PROFESSOR: OK, in that case we can jump into the early universe. So on the opening slide here I have a picture of the Planck satellite, which is a satellite that was launched a few years ago dedicated to measuring the cosmic background radiation. Cosmic background radiation is really our biggest clue for the early history of the universe. The Planck satellite is actually the third satellite to go up completely dedicated to measuring the cosmic background radiation. The first was called COBE and then WMAP and now Planck.

Planck is still in orbit. It actually is finished with its data-taking, although it's nowhere near finished with the analysis of that data. So they made one major data release last March, and there are still very important pieces of their data that I haven't looked at yet. And we'll be talking more about what exactly these satellites see. Onward.

I want to begin by talking about the standard Big Bang, which will in fact be the main focus of this course. We'll probably spend about 2/3 of the course or so talking about the standard Big Bang and then move on to topics like inflation. That actually is, I think, a very sensible balance, because as you'll see once we get under studying inflation, it's a fairly straightforward thing once you know the basic equations coming out of standard cosmology. So I think spending about 2/3 of the term or so on the conventional cosmology before we get to inflation is very sensible, because that will set up all the principles that we'll be using later to discuss more advanced topics like inflation.

The conventional Big Bang model is basically the theory that the universe as we know it began some 13 to 14 billion years ago. And now we even have a pretty precise number to replace this 13 to 14 billion years. This is based on the Planck

satellite combined with a few other pieces of information. The number is 13.82 billion years, plus or minus 0.05. So it's pretty well pinned down now, the age of the universe since the Big Bang.

I should add, though, I put in the qualifier "the universe as we know it." What that really means is that I want to leave it out, because we don't really know that the universe began with what we call the Big Bang. So we have a very good picture of the Big Bang, and we're very confident that it happened and that we understand what it looked like. But whether or not anything came before it is a much more open question which I think is really completely open.

I think we should not act like we know that the universe began with the Big Bang. And in fact later at the very end of the course when we discuss some of the implications of inflation and the multiverse, we'll see that there are strong suggestions that the Big Bang was perhaps not really the beginning of existence, but really just the beginning of our local universe, often called a pocket universe. OK.

In any case, what the Big Bang theory tells us is that at least our region of the universe 13.82 billion years ago was an extremely hot, dense, uniform soup of particles, which in the conventional standard Big Bang model filled literally all of space. And now we certainly believe it filled essentially all the space that we have access to uniformly. Now I should point out that this is contrary to a popular cartoon image of the Big Bang, which is just plain wrong. The cartoon image of the Big Bang is the image of a small egg of highly dense matter that then exploded and spewed out into empty space. That is not the scientific picture of the Big Bang.

And the reason is not because it's illogical. It's hard to know what's logical or illogical in this context. But simply based on what we see, if there was a small egg that exploded into empty space, you would certainly expect that today you would see something different if you were looking toward where the egg was versus looking the opposite direction. But we don't see any effect like that. When we look around the sky, the universal looks completely uniform, on average, in all directions to very

high degree of accuracy. I'll talk a little bit more precisely later.

So we don't see any sign of an egg having happened anywhere. Rather the Big Bang seems to have happened everywhere uniformly. OK.

The Big Bang describes a number of important things, and we'll be talking about this more as the course goes on. It describes how the early universe expanded and cooled-- and will be spending a fair amount of time understanding the details behind those words. The point is that the Big Bang really is a very precise model based on very simple assumptions. You basically assume that the early universe was filled by a hot gas in thermal equilibrium. And that this gas was expanding and being pulled back by gravity.

And from those simple ideas you can really calculate-- and we'll learn how to calculate-- how fast the universe would have been expanding at any given instant of time, what the temperature would have been at any given instant of time, what the density of matter would have been at any instant of time. So all the details really can be calculated from some rather simple ideas, and we'll have fun exploring that.

The Big Bang also talks about how the light chemical elements formed. And that actually is the main topic of Steve Weinberg's book "The First Three Minutes." Because that was more or less the time period during which the chemical elements formed. It turns out that most of the elements in the universe did not form in the Big Bang, but formed much later in the interior of stars. And those elements are then strewn out into space in supernova explosions and collected into later generation stars, of which our sun would be one.

