}_n W_n \Gamma h_v^{us} = \bar{\xi}_n W_n \Gamma h_v^{us}$, so the current is collinear gauge invariant. Under an ultrasoft gauge transformation $J_{\rm SCET} \to \bar{\xi}_n U_{us}^\dagger U_{us} W_n U_{us}^\dagger \Gamma U_{us} h_v^{us} = \bar{\xi}_n W_n \Gamma h_v^{us}$, so the current is also ultrasoft gauge invariant. Thus the leading order attachments of $\bar{n}\cdot A_n$ gluons that lead to the Wilson line W_n are necessary to obtain a gauge invariant result. Furthermore, by gauge symmetry the fact that the product $\bar{\xi}_n W_n$ appears in the operator will not be modified by loop corrections. We will take up what modifications can be generated by loop corrections in section 6.2 below.

Gauge symmetry forces gauge fields and derivatives to occur in the following combinations

$$\begin{aligned} in\cdot D &= in\cdot\partial + gn\cdot A_n + gn\cdot A_{us}\,, \\ iD_{n\perp}^\mu &= \mathcal{P}_\perp^\mu + gA_{n\perp}^\mu\,, \\ i\bar{n}\cdot D_n &= \overline{\mathcal{P}} + g\bar{n}\cdot A_n\,, \\ iD_{us}^\mu &= i\partial^\mu + gA_{us}^\mu\,. \end{aligned} \tag{5.13}$$

We see that gauge symmetry is a powerful tool in determining the structure of operators. It is reasonable to ask, is power counting and gauge invariance enough to fix the leading order Lagrangian $\mathcal{L}_{n\xi}^{(0)}$ for ξ_n? Only the operators $in\cdot D$ and $(1/\overline{\mathcal{P}})D_{n\perp}D_{n\perp}$ are $\mathcal{O}(\lambda^2)$ and have the correct mass dimension. The latter will have the correct gauge transformation properties once we include W_ns. Nevertheless, nothing so far rules out the operator

$$\bar{\xi}_n iD_{n\perp}^\mu W_n \frac{1}{\overline{\mathcal{P}}} W_n^\dagger iD_{n\,\mu}^\perp \frac{\bar{n}\!\!\!/}{2}\xi_n \tag{5.14}$$

which is gauge invariant and has the correct λ scaling. To exclude this term we need to consider another symmetry prinicple, namely reparameterization invariance.

5.3 Reparamterization Invariance

Our choice of the n and $\bar{n}$ reference vectors explicitly breaks Lorentz symmetry in SCET, much like v does in HQET. Part of this breaking is natural, SCET_{I} is describing a collimated jet which explicitly picks out a corresponding n-collinear direction about which the field theory is describing fluctuations. There is also a part of the symmetry that is restored by the freedom we have in choosing our n and $\bar{n}$ vectors, which is a reparameterization invariance (RPI). A second attribute of the reparameterization symmetry is the freedom we have in splitting momenta between label and residual components. We will explore these two in turn.

The only required property of a set of n, $\bar{n}$ basis vectors is that they satisfy

$$n^2 = \bar{n}^2 = 0, \qquad n \cdot \bar{n} = 2. \tag{5.15}$$

Consequently a different choice for n and $\bar{n}$ can yield a valid set of light-cone coordinates as long as our result still obeys (5.15). Specifically, there are three sets of transformations which can be made on a set of light-cone coordinates to obtain another, equally valid, set.

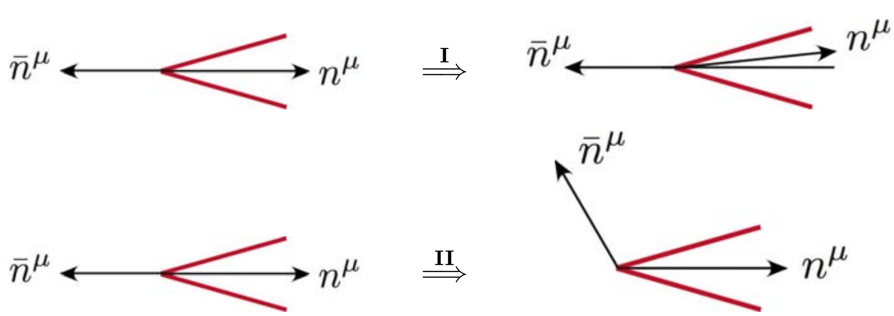
$$\begin{array}{ccc} \underline{\mathbf{I}} & \underline{\mathbf{II}} & \underline{\mathbf{III}} \\ n_\mu \to n_\mu + \Delta^\perp_\mu & n_\mu \to n_\mu & n_\mu \to e^\alpha n^\mu \\ \bar{n}_\mu \to \bar{n}_\mu & \bar{n}_\mu \to \bar{n}_\mu + \varepsilon^\perp_\mu & \bar{n}_\mu \to e^{-\alpha}\bar{n}_\mu \end{array} \tag{5.16}$$

