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PROFESSOR: So good afternoon. Our main topic today is to understand light forces, in particular the stimulated force using the dressed atom picture.

And I hope once you understand how light forces can be described in the dressed atom picture you will love the dressed atom picture, as most atomic physicists do. It is really the language we use, the intuition we apply to atomic systems. Whenever we have a laser beam which drives the atom it is, in most situations, more appropriate and better to use dressed states as your basis states and not the naked, the bare states, which are ground and excited state.

Now I know last week I quickly went through the dressed atom picture. Let me sort of go through it in a fly-over by not doing any of the derivation, but making a few comments here and there. Also, because I want that some of the results are fresh in your head when we apply it to mechanical forces of light.

So I told you that in the dressed atom picture we have two kinds of interactions, the atom interact with a vacuum, with a reservoir of empty modes with spontaneous emission. And we also interact with a laser field. And the new feature about the dressed atom picture is that we first solve this exactly, we re-diagonalize the Hamiltonian for that Hilbert space, and then we allow spontaneous emission to happen.

Now the derivation is pretty straightforward. After a number of assumptions, we simply came to the two-by-two matrix, where we have two levels of the atoms coupled with the Rabi frequency. I derived it here for you using the quantized electromagnetic field. But in your homework you derive the same two-by-two matrix using a classical electromagnetic field, which drives the atoms. And we've talked

about the similarities and differences last week.

If you have any questions, please raise your hand or interrupt me.

Now two-by-two matrix is fairly trivial. We can immediately diagonalize it. And we find Eigensolutions, which are just sine theta, cosine theta, amplitude, linear superpositions of the naked states. And the naked states are ground and excited states, state A and state B, with N or N plus 1 photons in the laser field.

So it's simple. It's just a two-by-two matrix. But the insight we can get out of it is pretty profound and pretty subtle.

So what we first discussed is-- and we will need that to understand forces and energy conservation-- we can now look at spontaneous emission. Spontaneous emission coupling to the empty modes of the vacuum can only connect an excited state B with a ground state A. And therefore, we have diachromatically those four combinations, represented here by those four arrows. And if you look for the matrix element, you collect cosine theta and sine theta factors. And that's what we did here.

So we immediately get in the dressed atom picture, an expression which tells us what are the matrix elements for the carrier, for the blue and for the red detuned sideband of the Mollow triplet. But then there's something else, and I edit this with the lecture notes in green. We can also, now, if you know what the transition rates are, we can set up rate equations and can solve them. And once you solve a set of differential equations for rate equations, you know what the populations are.

But now, today, I want to tell you one more subtlety. You know already that you cannot, in the most general situation, formulate an equation with just a population. You know that we have the master equation, we have the Optical Bloch Equation. This is an equation for the density matrix. And usually we get equations where the populations are mixed with coherences.

In other words, you cannot just say the diagonal matrix element of the density matrix is a population, and the change of population is what goes out of this state

and what arrives in this state. Coherences play a role.

But, and this is what the dressed atom picture does for you, in the limit that we have a strong Rabi frequency-- and this is the limit you want to consider here-- the matrix equation decouples into an equation for populations and an equation for coherences. And if something decouples that should tell you we all on the right track, we have the right description.

In this description things decouple, things are simpler. And that means this description is most appropriate for the physics we want to understand.

Or, to say it more specifically-- and all the details of course are in the book *Atom Photon Interaction*-- you can rewrite the Optical Bloch Equations not in the bare bases but in the dressed atom basis. And in the dressed atom basis you find this wonderful separation between populations and coherences.

So we have the simple rate equations. By setting the left hand side to 0 we find the steady state populations. This will be very important to understand the light forces. But you should just remember the steady state solution such as we know them. They're just exactly given by this angle θ , which tells us what kind of superposition of the bare states from the dressed states.

Remember in the case of θ equals 45 degrees, the dressed states are just symmetric and antisymmetric superposition of the naked states.

Now what is also important is those rate equations, or those rate equations have a time constant. The time constant is here, the relaxation time for populations. And we know exactly what it is.

It's a little bit more complicated to find the equivalent equation for the coherences. But just take my word, the density matrix in the dressed atom basis decouples into equation for population, which is simple, equation for coherences, little bit harder to derive but looks the same. And from this equation we have now an expression for what is the relaxation time for coherences.

And just to show you one non-trivial example, if you add up cosine square plus sine square you get 1. But based on what we derived we have to take cosine to the power of 4, sine to the power of 4. And this gives us effect of $1/2$, which will appear somewhere in an important place in a few minutes.

So with that we can now address, we know now something about the intensities of this Mollow triplet, because the intensity is nothing else than the population, what is the population in the initial state times the matrix element squared. But what is also - what comes out very naturally-- is that we have different widths. The features have different widths.

The sidebands have a different width from the central part of the Mollow triplet. And well, this is now very naturally interpreted at one has a width, which is a relaxation rate for the population. The other one, is the sidebands have a width which is a relaxation rate for the coherences.

I showed you, and this was just a reminder of a few weeks ago, that when we looked at the linear matrix equation, which are the Optical Bloch Equation, we discussed complex eigenvalues mathematically. And those complex eigenvalues, the imaginary part is the sideband frequency. And the real part was the widths.

What I have now, sort of-- I can get everything from the dressed atom picture. But what I've gotten in a very natural and simple way is this case, which is the case where the dressed atom picture is particularly simple. It's called the secular case, where the population and coherences decouple.

And so, what I just showed you, that the population relaxation time is γ over 2. This explains now why, in this case, the widths of the central feature is $1/2$ of what it is when we're on resonance. So everything is sort of nicely playing out here.

OK, any questions about dressed level populations, rate equations, the Mollow triplet? Because we want to apply it now, in a few minutes, to the stimulated light force.

But before I do that, let me show you one nice piece of physics, which can be

formulated both in the bare, in the uncoupled, and in the dressed state basis. And this is the process of spontaneous emission.

If an atom is in the laser field, it just emits photon, rapid fire photons. And I want to show you now, how we would describe it in the bare basis, how we describe it in the dressed basis. And we get two very different physical pictures out of it.

