Lecture 20: Electron-phonon coupling and electron lifetime

The formalism of second quantization is outlined. The phonon contribution to the specific heat is discussed. A simple argument shows that at low temperatures the specific heat goes as T^3 in the Debye model.

The electron-phonon coupling in the jellium model is discussed. It is simply the screened Coulomb interaction $v(\mathbf{r})$ between the charge modulation of the jellium $n(\mathbf{r})$ with the charge modulation of the electrons $\rho(\mathbf{r})$. The former is given by combining Eq. (4) and Eq. (8) of the last lecture while the Fourier transform of the latter is $\rho_{\mathbf{q}} = \sum_{\mathbf{k},\sigma} c^\dagger_{\mathbf{k}-\mathbf{q},\sigma} c_{\mathbf{k}}$. Putting everything together, we find

$$\begin{aligned} H_{e-pl} &= \int d\mathbf{r} d\mathbf{r}'\, n(\mathbf{r})\, v(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') \\ &= \sum_{\mathbf{q}} n_{\mathbf{q}} v_{\mathbf{q}} \rho_{\mathbf{q}} \\ &= \sum_q \sum_{k,\sigma} g_q \left(b_q + b^\dagger_{-q}\right) c^\dagger_{k+q,\sigma} c_{k,\sigma} \end{aligned} \tag{1}$$

where

$$g_q = \frac{4\pi e^2}{\kappa^2}\sqrt{\frac{n_0}{2M}}\frac{q}{\sqrt{\omega_q}} \tag{2}$$

Note the linear q term in the numerator of Eq. (2) comes from the gradient term in the relation between jellium density and displacement. This form of electron-phonon coupling is called the deformation potential and describes the coupling to longitudinal acoustic phonons.

The lifetime τ of an electron due to emission or absorption of phonons is written down using the Fermi golden rule. We show that if $kT \gg \hbar\omega_D$, $\frac{1}{\tau}$ is linear in T, i.e.,

$$\frac{1}{\tau} = 2\pi\lambda kT \tag{3}$$

where the important dimensionless electron-phonon coupling parameter is given by

$$\lambda = 2N(0)\int_{-1}^{1}\frac{d\cos\theta}{2}\frac{g_q^2}{\omega_q} \tag{4}$$

where $\mathbf{q} = 2k_F \sin\frac{\theta}{2}$ is the phonon vector which connects two points on the Fermi surface separated by the angle θ.

For the jellium model all the constants are known and Eq. (4) is easily evaluated to give $\lambda = \frac{1}{2}$. In real metals λ ranges from 0.2 to 0.3 for simple metals to more than unity for strong coupled metals such as Pb.

Reading: Marder 13.3.1, 13.3.2