where $\bar{n}\cdot\varepsilon^\perp = n\cdot\varepsilon^\perp = \bar{n}\cdot\Delta^\perp = n\cdot\Delta^\perp = 0$. The first two transformations are inifinitesimal. The third is a finite transformation (where the form is simple), but can be made infinitesimal by expansion in α. These transformations must leave a collinear momentum collinear in the same directions, so we can obtain the λ-scaling of these parameters by noting that:

$$\begin{aligned} &\lambda^2 \sim n\cdot p \to n\cdot p + \Delta^\perp \cdot p_\perp \Longrightarrow \Delta^\perp \sim \lambda^1 \\ &\lambda^0 \sim \bar{n}\cdot p \to \bar{n}\cdot p + \varepsilon^\perp \cdot p_\perp \Longrightarrow \varepsilon^\perp \sim \lambda^0 \\ &\alpha \sim \lambda^0 \end{aligned} \tag{5.17}$$

Thus only $\Delta^\perp$ is constrained by the power counting, while large changes are allowed for α and $\epsilon^\perp$. These RPI transformations are a manifestation of the Lorentz symmetry which was broken by introducing the vectors n and $\bar{n}$. The five infinitesimal parameters $\Delta^\perp_\mu$, $\varepsilon^\perp_\mu$, and α correpsond to the five generators of the Lorentz group which were broken by introducing the vectors n and $\bar{n}$. These generators are defined by $\{n_\mu M^{\mu\nu}, \bar{n}_\mu M^{\mu\nu}\}$ or in terms of our standard light-cone coordinates $Q_1^\pm = J_1 \pm K_2$, $Q_2^\pm = J_2 \pm K_1$, and K_3. Here $M^{\mu\nu}$ are the usual 6 antisymmetric SO(3,1) generators.

If we start with our canonical basis choice $n = (1,0,0,1)$ and $\bar{n} = (1,0,0,-1)$ then we can visualize the Type I and Type II transformations as changes in the directions orthogonal to the $\hat{z}$ direction



and we can visualize Type III transformations as boosts in the $\hat{z}$ direction. For Type I we can transform n by an $\mathcal{O}(\lambda)$ amount, into another vector within this collinear sector, without changing any of the physics. For Type II we recall that the auxillary vector $\bar{n}$ was chosen simply to enable us to decompose momenta, so their is a considerable freedom in its definition, and this corresponds to the freedom to make large transformations. (If we start with a more general choice for n and $\bar{n}$ that satisfies Eq. (5.15) then the picture for the Type-III transformation is more complicated than a simple boost.)

The implications of the Type III transformation for SCET operators are very simple, n and $\bar{n}$ must appear in operators either together, or with one factor of $\bar{n}/n$ in both the numerator and denominator. That is, in one of the combinations

$$(A\cdot n)(B\cdot\bar{n}), \qquad \frac{A\cdot n}{B\cdot n}, \qquad \frac{A\cdot\bar{n}}{B\cdot\bar{n}} \tag{5.18}$$

where A^μ and B^μ are arbitrary 4-vectors.