Well, in the bare basis, maybe just think of quantum Monte Carlo simulation, a photon has been emitted, and that means we are in the ground state. Now it takes a Rabi cycle before the ground state atom is in the excited state. And when it is in the excited state, it can emit again and goes back to the ground state. And then things start over.

So the picture you obtain here is the picture of antibunching, that the atom emits a photon, but then it takes the inverse Rabi frequency to load the gun again. And then it can fire again and emit the next photon. So therefore, the probability of getting another photon right after you've observed the first photon is reduced, or T equals 0, it is exactly 0. And this is antibunching, the photons are antibunched.

So this is how we would describe the radiative cascade, how we go from N to N minus 1, to N minus 2, how we lose photons from the laser field and dump them into the empty modes of the vacuum. That's how we describe it in the uncoupled basis.

Well in the dressed basis we have sort of diagonalized the atoms interacting with the strong laser field. And now we would have the following picture.

We have this manifold here. We can emit from this manifold. And we can go to the upper or lower dressed state. And then, the next thing will happen again.

But what we realize here, if we are in the upper dressed state, we can either emit on the carrier or we can emit a blue detune sideband. But once we have emitted a blue detune sideband, the next sideband has to be red detuned because we are now in the lower dressed state, and we need a lower sideband to go back to the upper dressed state.

So what this picture gives us, it gives us the sidebands because we are resolving the splitting in this picture. And secondly, it tells us that there is an alternation between the blue detuned photons and the red detuned photon.

Now you would say, well aren't those two pictures really different? In one case we have antibunching, and all of the photons are the same. And over here I talk about an alternation between the blue and the red sideband. Well again, the two pictures are complementary.

In this case, if I want to resolve the antibunching I need a temporal resolution which is better than the inverse Rabi frequency. But if my temporal resolution is better than the inverse Rabi frequency then I don't have the spectral resolution to observe the sidebands, and vice versa. If I want to resolve the sidebands, if I want to distinguish the red from the blue detuned sideband of the Mollow triplet I cannot run the experiment with the spectral resolution, which would be required to observe the antibunching.

So these are sort of two different pictures. One is sort of in terms of energy eigenstates when we have spectral resolution. And this other picture is more appropriate when we're interested in the short time domain.

So in that sense, you need both. You need the bare basis. You need to dressed basis. And you have to just know for which physical process, which of them allows the more direct description. Questions?

OK, so this was partly review, partly preparation for the next big chapter, namely we want to understand the dipole forces within the dressed atom picture. The book *Atom Photon Interaction* has a nice summary of it. But the more detailed picture, and I heavily draw from this, is in the truly seminal article by Jean Dalibard and Claude Cohen-Tannoudji. I've linked this article to our website.

So I've summarized here pretty much what I just told you. This is sort of the summary from the previous section. What we have to know is to understand light forces in the dressed atom picture is number one, the energy speaking between the

two. The splitting between the two is the generalized Rabi frequency.

If there's anything which is not-- well I shouldn't say 100%-- but at least 90% clear to you, ask me question. I mean these are the results we really need for the following discussion.

So number one is the dressed energy levels are split by the generalized Rabi frequency. Number two is with this angle θ , which came from the solution of diagonalizing the two-by-two matrix, we have now the steady state populations in the upper and in the lower dressed state.

And the third thing is when we have a population when we are not in steady state, then the steady state populations are obtained with the time constant, which is given here. The time constant is not just γ , it's γ times these trigonometric functions.

So these are simple ingredients. But by using those ingredients we can get a very nice formulation for the stimulated force.

So what I've shown here is, and this is the situation we want to focus on for a while. Assume you focus a laser beam. Then plot it as a function of position. The atom is outside the laser beam, it goes into the laser beam where it experiences the electromagnetic field. And if it goes further it has crossed the laser beam and is out of the laser beam again.

So what happens is the splitting between the two dressed energy levels is the generalized Rabi frequency. And this is outside the laser beam, simply the detuning. But then when the laser beam is on, the generalized Rabi frequency gets larger. We have a larger splitting between the dressed energy levels, and so on.

What is also important is that if the laser is red detuned, which is here, the lower dressed level correlates, corresponds to the ground state. Whereas for blue detuning the ground state in this limit, the ground state correlates with the upper dressed level. So it's important. So therefore an atom in this situation-- because atoms are always more in the ground state than in the excited state-- an atom will

have more population in the upper dressed level. And here it will have more population the lower dressed level.

So what we want to do now is we want to rewrite the stimulated light force, or the dipole force, as we want to express it through the forces acting on the dressed atom level. Well, I told you that we have a splitting of the dressed energy levels by the generalized Rabi frequency. So therefore, if I just cover that, the energy level of the two dressed levels is $\hbar\omega/2$ plus minus the generalized Rabi frequency.

And the force if an atom is in one of the dressed levels is simply the gradient of that. So therefore we have the picture that the two dressed energy levels, atoms in those two dressed levels, always experience opposite forces.

Then an expression for the net force is that the average dipole force is the force in dressed level one times the probability that the atom is in dressed level one. And the same for two. And if you sum it up we get exactly the expression we have derived earlier for the stimulated light force.

But let's just sort of look at it in the following picture. If we are blue detuned, and let's assume we are not saturating the transition, so therefore the atom will be mainly in the ground state. So you have more population in that state.

But, well, if the laser power is 0, we have 100% population here. But when the atom moves into the laser beam there is a superposition of ground and excited state. But that also means there's a superposition of upper and lower dressed level.

But we have always, of course, more population in the dressed level which connects to the ground state because an atom will-- well, it also comes out of the solution. But it's natural, just assume you make the laser beam weak, you start in the ground state and everything is sort of connected. The ground state is simply dressed by a small excited state contribution.

So that explains it, why for blue detuned light, the physics is mainly in the upper dressed level. And we see the repulsive dipole force of a blue detuned laser.

Whereas, for red detuned light, the dominant population is in the lower dressed

level, and we have an attractive force.

Well that's simple. We haven't gained so much, but you'll see, we can now rapidly gain insight by take it to the next level. Let me first mention a trivial example. And this is when we are on resonance.

