Holography Duality (8.821/8.871) Fall 2014
Assignment 5

Nov. 26th, 2014   **Due Friday, Dec. 5th, 2014**

- Please remember to put **your name** at the top of your paper.

Note:

- The computation of a Wilson loop (which contains couplings to scalar fields) in $\mathcal{N} = 4$ SYM using AdS/CFT was first discussed in

  "Macroscopic strings as heavy quarks: Large-N gauge theory and anti-de Sitter supergravity," by S-J. Rey, J-T. Yee, arXiv:hep-th/9803001

  "Wilson loops in large N field theories," by J. Maldacena, arXiv:hep-th/9803002.

  It is rather amusing that the two papers happened to appear back-to-back on the same day (apparently without prior coordination).

- The calculation of problem 3 of this pset was first given in

  "Wilson Loops in the Large N Limit at Finite Temperature," A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankielowicz, hep-th/9803137.

  "Wilson-Polyakov Loop at Finite Temperature in Large N Gauge Theory and Anti-de Sitter Supergravity," S-J. Rey, S. Theisen, J-T. Yee, hep-th/9803135.

  The following paper contains various generalization with applications to heavy ion collisions:

  "Wilson loops in heavy ion collisions and their calculation in AdS/CFT," by H. Liu, K. Rajagopal, U. Wiedemann arXiv:hep-ph/0612168.

Problem Set 5

1. **Massless and massive vector fields in AdS (30 points)**

   Consider a *massive* vector A_M in AdS_{d+1} which is dual to some current operator J^μ in the boundary theory. We use the index notation $x^M = (z, x^\mu)$ where z is the direction perpendicular to the boundary with $z = 0$ the boundary. Take the bulk action for A_M to be

   $$S = -\int d^{d+1}x \sqrt{-g} \left(\frac{1}{4}F_{MN}F^{MN} + \frac{1}{2}m^2 A_M A^M\right) \tag{1}$$

   (a) Consider $m^2 = 0$ first, i.e the Maxwell action. Choose gauge $A_z = 0$ and work out the asymptotic behavior for A_μ near the boundary $z = 0$.

   (b) From the asymptotic behavior found in (a) show that J^μ corresponding to a Maxwell field has scaling dimension $d - 1$. Note that A_μ and A^μ have different asymptotic behavior. Does it matter which one we use to find the dimension of J^μ?

   (c) Now consider $m^2 \neq 0$. Work out the asymptotic behavior of A_M near the boundary.

   (d) From the results of (c) find the scaling dimension of the corresponding operator J^μ.

   (e) In the massive case, what happens to component A_z?

   (f) Discuss the differences between the massive and massless case.

2. **AdS propagators (40 points)**

   In this problem we consider AdS in Euclidean signature, i.e. the metric can be written as

   $$ds^2 = \frac{R^2}{z^2}\left(\delta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \ . \tag{2}$$

   (a) While in Lorentzian signature the Poincare coordinates do not cover the full AdS, show that in Euclidean signature, Poincare coordinates in the metric (2) do cover the whole Euclidean AdS. In particular, show that $z = \infty$ (for any x^μ) corresponds to a *single boundary point*. (In contrast, in the Lorentzian signature, $z = \infty$ corresponds to a degenerate horizon an infinite proper distance away.)

   (b) For a massive scalar field ϕ with mass m, find the explicit form of the boundary-to-bulk propagator $K(z, x; x')$.

(c) Show that the bulk-to-bulk propagator $G_B(z, x; z', x')$ and the boundary-to-bulk propagator $K(z, x; x')$ satisfies the relation

$$\lim_{z' \to 0} G_B(z, x; z', x') = \frac{z'^\Delta}{2\nu} K(z, x; x') \tag{3}$$

where ν and Δ are as we defined in lecture. The above relation can be shown without using the explicit forms of G_B and K. (But it is ok if you manage to find an explicit expression for G_B. If you do it this way you don't have to find G_B yourself. It is ok to quote literature.)

(d) Suppose ϕ_n is the bulk scalar field dual to a boundary operator $\mathcal{O}_n$. Using (3), argue that

$$\begin{aligned}
&\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle_{\text{CFT}} \\
&= \lim_{z_1 \to 0} 2\nu_1 z_1^{-\Delta_1} \cdots \lim_{z_n \to 0} 2\nu_n z_n^{-\Delta_n} \langle \phi_1(z_1, x_1) \cdots \phi_n(z_n, x_n) \rangle
\end{aligned} \tag{4}$$

where $\langle \phi_1(z_1, x_1) \cdots \phi_n(z_n, x_n) \rangle$ is the bulk correlation function for fields $\phi_1, \cdots \phi_n$.

3. **Color screening at a finite temperature (30 points)**

Calculate the thermal expectation value for a rectangular Wilson loop with length $\mathcal{T}$ in the time direction and L in one of the spatial directions for $\mathcal{N} = 4$ SYM theory at a finite temperature. Assume that $\mathcal{T} \gg L$. Show that its behavior implies that color charges are screened at large distance.

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