operator $\bar{\xi}_{n,p}S^{\dagger}\Gamma^{\mu}Wh_v$ which is not equivalent to (10.6) because S and W do not commute. This apparent problem is solved by considering the remaining two diagrams of the same order as this one

Diagrams.

These diagrams both yield the current

$$\text{Fig}() = \text{Fig}() = \frac{g^2}{2}if^{abc}T^c\frac{n^{\mu}}{n\cdot q^s}\frac{\bar{n}^{\nu}}{\bar{n}\cdot q_c}\bar{\xi}_{n,p}\Gamma h_v. \tag{10.8}$$

Adding the three graps together, revereses the order of the color indices (by virtue of $[T^a, T^b] = if^{abc}$) to give

$$\text{Fig}() + \text{Fig}() + \text{Fig}() = -g^2\frac{n^{\mu}}{n\cdot q_s}\frac{\bar{n}^{\nu}}{\bar{n}\cdot q_c}\bar{\xi}_{n,q}T^b\Gamma T^a h_v \tag{10.9}$$

which is the correct ordering for the gauge invariant current in (10.6). This procedure can be extended to all orders as in (Refererence).

We may construct SCET operators by another method using SCET$_I$. The basis of the procedure comes from the fact that soft-modes in SCET$_{II}$ and usoft modes in SCET $_I$ have the same momentum; it is only the collinear fields which have distinct momentuma. The exact procedure for obtaining SCET$_{II}$ is

1. Match QCD onto SCET$_I$

2. Redefine fields with the usoft wilson line Y_n so that usoft interactions are only present in currents

3. Match SCET$_I$ onto SCET$_{II}$ by taking $Y_n \rightarrow S_n$.

As an example of the above procedure we may construct the SCET$_{II}$ current postulate above.

1. Matching QCD onto SCET$_I$
$$J = \bar{u}\Gamma^{\mu}b \ \rightarrow \ J_I = (\bar{\xi}_nW)\Gamma^{\mu}h_v \tag{10.10}$$

2. Redefining fields so that usoft interactions are only present in currents
$$J_I = (\bar{\xi}_n^{(0)}W^{(0)})\Gamma^{\mu}Y^{\dagger}h_v \tag{10.11}$$

3. Matching SCET$_I$ onto SCET$_{II}$ by taking $Y_n \rightarrow S_n$.
$$J_{II} = (\bar{\xi}_n^{(0)}W^{(0)})\Gamma^{\mu}S^{\dagger}h_v \tag{10.12}$$

11 SCET$_{\text{II}}$ Applications

(**ROUGH**) In this section we will apply the SCET$_{\text{II}}$ formalism developed in previous sections to various processes to illustrate the formalism

- $\gamma^*\gamma \rightarrow \pi^0$

- $B \rightarrow D\pi$

- The Massive Gauge Boson Sudakov Form Factor

- p_T distribution in Higgs production

- Jet broadening

A distinguishing feature of these processes is whether they involve a new type of divergence that requires a renormalization procedure, known as rapidity divergences. The first two processes do not, while the last three do. We will discuss these divergences in detail for the massive gauge boson form factor, and then be very brief about the last two examples.

11.1 $\gamma^* \gamma \to \pi^0$

11.2 $B \to D\pi$

(**ROUGH**) As another exclusive scattering process, we analyze $B \to D\pi$. We may use the SCET framework here because the hard scales $Q = \{m_b, m_c, E_\pi\} \gg \Lambda_{QCD}$. At the scale $\mu \sim m_b$ the QCD operators represented by the weak Hamiltonian are

$$H_W = \frac{4G_F}{\sqrt{2}} V_{ud}^\dagger V_{cb} [C_{\mathbf{0}}^F(\mu_0) O_{\mathbf{0}}(\mu_0) + C_{\mathbf{8}}^F(\mu_0) O_{\mathbf{8}}(\mu_0)] \tag{11.1}$$

where

$$O_{\mathbf{0}} = [\bar{c}\gamma^\mu P_L b][\bar{d}\gamma_\mu P_L u] \tag{11.2}$$

$$O_{\mathbf{8}} = [\bar{c}\gamma^\mu P_L T^a b][\bar{d}\gamma_\mu P_L T^a u]. \tag{11.3}$$

We want to factorize the matrix element $\langle D\pi | O_{\mathbf{0},\mathbf{8}} | B \rangle$. We can represent this factorization diagrammatically as (INSERT FIG) where there are no gluons between π quarks and B/D quarks. For this process we expect a $B \to D$ form factor (Isgur-Wise form factor) and a pion wavefunction/distribution. This factorization will be possible because the particles B and D have soft momentum scaling and π has collinear scalings. Specifically $p_c^2 \sim \Lambda^2$ and we therefore use SCET$_{\mathrm{II}}$ to describe this process.

First, matching the QCD Hamiltonian onto SCET we need the operators

$$Q_{\mathbf{0}}^{1,5} = [\bar{h}_v^{(c)} \Gamma_h^{1,5} h_v^{(b)}] \, [\bar{\xi}_{n,p}^{(d)} W \Gamma_l C_0(\overline{\mathcal{P}}_+) W^\dagger \xi_{n,p}^{(u)}] \tag{11.4}$$

$$Q_{\mathbf{8}}^{1,5} = [\bar{h}_v^{(c)} \Gamma_h^{1,5} T^a h_v^{(b)}] \, [\bar{\xi}_{n,p}^{(d)} W \Gamma_l C_8(\overline{\mathcal{P}}_+) T^a W^\dagger \xi_{n,p}^{(u)}] \tag{11.5}$$

where $\Gamma_h^1 = \frac{\not{n}}{2}$, $\Gamma_h^5 = \frac{\not{n}}{2}\gamma_5$ and $\Gamma_l = \frac{\not{\bar{n}}}{4}(1-\gamma_5)$. Note that the two operators $O_{\mathbf{0}}$ and $O_{\mathbf{8}}$ can both produce any of the $Q_{\mathbf{0},\mathbf{8}}^{1,5}$ operators. Now, implementing field redefinitions to factor usoft effects (remember we can start with SCET$_{\mathrm{I}}$ to derive SCET$_{\mathrm{II}}$ results) we have

$$\xi_{n,p} = Y \xi_{n,p}^{(0)}$$

$$W = Y W^{(0)} Y^\dagger$$

These redefinitions are easily implemented in $Q_{\mathbf{0}}^{1,5}$. They simply take

$$[\bar{\xi}_{n,p}^{(d)} W \Gamma_l C_0(\overline{\mathcal{P}}_+) W^\dagger \xi_{n,p}^{(u)}] \to [\bar{\xi}_{n,p}^{(d)(0)} W^{(0)} \Gamma_l C_0(\overline{\mathcal{P}}_+) W^{(0)\dagger} \xi_{n,p}^{(u)(0)}] \tag{11.6}$$

where we used the fact that Y commutes with the wilson coefficient $C_0(\overline{\mathcal{P}}_+)$. This argument cannot be applied to $Q_{\mathbf{8}}^{1,5}$ because Y, containing generators of its own, does not commute with T^a. However, by making use of the color identity

$$T^a \otimes Y^\dagger T^a Y = Y T^a Y^\dagger \otimes T^a \tag{11.7}$$

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