In order to derive the complete set of transformation relations we must also determine how $p_\perp^\mu$ transforms. Recall that the definition of $p_\perp$ depends on n and $\bar{n}$, since it is orthogonal to n and $\bar{n}$, satisfying $n\cdot p_\perp = 0 = \bar{n}\cdot p_\perp$. We can work out its transformation by noting that the four vector p^μ does not depend on the basis for coordinates. Using the Type-I transformation as an example

$$p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}n\cdot p + p_\perp^\mu \implies \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}n\cdot p + p_\perp^\mu + \frac{\Delta_\perp^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2}\Delta_\perp\cdot p_\perp + \delta_{\rm I}(p_\perp^\mu) = p^\mu\,. \tag{5.19}$$

Thus $p_\perp^\mu$ must transform as

$$p_\perp^\mu \xRightarrow{\mathbf{I}} p_\perp^\mu - \frac{\bar{n}^\mu}{2}\Delta_\perp\cdot p_\perp - \frac{\Delta_\perp^\mu}{2}\bar{n}\cdot p\,. \tag{5.20}$$

The projection relation $(\not{n}\bar{\not{n}}/4)\xi_n = \xi_n$ also implies that $\xi_n \to [1 + (\not{\Delta}^\perp\bar{\not{n}})/4]\xi_n$. Similar relations can also be worked out for type-II transformations, for example

$$p_\perp^\mu \xRightarrow{\mathbf{II}} p_\perp^\mu - \frac{n^\mu}{2}\varepsilon_\perp\cdot p_\perp - \frac{\varepsilon_\perp^\mu}{2}n\cdot p\,. \tag{5.21}$$

Summarizing all the type-I and type-II transformations on vectors and fields (using D^μ as a typical vector) we have

$$\begin{array}{c|c}
\underline{\mathbf{I}} & \underline{\mathbf{II}} \\
n\to n+\Delta^\perp & n\to n \\
\bar{n}\to\bar{n} & \bar{n}\to\bar{n}+\varepsilon^\perp \\
n\cdot D\to n\cdot D+\Delta^\perp\cdot D^\perp & n\cdot D\to n\cdot D \\
D_\mu^\perp\to D_\mu^\perp-\frac{\Delta_\mu^\perp}{2}\bar{n}\cdot D-\frac{\bar{n}^\mu}{2}\Delta^\perp\cdot D & D_\mu^\perp\to D_\mu^\perp-\frac{\epsilon_\mu^\perp}{2}n\cdot D-\frac{n^\mu}{2}\epsilon^\perp\cdot D \\
\bar{n}\cdot D\to\bar{n}\cdot D & \bar{n}\cdot D\to\bar{n}\cdot D+\epsilon^\perp\cdot D^\perp \\
\xi_n\to\left(1+\frac{1}{4}\not{\Delta}^\perp\bar{\not{n}}\right)\xi_n & \xi_n\to\left(1+\frac{1}{2}\not{\epsilon}^\perp\frac{1}{i\bar{n}\cdot D}i\not{D}_\perp\right)\xi_n \\
W_n\to W_n & W_n\to\left(1-\frac{1}{i\bar{n}\cdot D}\epsilon^\perp\cdot iD^\perp\right)W_n
\end{array} \tag{5.22}$$

For type-III transformations $p_\perp^\mu$ does not transform, and neither does W_n.

We can show that our leading order SCET Lagrangian

$$\mathcal{L}_{n\xi}^{(0)} = \xi_n in\cdot D\frac{\bar{\not{n}}}{2}\xi_n + \bar{\xi}_n i\not{D}_{n,\perp}\frac{1}{i\bar{n}\cdot D}i\not{D}_{n\perp}\frac{\bar{\not{n}}}{2}\xi_n \tag{5.23}$$

is invariant under these transformations. Under a type-I transformation we have

$$\begin{aligned}\delta_{\mathrm{I}}\mathcal{L}^{(0)}_{n\xi} &= \delta_{\mathrm{I}}\left(\xi_n in\cdot D\frac{\bar{\not{n}}}{2}\xi_n\right) + \delta_{\mathrm{I}}\left(\bar{\xi}_n i\not{D}_{n,\perp}\frac{1}{i\bar{n}\cdot D}i\not{D}_{n\perp}\frac{\bar{\not{n}}}{2}\xi_n\right) \\ &= \bar{\xi}_n i\Delta^{\perp}\cdot D^{\perp}\frac{\bar{\not{n}}}{2}\xi_n - \bar{\xi}_n i\Delta^{\perp}\cdot D^{\perp}\frac{\bar{\not{n}}}{2}\xi_n \\ &= 0\end{aligned} \tag{5.24}$$