When we are on resonance we have equal population in the two dressed states, they are resonantly mixed. And therefore if the population in the two dressed states is the same, but the atoms experience opposite forces, that means the dipole force average out to 0.

In a few minutes I want to calculate for you what the heating is. The dipole force is 0. But you know the atom in this radiative cascade is making transitions between upper and lower dressed level. So what does it mean for the atom?

When it is in one dressed level it experience the force from this side. If the atom is in the other dressed level, it's experience a force from that side. So the atom is actually subject to an average force which is 0. But if you take these picture literally, and you can, the atom is really whacked left, right, left, right, left, right. And there is fluctuations.

And we can use those fluctuating forces to immediately calculate what is the heating. The force average is 0, but the fluctuations of the force cause heating. So that's immediately the next step of physical insight we will get out of this picture.

But before I do that I want to do something else. I want to discuss what happens now to the force when the atom is slowly moving. Remember, there is still one phenomenon which I owe you an explanation for. And this is when we discussed molasses.

We always had the effect in optical molasses with a spontaneous light force that we need red detuning of the laser beam to get cooling. The simple picture is the atom absorbs a red detuned photon and emits away a resonant photon. And then that energy difference is taken away from the kinetic energy, and this is cooling.

But then I told you that when you go to higher power of the standing wave, the sine of alpha of the friction coefficient reverses. And that means cooling is now done by blue detuned light, heating is done by red detuned light. And that's what we want to understand.

So in order to get any cooling effect, or heating effect, we have to now change the equation we had for the average dipole force. Remember also, the average dipole force, we did that earlier, can be derived from a potential. It's a gradient over potential. So therefore, when you fly through a laser beam you go down and up or up and down the potential, but nothing has happened to you.

But the assumption we had for that is that the dipole force was calculated with a steady state population. So now we want to say if the atom is slowly moving it may not be fully adiabatic, and we want to calculate the first correction to the non-adiabaticity.

So in other words, we have the following picture. When an atom flies through the laser beam it needs a certain time to adjust its population. And the question, what is this time? Well, it is the relaxation time for the populations among the dressed states to adjust. And this is exactly the inverse of the rate γ_{pop} , which we have calculated earlier.

So the physical picture now is that in the dressed level one and two, the population at any given moment is almost the steady state. But there is a time lag, a time lag by the time it takes to relax. So in other words, the atom is always a little bit behind in adjusting to the laser field because it takes a finite amount of time.

So let's now try to understand, in this picture which we have obtained with a laser beam the two dressed levels and the atom flying through, let's try to understand what this extra lag time means.

So the atom is flying into the laser beam. It starts out in the ground state, which means 100% in the upper dressed energy level. Now, in the laser beam the higher the laser power is, the more there is a mixture of the other dressed energy level.

But what happens is, when the atom is here, when it just assume it flies in, and there was not enough time to adjust the population, that would mean that the atom-- and this is indicated with the red dots-- that the atom has a little bit of more population in the upper dressed level than it would have in equilibrium

But what does it mean for the force now, if there is more population here than there? Well, the more population you have in the upper state, the more you're climbing up a hill, the more you get a force to the left. So this red arrow shows due to the non-adiabaticity, due to the time lag, there is a little extra force to the left hand side.

So this is on the up hill. Let's now understand what happens on the down hill. Well, we just have to keep track of signs.

Here, the atom, in a fairly high laser field, is in the upper dressed level with a mixture in the lower dressed level. But now it flies out of the laser beam, but it cannot immediately adjust its population. And that means it wants to have now more population in the upper dressed level but it hasn't fully adjusted.

So therefore, instead of having the blue bullet, which represents equilibrium, the atom has the red bullet. It has a little bit lower population in that, and higher population there. But lower population here and higher population here means-- if you have a little bit higher population than you should have here it means you are climbing up the hill, and there is an extra force, which is again, to the left.

So we realize the n the time lag, means when an atom goes up the hill there's a little bit extra force to the left. And when it goes down the hill there is a little bit extra force to the left. And after it has flown through the steady state part of the force roller coaster up down, which can be derived from a potential, has integrated to 0. But what is left is this non-adiabatic component of the force which is slowing down the atom.

In other words, I have shown you in this simple picture that the time lag in adjusting the dressed energy population means that there is a small differential force which is

slowing down the atom. And what I assumed here was blue detuning. So we have cooling, we have friction for blue detuned light when it comes to the stimulated force.

Now this picture, this equation that the population is a steady state at the given position minus a time lag, we can solve it. And I'm not doing it here because it involves a few mathematical equations. But by just using these answers, and the rest is straight forward, you can then arrive at an expression for the stimulated for the dipole force.

Colin?

AUDIENCE: So, in the spirit of last time, keeping track of the energy, the energy here would be going into the fact that now I have-- if the atom's a harmonic oscillator it's a little behind the drive field?

PROFESSOR: Yes.

AUDIENCE: So you imprint some fluctuation on the laser beam, and that's where the energy's being carried off, in the laser beam?

PROFESSOR: Well the energy's carried off by the laser beam. I have actually prepared something which I want to show you in 10 or 15 minutes, where the energy goes. So let's maybe do that in a moment.

So this is a physical picture. And I think what is also important I want to later tell you, for other laser cooling mechanisms, which are too complicated to describe, but they also have a time lag. And I think, I try to teach you basic concepts by using this simplest physical picture I can have. And what can be simpler than two levels of an atom and one laser beam?

But what you understand here with this stimulated force, which is also called blue molasses, is for the first time how a time lag in adjusting to the local laser field, how that leads to cooling.

I'm summarizing here, simply taking this picture, this equation, and you know,

turning the crank on it. The paper by Jean Dalibard and Claude Cohen-Tannoudji calculates what is the dipole force in steady state. We get almost the same result with the Optical Bloch Equations, but we are discussing here the limit that the Rabi frequency is larger than gamma. So therefore, if you're missing occurrences of gamma, they don't appear here because we've always assumed omega is larger than gamma in this picture.

