Holography Duality (8.821/8.871) Fall 2014
Assignment 1

Sept. 9, 2014 **Due Thursday, Sept. 25, 2014**

- Please remember to put **your name** at the top of your paper.

Note:

- Some review papers which discuss the motivation for the holographic principle include:
  - D. Bigatti and L. Susskind, "TASI lectures on the Holographic Principle," arXiv:hep-th/0002044.
  - J. D. Bekenstein, "Black holes and information theory," arXiv:quant-ph/0311049.
  - R. Bousso, "The holographic principle," arXiv:hep-th/0203101, Sec. I-III.
- There are many books and reviews on black holes and black hole thermodynamics. Here are some examples:
  - Classical geometry and physics of a Schwarzschild black hole is nicely discussed in Misner, Thorne and Wheeler, "Gravitation," Chapt. 31-33.
  - Penrose diagrams are described in Sec. 5.1 of Hawking and Ellis, "The large scale structure of pacetime."
  - Chapt. 6 and 12 of Wald, "General Relativity."
  - Lecture notes by Ted Jacobson which can be found at http://www.glue.umd.edu/%7Etajac/BHTlectures/lectures.ps

Problem Set 1

1. **Matrix element identities in the proof of Weinberg-Witten Theorem (20 points)**

   Consider a massless particle of spin-j, whose single-particle states can be written as $|k,\sigma\rangle$, with $k \equiv k^\mu = (k^0, \vec{k})$ the four-momentum and $\sigma = \pm j$ the helicity of the particle. We normalize the states as

   $$\langle k,\sigma|k',\sigma'\rangle = \delta_{\sigma,\sigma'}\delta^{(3)}(\vec{k}-\vec{k}') \ . \tag{1}$$

   J^μ is a Lorentz-covariant conserved four-current with charge $Q = \int d^3x\, J^0$. $T^{\mu\nu}$ is a conserved Lorentz-covariant energy momentum tensor with the energy-momentum four-vector $P^\mu = \int d^3x\, T^{0\mu}$. Show that Lorentz invariance requires that

   $$\lim_{k'\to k} \langle k^\mu,\sigma|J^\mu|k'^\mu,\sigma\rangle = \frac{q}{(2\pi)^3}\frac{k^\mu}{k^0} \tag{2}$$

   and

   $$\lim_{k'\to k} \langle k^\mu,\sigma|T^{\mu\nu}|k'^\mu,\sigma\rangle = \frac{k^\mu k^\nu}{(2\pi)^3 k^0} \tag{3}$$

   where q is the Q-charge of the particle.

   (*You may find it useful to consult Weinberg "The Quantum Theory if Fields" Vol. I Sec. 2.5.*)

2. $SU(2)$ **Yang-Mills theory as a** $U(1)$ **theory (20 points)**

   Consider an $SU(2)$ Yang-Mills theory with gauge vector bosons $A^a_\mu, a = 1,2,3$. We can also interpret this theory as a $U(1)$ gauge theory with the generator of the $U(1)$ symmetry given by $\frac{\sigma^3}{2}$. In this interpretation the corresponding $U(1)$ gauge field is A^3_μ and $B_\mu = \frac{1}{\sqrt{2}}(A^1_\mu + iA^2_\mu)$ is a complex massless vector field with a nonzero $U(1)$ charge. Show that this theory does not have a Lorentz-covariant conserved four-vector current corresponding to the $U(1)$ symmetry.

3. **Falling into a black hole! (30 points)**

   Consider an observer free falling toward a black hole following along the radial direction.

   (a) Show that to reach the Schwarzschild radius, it takes a *finite amount of proper time* but *an infinite amount* of coordinate time.

   (b) Show that once the observer is inside the Schwarzschild radius, it cannot send any message to his friends outside the horizon, but can still receive messages from them.

(c) Show that once the observer is inside the Schwarzschild radius, he cannot avoid hitting the black hole singularity at $r = 0$. In particular, he will reach (and thus will be killed by) the singularity in a proper time

$$\tau = \pi G_N M = 1.54 \times 10^{-5} \times \frac{M}{M_{\text{Sun}}} \text{sec} \ , \tag{4}$$

where M is the mass of the black hole and M_{Sun} is the mass of the Sun. Even for the supermassive black hole in our galaxy whose $M \approx 4 \times 10^6 M_{\text{Sun}}$, the observer could only live for 1 minute after crossing the horizon.

4. **Black hole evaporation (30 points)**

Assuming that a Schwarzschild black hole of initial mass M radiates into *vacuum* n species of (non-interacting) massless bosonic particles as a perfect black body.

(a) Show that the entropy of system (i.e. black hole plus radiation) increases.

(b) Estimate the lifetime of the black hole.

8.821 / 8.871 String Theory and Holographic Duality
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