where to obtain the second line we used $\bar{\not{n}}^2 = 0$, the orthogonal properties of the 4-vectors, and ignored quadratic combinations of the $\Delta^{\perp}$ infinitesimal. Hence the SCET quark Lagrangian obtained from tree level matching is indeed invaraiant under δ_{I}. However, this Lagrangian is not completely determined by invariance under δ_{I}. For example, the term we encountered at the end of the gauge symmetry section transforms as

$$\delta_{(I)}\left(\bar{\xi}_n iD^{\perp}_{\mu}\frac{1}{i\bar{n}\cdot D}iD^{\perp\,\mu}\frac{\bar{\not{n}}}{2}\xi_n\right) = -\bar{\xi}_n i\Delta^{\perp}\cdot D\frac{\bar{\not{n}}}{2}\xi_n \tag{5.25}$$

which is the same transformation as for the second term in (5.24). Consequently, we may replace the second term with this new term with no violation of power counting, gauge symmetry, or RPI type-I. This ambiguity is only resolved by using invariance under RPI of type-II. The detailed calculation is given in [7] with the final result that our Lagrangian $\mathcal{L}^{(0)}_{n\xi}$ remains invariant under δ_{II} while the term given in (5.14) does transforms in a way that can not be compensated by any other leading order term in the Lagrangian. Therefore our SCET$_{\mathrm{I}}$ Lagrangian $\mathcal{L}^{(0)}_{n\xi}$ is unique by power counting, gauge invariance, and reparameterization invariance. This also implies that its form is not modifed by loop corrections. In general type-III RPI will restrict operators at the same order in λ, type-I restricts operators at different orders in λ, and type-II will restrict operators at both the same and different orders in λ.

Reparameterization invariance also manifests itself in the ambiguity of label and residual momenta decomposition. We can separate the total momenta

$$\bar{n}\cdot p = \bar{n}\cdot(p_\ell + p_r) \qquad p^{\mu}_{\perp} = p^{\mu}_{l\,\perp} + p^{\mu}_{r\,\perp} \tag{5.26}$$

into p_ℓ and p_r in different ways as long as we maintain the power counting. Specifically, a transformation that takes

$$\mathcal{P}^{\mu} \to \mathcal{P}^{\mu} + \beta^{\mu} \qquad i\partial^{\mu} \to i\partial^{\mu} - \beta^{\mu} \tag{5.27}$$

implements this freedom. The transformation on $i\partial^{\mu}$ is induced by the β-transformation of the fields, for example

$$\xi_{n,p}(x) \to e^{i\beta(x)}\xi_{n,p+\beta}(x)\,. \tag{5.28}$$

The set of these β transformations also determines the space of equivalent decompositions $\mathcal{I}$ that we mod out by when constructing pairs of label and residual momenta components (p_ℓ, p_r) in $\mathbb{R}^3\times\mathbb{R}^4/\mathcal{I}$. Invariance under this RPI requires the combination

$$\mathcal{P}^{\mu} + i\partial^{\mu} \tag{5.29}$$

to be grouped together for collinear fields. Since $\overline{\mathcal{P}}$ and $i\bar{n}\cdot\partial$ (and $\mathcal{P}^{\mu}_{\perp}$ and $i\partial^{\mu}_{\perp}$) appear at different orders in the power counting, this RPI connects the Wilson coefficients of operators at different orders in λ.

A natural question is how to gauge the connection between label and residual derivatives in (5.29). Recall that the gauge transformations for derivatives are

$$\begin{array}{rcc} & \text{collinear} & \text{ultrasoft} \\ iD_{n\perp} \to & U_c iD_{n\perp}U_c^{\dagger} & U_{us}iD_{n\perp}U_{us}^{\dagger} \\ i\bar{n}\cdot D_n \to & U_c i\bar{n}\cdot D_n U_c^{\dagger} & U_{us}i\bar{n}\cdot D_n U_{us}^{\dagger}m \\ in\cdot D \to & U_c in\cdot DU_c^{\dagger} & U_{us}in\cdot DU_{us}^{\dagger} \\ iD^{\mu}_{us} \to & iD^{\mu}_{us} & U_{us}iD^{\mu}_{us}U_{us}^{\dagger} \end{array}$$