Now I want to apply this picture now, not just going through one hump of the laser field, but going through a standing wave. So we have a standing wave of light, which is blue detuned. Of course, in a standing wave of light, if you ever reach over one optical wavelengths, you go up the hill, you go down the hill, the lambda average dipole force is 0.

But now you find that if you put in this time lag, you find a friction force, with a friction coefficient alpha. And this friction coefficient is cooling for blue detuned light.

Now I don't want to elaborate on this expression. But if you take the treatment of Jean Dalibard and Claude Cohen-Tannoudji, which is very transparent. It reads wonderfully easily, as the book *Atom Photon Interaction*. In the limit of large detuning, which is the simplest possibly limit, you find the friction coefficient which looks highly nontrivial. It depends to the sixth power of the Rabi frequency and such.

The only reason why I put it here, in 20 minutes I want to give you a very, very simple picture where I derive that for you. So that's just a check mark.

Any questions right now about-- because I want to go on-- about the stimulated force in the dressed atom picture? And if we allowed deviations from the steady state population we get a cooling effect, and this explains the effect of blue molasses. And we've applied it to a standing wave.

Cody?

AUDIENCE:

Just [INAUDIBLE] the upper alpha are a friction constant and the lower one is the polarizability? Or, something.

PROFESSOR: No, oh, thanks for asking. Alpha, yeah, we [INAUDIBLE]. Here it's the friction coefficient. Here, remember when we had the light force-- the spontaneous light force and the stimulated light force-- one was directed along a vector alpha, and the other one along a vector beta. Alpha was pointing into the direction of the gradient of the laser intensity, and beta was the phase gradient of the laser beam.

So those two have nothing to do-- this is friction coefficient. This is the direction for the stimulated light force. And yes, a little bit later today, I will use alpha again, but for the polarizability.

Yes.

AUDIENCE: The [INAUDIBLE] track the atoms in the [INAUDIBLE] standing wave, [INAUDIBLE].

PROFESSOR: You're talking about trap in here, that if you have a standing wave of blue detuned light-- well if the standing wave is infinitely extended then the atom is always in the standing wave. But if you use two Gaussian laser beams to form a standing wave, there is always a net force to expel the atoms away from blue detuned light, because blue detuned light has an average dipole force which is repulsive.

So whenever you want to manipulate atoms with blue detuned light you need an additional magnetic trap, or an additional red detuned optical trap. Because whatever you want to do with atoms, if you want to do it with atoms in steady state you have to keep the atoms together.

And what keeps the atoms together is red detuned optical traps, or magnetic traps. The blue detuned light can be used to create a blue detuned lattice. Or it can be used at least, if you want to do laser cooling with blue detuned light it can be used for laser cooling, as I've started to describe for you now.

But yes, I mean, in that sense as an experimentalist, you can run experiments where you have only red detuned light-- red detuned light for trapping, red detuned light for cooling. But if you use blue detuned light for a standing wave or for blue molasses, in addition you will always use red detuned light to keep the clouds together.

So pretty much every experiment usually has some red detuned light for confinement. And you have to then match blue detuned light with red detuned light to get the two things you want.

We can now use the dressed atom picture to calculate the heating. Remember the final temperature you can reach was always the ratio between a heating coefficient, which is actually the momentum diffusion coefficient, over alpha. So right now we have understood the alpha part. We know the physical picture, why alpha provides cooling, why we have a friction force for blue detuned light.

But let's now take it to the next step, and calculate what is the heating. And I mentioned it already in the context of simple molasses, which we discussed, I think, two weeks ago, that we describe heating by the momentum diffusion coefficient.

Well the momentum diffusion coefficient tells you that P^2 is simply growing as a function of time. If you have a cold cloud and you heat it up, you can describe the heating by the temporal growth of P^2 . If you don't like momentum, divide P^2 by $2m$ you have the kinetic energy, which increases.

So it is this momentum diffusion coefficient. And if you have a cloud which moves with an average momentum, you only want to have the P^2 minus P_{average}^2 . But if you think about a cloud which is being cooled, the second term is 0.

So the momentum diffusion coefficient is the derivative of P^2 . But the derivative of P is simply the force. So therefore we can absorb the derivative by replacing P at least one of the P 's by the force. And we can now get rid of the second P of the second occurrence of momentum by saying well momentum is derivative of force. Sorry, derivative of momentum is force, or momentum is the integral of force.

So we can exactly rewrite that momentum diffusion coefficient by an integral over correlation function of forces. And then we have the nice physical picture that with increase of kinetic energy, the momentum diffusion coefficient comes because we have fluctuations of the force. Yes?

AUDIENCE: With the last time, the previous page the factor [INAUDIBLE].

PROFESSOR: The factor of 2 here?

AUDIENCE: No, the next slide.

PROFESSOR: Here is 2.

AUDIENCE: For both terms.

PROFESSOR: Here is 2. And here it disappears, because we have a factor of 2 and a factor of 2, and the two cancel each other.

AUDIENCE: No, I meant, there's a 2 multiplied the first average. And there's a 2 for the second one. Are those for the first term?

PROFESSOR: I think this is correct.

AUDIENCE: [INAUDIBLE].

PROFESSOR: What happens is P-- you use--

AUDIENCE: [INAUDIBLE].

AUDIENCE: No, it's supposed to be--

PROFESSOR: Wait, is one missing? Maybe [INAUDIBLE]. I think you're right.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Let's assume this is what it should be. So here is the reminder, the kinetic energy. If you have a process which causes momentum diffusion, the momentum diffusion coefficient gives you the heating rate.

And the cooling rate is given by our friction coefficient alpha. And as we discussed in the context of molasses-- of red detuned molasses, but it applies to any cooling scheme-- the finite temperature is when you have detail balance between heating

and cooling And therefore its ratio of the momentum diffusion coefficient over the friction coefficient. Kensie?

AUDIENCE: In the previous step, what was the significance of choosing f of 0 basically? Like, the time arguments were 0 and D . So we said that f of 0 was DPDT?

PROFESSOR: Oh, we assumed that the laser beam's Hamiltonian is invariant against time translation, and we simply evaluate that at T equals 0. I mean, that's of the argument you have fluctuating forces. And you could have fluctuating forces at time T and time T prime. But then you use the argument that in a steady state solution the correlation function does not depend on T and T prime separately, but only on the difference. And then you simply set one argument to 0, and the other argument becomes a time difference.

Other questions?

So now, by understanding that momentum diffusion is nothing else than the integral over the force fluctuation, we can now use this physical picture which I gave you earlier that an atom which cascades through upper, lower, upper, lower dressed states will experience opposite but equal forces, from the right and from the left.

So I can simply evaluate that by assuming we are on resonance. On resonance the force is \hbar over 2 times the derivative of the generalized Rabi frequency. And the force correlation, what is it? Well, it's the force f of 0 times f of T .

But as long as the atom is in one dressed state the force doesn't change. So I simply take here the force, I take the force squared, and then when I integrate over that, well the moment the atom makes a transition there is no longer any correlation between the force. Everything becomes random and the integral averages out to 0.

So in other words, that's the way how you should look at this integral. You start with the force f_0 , as long as the force is correlated. Because the atom hasn't done spontaneous emission, you have sort of f of 0, f of 0. And you integrate it, nominally to infinity. But you simply integrate it until some spontaneous emission randomizes the system.

So therefore, this integral over the correlation function is nothing else then the force squared times the typical time scale for a transition. So if you put that in on resonance, we have the force here, we square the force. And then we multiply with the spontaneous emission rate, which for resonance system is $\gamma/2$. And with that we get now the heating coefficient, the momentum diffusion coefficient for an atom which is exposed to resonant light due to the stimulated light force.

The paper, the references I've given you, is simply taking this augment and extending that to arbitrary detuning. For arbitrary detuning, I told you what happens on resonance. The atom spends equal amount in each dressed state. If you detune the atom spends more time in one dressed state, then quickly in the other dressed state, and then more time in the other dressed state again. So you can, by simply understanding what are the rates for cascading between the dressed levels, you could pretty much write down the expression, find it, and here is the solution taken out of the paper.

So we have to correct prefactor, but then if the detuning is non-vanishing we get a contribution from the second factor.

Colin?

AUDIENCE: Are you assuming the exponential correlation-- exponential behavior to the correlation. Because then you might have different prefactors, if it were something like if you were in a [INAUDIBLE].

PROFESSOR: Well what happens is the following. I think if you're in one dressed state and the force pushes you in one direction and you integrate over it you have an exponential decay. But instead of integrating over the exponential decay you can just take the force squared and multiply it with the average time the atom stays. In other words, if I have an exponentially decaying function I can always approximate it by the function of T equals 0 times the time.

And what you are asking me now is is the time the one over E time, or is it an average time? What is the correct time to use here?

AUDIENCE: I guess when the time scale depends on how the correlations decay for [INAUDIBLE] different.

PROFESSOR: Yes, OK, I agree. The shape of the decay of the correlation function would matter. I'm not going into that. But what I'm sort of assuming here is that the correlation function indicated with time is the correlation function of T equals 0 times the correlation time. And how the correlation time is related to the exponential time, or if there's a small prefactor and such, that may depend on details. Yes, you're right. I've swept this under the rug.

How are we doing? Good. Let me just summarize.

The dressed atom picture has given us two pieces of major insight into the stimulated force. The first one is that the atom experience is always opposite forces in the two dressed levels. And it is the population imbalance between the two dressed levels which results in a net force. And this requires either red detuning, then the force is repulsive, or blue detuning-- sorry, red detuning then the force is attractive, or blue detuning then it's a force repulsive.

Second, with this time lag thing, we understand that time lag, non-adiabaticity, is the way to get cooling. And the third thing we have learned, when the atom does a radiative cascade the force-- when it goes from the upper to the lower dressed level-- the force is suddenly reversing it's sign. And this causes fluctuations, this shakes up atom, and this causes heating. And this has nothing to do with the other heating mechanism we discussed earlier, namely every time there is spontaneous emission there's a random recoil.

In strong laser fields the fluctuations of the stimulated force, which has nothing to do with spontaneous emission, are much, much larger than the photon recoil. So therefore for strong laser fields, it is this alternation of the force when atoms switch dressed energy levels, which is responsible for most of the heating.

I want to come now to another physical situation, which is actually wonderful. You

learn about Sisyphus cooling. Everybody in cold atoms speaks about Sisyphus cooling because it's a very, very elegant cooling scheme.

We usually apply in the laboratory Sisyphus cooling when we do polarization gradient cooling. I will say you a few things later. But this is much, much harder to understand than the Sisyphus cooling, which takes place for a two level atom moving through a blue detuned standing wave.

So in other words, I try to explain now to you what Sisyphus cooling is. The relevant application is multi-level polarization gradient cooling, which I give you an idea, but it's hard to fully describe. But here, in our current discussion on the two level atom interacting with one laser beam, or one standing wave, we find the simplest physical realization of Sisyphus cooling.

So just to make sure that you are following the argument, I have so far explained to you why blue detuned light cools in the limit of very small velocities. Very small velocity means we did a first order Taylor expansion in this lag time. But now I want to discuss with you a different regime, namely that the atom moves with a velocity such that it can go up and down several periods of the standing wave before it does spontaneous emission.

So before when we discussed the friction coefficient, the velocity was infinitesimal. But now I assume that the velocity is such that the atom can surf a few waves and then it will spontaneously emit. And let me assume that we are blue detuned.

So the atom, let's say here, where the splitting is minimum, well this is where we don't have light. The generalized Rabi frequency is just the detuning. So we are in a node of the standing wave. And then the atom starts in the ground state, and it experiences the periodic potential due to the standing wave.

But we want to now ask, OK, when will spontaneous emission happen in this picture? Well, it cannot happen here, when the atom is in the ground state, when there is not light. But where there is light, when the atoms move to an anti-node, there is a mixture, indicated in red, of the excited state. So in other words, when an

atom does this roller coaster, it will most likely emit a photon-- either the carrier or one of the sidebands-- when it is at the maximum of the potential corresponding to an anti-node where it experiences the light.