The most natural guess for the gauging of (5.29) would be

$$iD^\mu_{n\perp} + iD^\mu_{us\perp}\,, \qquad i\bar n\cdot D_n + i\bar n\cdot D_{us}\,. \tag{5.30}$$

However, with the above transformations these combinations do not have uniform transformations under the gauge symmetries, since D_{us} does not transform under U_n. We can rectify this problem by introducing our Wilson line W_n into the combination of these derivatives. The unique result which preserves the SCET gauge symmetries without changing the power counting of the terms is

$$iD^\mu_\perp \equiv iD^\mu_{n\perp} + W_n iD^{us,\mu}_{n\perp} W_n^\dagger \tag{5.31}$$

$$i\bar n\cdot D \equiv i\bar n\cdot D_n + W_n i\bar n\cdot D_{us} W_n^\dagger\,, \tag{5.32}$$

where W_n transforms as $W_n \to U_n W_n$. Stripping off the regular derivative terms, the extra multi-gluon terms appearing in the formulae like $A^\mu_\perp = A^\mu_{n\perp} + A^\mu_{us\perp} + \ldots$ are the terms we denoted by ellipses in (4.9). These terms are necessary to form gauge invariant subleading operators.

Like in HQET, the RPI in SCET connects the Wilson coefficients of leading and λ-suppressed Lagrangians and external currents and operators. As an example, applying the connection to the term $\bar\xi_n i\not{D}_{n,\perp} W_n(1/\overline{\mathcal{P}})\, W_n^\dagger i\not{D}_{n,\perp}\xi_n$ in $\mathcal{L}^{(0)}_{n\xi}$ yields the subleading Lagrangian that couples collinear quarks to $A^{us}_\perp$ gluons,

$$\mathcal{L}^{(1)}_{n\xi} = (\bar\xi_n W_n) i\not{D}^{us}_\perp \frac{1}{\overline{\mathcal{P}}} (W_n^\dagger i\not{D}_{n,\perp}\xi_n) + (\bar\xi_n i\not{D}_{n,\perp} W_n)\frac{1}{\overline{\mathcal{P}}} i\not{D}^{us}_\perp (W_n^\dagger \xi_n). \tag{5.33}$$

The complete set of $\mathrm{SCET_I}$ Lagrangian interactions up to $\mathcal{O}(\lambda^2)$ can be found in Ref. [10].

5.4 Discrete Symmetries

After considering the residual form of Lorentz symmetry encoded in reparameterization invariance it is natural to consider how our SCET fields transform under C, P, and T transformations. In this case we will satisfy ourselves with the transformations of the collinear field $\xi_{n,p}$. We have

$$\begin{aligned} C^{-1}\xi_{n,p}(x)C &= -[\bar\xi_{n,-p}(x)\mathcal{C}]^T \\ P^{-1}\xi_{n,p}(x)P &= \gamma_0\xi_{\bar n,\tilde p}(x_P) \\ T^{-1}\xi_{n,p}(x)T &= \mathcal{T}\xi_{\bar n,\tilde p}(x_T) \end{aligned} \tag{5.34}$$

where $n = (1,0,0,1)$, $\bar n = (1,0,0,-1)$, $p \equiv (p^+, p^-, p^\perp)$, $x \equiv (x^+, x^-, x^\perp)$, $\mathcal{C}$ is the standard matrix induced by charge conjugation symmetry, and we have defined $\tilde p = (p^-, p^+, -p^\perp)$ as well as $x_P = (x^-, x^+, -x^\perp)$ and $x_T = (-x^-, -x^+, x^T)$.

5.5 Extension to Multiple Collinear Directions

For processes with more than one energetic hadron, or more than one energetic jet our list of degrees of freedom must include more than one type of collinear mode, and hence more than one type of collinear quark and collinear gluon. When two collinear modes in different directions interact, the resulting particle is offshell, and does not change the formulation of the leading order collinear Lagrangians. Therefore the Lagrangian with multiple collinear directions is

$$\mathcal{L}^{(0)}_{\mathrm{SCET_I}} = \mathcal{L}^{(0)}_{us} + \sum_n \left[\mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{ng}\right]. \tag{5.35}$$

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