OK, when the atom emits on the carrier it goes from the upper dressed level to the upper dressed level. Then sort of nothing has happened because the atom is in the same dressed level. Let me now discuss the case when something happens. And this is when the atom decays from the upper dressed level to the lower dressed level.

And at least for weak excitation the lower dressed level corresponds to the excited state. In other words, when there is no light, the upper dressed level was ground state, the lower dressed level is excited state.

But what happens now is actually interesting, the atom had to climb up the hill. But the spontaneous emission to the lower dressed level takes the atom to the bottom. So you can see, on average, it has climbed a hill here, but now it is at the bottom.

And now we're going to ask how does the atom get out of the lower dressed level? Well, the lower dressed level is mainly excited-- is 100% excited if there is no light. But if there is light, and light means the generalized Rabi frequency is larger, the excited state has now in it mixture of the ground state because the light mixes ground and excited state.

So now, of course, whenever there is red you can spontaneously emit. But there is a probability that the atom is more likely to emit when you are in a purely excited state, and not in a mixed state.

So now, talking about the lower manifold, the atom does it's roller coaster. It has a velocity that it can cover several periods of the standing wave in one lifetime. But there is a higher probability for the atom now to emit when it is on top of the hill.

So that's remarkable. We have the upper dressed level. We have the lower dressed level. The atom is doing the roller coaster.

But whenever it emits a sideband, whenever it switches state, it mainly does it by emitting at the top of the hill. And after the spontaneous emission it finds itself at the bottom of the hill.

So on average the atom is climbing more up hills than down hills. And this is called Sisyphus cooling. I think you've all heard about the Greek myth of King Sisyphus, who was condemned. He challenged the gods and he got his punishment. And his punishment was that he always has to roll a stone up the hill. And when he's done the stone falls down and he has to roll the stone up.

So in other words, we have condemned the atoms to the same verdict. They always have to go up hill to work. Then they fall down hill with the help of a spontaneous photon. But then they have to climb up hill. And it is the uphill climb where the experience and net friction force, which cools the atoms down and reduces their kinetic energy.

So this is Sisyphus cooling. And since I need it in a few minutes, I just wrote down for you, based on this simple and wonderfully elegant picture, how much energy does the atom lose per unit time? What is the cooling rate?

Well it's clear that the cooling rate comes when the atom switches from one dressed energy level to the next, because nothing happens on the carrier. So it is the transition state between dressed level one and dressed level two. And then we multiply with μ naught the depths of the optical lattice, because it is this energy difference which is extracted from the atom.

You may ask, OK what is time limiting? It's really the transition rate from the first dressed level, which is mainly ground state, to the other dressed level, which is mainly exciting state. Because once the atom is in this level, which is mainly the excited state, it will pretty much do the next transition at a rate which corresponds to the spontaneous emission rate.

So in other words, if you're not fully on resonance, or if you're not situated in laser power, the rate, one, two, is slower than the rate two, one. And therefore this rate is

the rate limiting step.

Let me show you just a few pictures. What I'm just explaining to you was explored very early on, just have to sort of put you back that it was in the mid '80s, '82, '83, '84, that people developed [INAUDIBLE] slowing. It was in '85 that optical molasses with red detuned light was demonstrated for the first time. It was in '86 that the dipole trap was demonstrated by Steve Chu. And in '87 that Steve Chu and Dave Pritchard introduced the magneto optic trap.

So the mid to late '80s were fantastic times, where one important technique was realized and demonstrated. And this was actually the first major experimental paper of a new laser cooling group, which was founded by Claude Cohen-Tonnoudji at the *Ecole Normale Supérieure* in Paris. And it was exactly about cooling atoms with stimulated emission, and this is blue molasses.

And, I mean, you exactly see here out of this paper, the Sisyphus cooling mechanism. And addressing the question about red detuned light here, this was actually beam experiment. You had a blue detuned standing wave, and you were sending an atomic beam through. And if this atomic beam was cooled, the divergence of the atomic beam was reduced.

So this experiment had only one blue detuned standing wave and nothing else, because it was not a trapping experiment. It was a beam collimation experiment. And what is shown here is that you have the original atomic beam profile. And for blue detuned light you make it narrower, so this is cooling. Whereas for red detuned light you have a defocusing, which corresponds to heating.

Any question?

I want to wrap up the discussion of the stimulated light force by discussing pretty much everything again, but in the simplest possible limit. That's actually something I haven't found in any text books. But when I derived it for myself it helped me to really understand it. It's sort of the perturbative result of the dressed atom picture.

So everything is simple. And by telling you, look at this. You know this is just EC

stock shift. But in the dressed atom picture it's a generalized Rabi frequency, you suddenly understand the trivial perturbative result and how it translates into the dressed atom picture.

So what I'm presenting you now in the next 10 minutes, it provides a lot of insight because it connects simple pictures. So I'm discussing things here in the limit that the detuning is large. The detuning is larger than the Rabi frequency, and spontaneous emission rate is the smallest of the three rates.

In that case, I can simply do perturbation theory, and I want to show you. Also, because this is pedagogical, I just want to show you how the effects come together. I neglect all factors on the order of unity. And I set \hbar to 1.

So let's assume we have a standing wave. A standing wave is $\cos KX$. So we have an electric field. But we describe the electric field with a Rabi frequency. So the Rabi frequency forms a standing wave.

Now what are the two dressed states? The dressed states 1 and 2 are the bare states, ground and excited state. And then there a perturbative mixture. And in first order perturbation theory we take the matrix element over the detuning.

So one dressed state is the ground state with a little bit of excited state mixed. The other dressed state is the excited state, minus sign, little bit of ground state mixed. These are our two dressed states in the trivial limit that we can apply perturbation theory.

What are now the transition rates between the two dressed states? Well, we want to make spontaneous emission from dressed state 1 to dressed state 2. We have to go from this small mixture, which is the excited state mixture, to the ground state here. So therefore we get the product of the two amplitudes, and then we multiply with γ .

Of course, most of-- yeah. So this is the rate to go from dressed state 1 to dressed state 2.

What is the inverse rate to go back from dressed state 2 to dressed state 1? Well, you go from the excited state to the ground state, multiply with γ , and that's what you get.

Sure we have, in perturbation theory, the coefficient is 1. It's only in higher order perturbation theory that the coefficient of the bare state becomes less than 1.

So isn't that simple? We just discussed it with all the cosine to the 4, sine to the 4. But now in perturbation theory this is the rate for going from dressed state 1 to 2. And this is the rate to go the other way around.

So what is our dressed state potential? Well, the dressed state potential is simply the EC stock shift, which is opposite for the ground and for the excited state. And the EC stock shift in perturbation theory is nothing else than matrix elements squared over detuning.

So this is μ naught. This is the potential of the standing wave experienced by the atom.

Now we want to do Sisyphus cooling. What is the cooling rate in Sisyphus cooling?

Well, remember, Sisyphus cooling we assume that the atom is fast enough to go over several hills and valleys. The cooling rate is determined by the rate at which the atoms switch dressed levels times μ naught. And I can scroll back, but μ naught was the EC stock effect, and γ one, two was this expression we had before. So therefore, in the simple picture, while everything is perturbative, we have a nice expression for the cooling rate.

Well, now let's get something non-trivial out of it. What is the friction coefficient?

Well, in many situations I've plotted for you, the force versus velocity. Now I want to plot for you the cooling rate versus velocity. That means its force times velocity.

And as we just discussed, when the velocity is sufficiently fast that the atom can go over several hills and valleys in a spontaneous lifetime, the cooling rate will saturate because it is limited by the rate at which the atom switches between dressed levels.

But now the question is, if we want to understand what happens at low velocities, how should we connect the two? Well, you can say we know, for analytical reasons, that the force versus velocity has to be linear. So the cooling rate, which is another power of velocity, should be quadratic.

And since I neglect all factors on the order of unity, this is now the force is friction coefficient times V . So therefore, what I do in red here is a parabolic approximation, which is α the cooling rate times V squared.

And now, if I want to know what is α , I just sort of connect this parabola with the saturated value, which I know. And I know the transition happens at the velocity when the atom moves further than one wavelength in one spontaneous emission time.

So therefore, based on this very physical picture, and on the inside how low velocities and high velocities connect together, I can actually obtain the friction coefficient in this blue detuned standing wave by taking the cooling rate and dividing it by the velocity at which the atom moves one wavelength per spontaneous emission time.

And if I do that, I obtain this result, which I have already quoted earlier to you from the paper of Cohen-Tonnoudji and Jean Dalibard. It looks fairly non-trivial with power to the 6, power to the 5. But this is now the simple perturbative results for the friction coefficient.

Now we have the friction coefficient. Let's take it further. I want to give you, in this perturbative limit, I want to give you the momentum diffusion coefficient. And then we will find in the perturbative limit what is the lowest temperature to which we can cool. So pretty much I'll do everything as we've done before, but I use the simple perturbative limit.

So again, in order to calculate the diffusion coefficient-- the momentum diffusion coefficient-- and the heating we need the fluctuations of the force. And now, just remember, we are in the perturbative limit. The atom is mainly in the ground state,

which is the upper dressed level. It has only a little excited state mixture, so it will mainly stay in this state. But then with this excited state mixture and this, power of the Rabi frequency, it makes the transition to the other dressed state.

But the other dressed state is almost 100% excited state. So it will leave the other dressed state almost immediately. So this could be a Quantum Monte Carlo wave function result that the atom experiences the force in the upper dressed level. Then it goes to the lower dressed level, which is mainly the excited state, just 4×1 over γ . And then it returns.

So the picture now is that the atom for almost all the time experiences the steady state force, which is the blue dashed line. And there are only, when it goes to the other dressed state, these small spikes where the force has changed sign.

And if you now calculate the fluctuations of the force-- the integral of f^2 -- once we calculate the square the small deviations from the average force does not contribute. What contributes are the spikes.

So all I do is now I take the 4^2 in one of the spikes. And the force is nothing else than the derivative-- remember we have a standing wave $a \mu \sin(kx)$. The derivative of the standing wave is the force. But if you have a sinusoid lattice the force is K times the amplitude of the lattice.

I square it, so this is the spike squared. Then I have to multiply with the time $1/\gamma$, over which the force stays the same. So this is, you can see, the correlation time of the force.

But then I have to multiply, also, when I do an average over many, sort of, you can see, trajectories, I have to average over the probability that-- I have to multiply with the probability that the atom is in the dressed state 1 and not in the dressed state 2.

It may take you a few minutes just to think about the combinatorics. But in the end all I do is I take the 4^2 , I multiply with correlation time when the atom is in state 2, and get the probability that the atom is in state 2. And when the atom is in state 1, in my ensemble, it doesn't contribute at all, because in state 1 you're so

close to the average force that it doesn't contribute with the fluctuation.

Anyway, it's simple but subtle. But it's a one-liner. And now based on this perturbative picture we have perturbatively exact an expression for the heating rate.

But now it's interesting, the rate $\gamma_{1,2}$, the rate to switch between dressed energy levels always also appears in the expression for the friction coefficient. So therefore, if you're now asking what is the ultimate temperature to which we can cool the atom? It is the ratio of this analytic result for the heating coefficient divided by the analytic result for the friction coefficient. Pretty much everything cancels out, and what remains is μ_{naught} .

And this is a remarkable result. It's highly non-trivial, and I really enjoyed showing to you how simply it can be derived by-- it's really just perturbation theory. But you have to put in the right concepts from the dressed atom picture.

So what we learn is the following. If you have a standing wave, and we cool with a stimulated force, the lowest temperature is-- with a prefactor, which I haven't calculated-- the amplitude of the standing wave. And that means that it is impossible to ever localize atoms within a standing wave.

Laser cooling cannot be used to create atoms in an optical lattice. The temperature of the atoms is always comparable to the depths of the lattice. So therefore the atoms will never be localized in one well of the lattice.

And this is generally valid. The assumption we have made here is that we have a two level atom.

A lot of you work with atoms to optical lattices. And you often use evaporative cooling, take a Bose Einstein condensate, which is pretty much a [INAUDIBLE] temperature put it in optical lattice. But before '95, people studied laser cooling and atoms in optical lattices. And this was only possible because atoms can be cooled to lower temperatures than we have derived here because they are not two level atoms.

So laser cooling in a standing wave, localization of atoms in a standing wave by laser cooling, is only possibly by physics beyond what we have discussed.

Colin?

AUDIENCE: If you use something like Raman's sideband cooling, you could-- like, yeah, you'd kind of need three levels for that, but you eliminated the third. Really, it's just that it's two levels, right?

PROFESSOR: If you do Raman's sideband cooling, he posted-- I think, in the next 10 minutes, I cover what you want to see. Because now it involves two ground states. And I want to show you, if you have two ground states, to Hyper fine-- Raman's sideband cooling-- if you have two different ground states there is no physics. And I briefly want to touch upon that.

We'll just kind of fill in one thing here, and it is, well, if you take everything I've told you seriously, you would say OK, too bad. We cannot localize atoms in an optical lattice because the temperature is always comparable to the amplitude of the lattice. But by making the lattice smaller and smaller and smaller I can reach lower and lower temperatures.

Well, yes, to some extent that's possible. But what we have done here is we've done approximations. We've only focused on the stimulated force. We have not looked at the recoil which comes in spontaneous emission.

And therefore, the moment we would make the optical lattice smaller than γ then we have to include heating by the spontaneous light force. And the result will be that blue molasses cannot reach lower temperatures than red detuned molasses. And for red detuned molasses we have already discussed the famous Doppler limit, which is $\gamma/2$.

So that's what I wanted to present to you with the stimulated force. First, a more general discussion using the real dressed atom picture. But now in the end, to fly over using sort of the baby dressed atom picture, which is simple perturbation theory.

OK, so we've done the hard work. Now we want to have some fun discussing the physics. And I'm realizing I've only five, six, seven minutes left. I want to discuss three points. One is I want to give you all the different pictures to find the potential of a dipole trap, because we've encountered several.

The second thing is I want to have a discussion with you if the stimulated force is due to electric or magnetic forces. And eventually, and we'll be running out of time, I have to do that on Wednesday to discuss the question of energy conservation. If you cool the atoms where does the energy go?

So if you use the stimulated force, it gives rise to dipole potential. And using the Optical Bloch Equations or the dressed atom picture, we found this expression for the dipole potential. And there are at least four ways how we could have derived it.

One is the Optical Bloch Equations, we did that. The dressed atom picture, we did that today. Another picture is very straightforward and simple. It just says that the potential is the electric potential of a dipole.

And then all you have to know is from the EC polarizability, if you drive an atom with a laser field below resonance-- I mean that's just harmonic oscillator-- below resonance the harmonic oscillator is in phase with the force. Above resonance, if you drive a harmonic oscillator fast, the response of the harmonic oscillator is in opposite, it's out of phase.

So therefore, if your potential is minus the E , you are in phase-- it is a negative attractive potential. If the harmonic oscillator is driven above resonance there is another minus sign, you get plus the E , which becomes a repulsive potential. So it's, look and say, just the physics of a harmonic oscillator.

There's one thing I like, which is sometimes not taught in an atomic physics course because it is the force on a macroscopic dielectric object. So let's assume we have an atom-- no, we don't have an atom. We have something which is bigger. Let's assume a tiny polystyrenes sphere. You can assume a small sphere of glass.

What happens if I shine a laser beam on a small sphere of glass? Well, let me assume that the laser beam is-- well, if the laser beam hits this sphere right on, it's symmetric, there is no force. So what we want to do is we want to have a laser beam, shown here by this arrow, which has a maximum here so the profile is like this. And we want to discuss whether this sphere is sucked into the laser beam, this would be an attractive force, or whether this sphere is repelled.

And all we want to use is optics, the optics of refraction. When we have a laser beam which is weak, medium, strong, strong here, and for red detuning we have an index of refraction which is larger than 1. So the sphere acts like a lens. So that leaves those photons up and in, and those photons up and in.

And when a photon is bent in, the momentum of the photon has changed. And the momentum, of course, the opposite momentum transfer, is imparted to the sphere. So in this case, we have sort of-- we have more photons in this part of the laser beam, which are bent up, than photons are bent down. And this means the net force is that the sphere is sucked into the laser beam. And if you go for blue detuned light, where the index of refraction is smaller than 1, it is the opposite.

So the picture which we have discussed here was for atoms from microscopic objects. But you have, actually, similar physics of a stimulated force for mesoscopic and macroscopic objects. And then what matters whether you trap or anti-trap, is whether the index of refraction is larger than 1 or smaller than one.

And you know, two level system for blue detuned light has an index of refraction which is smaller than 1. And for red detuned light our atomic clouds have an index of refraction which is larger than 1.

But this picture that you can use the same set up, the same focus beam and a similar dipole force to trap larger objects is actually exploited in a different field of physics, in biology, in the form of optical tweezers.

When actually in the early '80s, when Steve Chu came to Bell Labs and had discussions with Art Ashkin, they actually discussed optical trapping, both of atoms

and both of biological objects, little microspheres, or cells. And it's sort of remarkable that both approaches were realized within the same year. So atoms were trapped in '86. And macroscopic objects were manipulated, I think it's exactly the same year.

And as you know, the optical tweezers is now an important technology in biology and biophysics. For instance, you can focus a laser beam and manipulate objects within a living cell.

So I think I should stop here, but maybe take your questions. Then let me conclude with an outlook. We have this week and next week, and then the semester is over.

So this week we have one class on Wednesday. I want to finish this discussion and teach about magnetic trapping and evaporative cooling. And the last week, because of a makeup class on Friday, we have three more classes, Monday, Wednesday, Friday. By the way, I just learned that the Friday class is in these room.

And on Monday, Wednesday, Friday we talk about, well, it's really featured pick about Bose gasses, lecture one, Fermi gasses, lecture two. And the third lecture is about quantum logic with ions.

OK, see you Wednesday.