So the stuff that we're made out of was actually not produced in the Big Bang, but rather was produced in the interior of some distant star that exploded long ago. And maybe many stars, whose material collected to form our solar system. Nonetheless, most of the matter in the universe-- as opposed to most of the different kinds of elements-- did form in the Big Bang. Most of the matter in the universe is just hydrogen and helium.

About five different isotopes of hydrogen, helium, and lithium were primarily formed in the Big Bang, and because we do have this detailed picture of the Big Bang that we'll be learning about, it's possible to actually calculate the predicted abundances of those different isotopes. And the predictions agree very well with the observations. And this is certainly one of the major confirmations we have that this picture of the Big Bang is correct. We can predict what the abundance of helium-3 should be, and we measure it, and it agrees. It's rather marvelous.

Finally-- and this subject we're not going to talk about much because it goes beyond the level of complexity that the course is going to have-- but finally the Big Bang does discuss how the matter ultimately congealed to form stars, galaxies, clusters of galaxies. We'll talk about that a little bit, but we won't really follow that very far. That is still in principle a work in progress. People do not understand everything about galaxies. But the general picture of that-- it started out with an almost uniform universe, and then the lumps congealed to form the galaxies and the structures-- we say certainly seems to be a working picture. And one can understand a lot about the universe from this very simple picture.

OK what I want to talk about next is what the conventional Big Bang theory does not talk about, where new ideas like inflation come in.

First of all, the conventional Big Bang theory does not say anything about what caused the expansion. It really is only a theory of the aftermath of a bang. In the scientific version of the Big Bang, the universe starts with everything already expanding with no explanation of how that expansion started. That's not part of the Big Bang theory. So the scientific version of the Big Bang theory is not really a theory of a bang. It's really the theory of the aftermath of a bang.

In addition, and maybe in a similar vein, the conventional Big Bang theory says nothing about where all the matter came from. The theory really assumes that for every particle that we see in the universe today, there was at the very beginning at least some precursor particle, if not the same particle, with no explanation of where all those particles came from. In short, what I like to say is that the Big Bang says

nothing about what banged, why it banged, or what happened before it banged. It really has no bang in the Big Bang. It's a bangless theory, despite its name.

Inflation, it turns out, fills in possible answers, very plausible answers, for many of these questions. And that's what I'll be talking about mainly for the rest of today. And as I said, in terms of the course, it's where we'll be aiming to get about 2/3 of the way through the semester.

What is cosmic inflation? It's basically a minor modification, in terms of the overall scheme of things, of the standard Big Bang theory. And the best word to describe it is a word that I think was invented by Hollywood-- inflation is a prequel to the conventional Big Bang theory. It's a short description of what happened before, immediately before, the Big Bang. So inflation really is an explanation of the bang of the Big Bang in the sense that it does provide a theory of the propulsion that drove the universe into this humongous episode of expansion which we call the Big Bang.

And it does it in terms of something that I like to think of as a miracle of physics. When I use the word "miracle" in this context-- referring to a miracle in the scientific sense-- simply something that's so surprising that I think it merits being called a miracle even though it's something that's a part of the laws of physics. There are just a few features of the laws of physics that are actually crucial to inflation-- I'll be talking about two of them-- which I consider miracles simply because-- well, mainly because when I was an undergrad nobody talked about them at all. They were just not part of physics then, even though they really were. They just weren't parts of physics people noticed and talked about.

So the miracle of physics I'm talking about here is something which actually is known since the time of Einstein's general relativity-- that gravity is not always attractive. Gravity can act repulsively. And Einstein introduced this in 1916, I guess, in the form of what he called his cosmological constant. And the original motivation of modifying the equations of general relativity to allow this was because Einstein thought that the universe was static. And he realized that ordinary gravity would cause the universe to collapse if it were static. It couldn't remain static. So he

introduced this cosmological constant term to balance the overall attraction of ordinary gravity to be able to build a static model of the universe.

As you'll soon be learning, that's dead wrong. That's not what the universe looks like at all. But the fact that general relativity can support this gravitation repulsion, still being consistent with all the principles that general relativity incorporates, is the important thing which Einstein himself did discover in this context. And inflation takes advantage of this possibility, within the context of general relativity, to let gravity be the repulsive force that drove the universe into the period of expansion that we call the Big Bang.

And in fact when one combines general relativity with conventional ideas now in particle physics, there really is a pretty clear indication, I should say-- not quite a prediction-- but a pretty clear indication that at very high energy densities, one expects to find states of matter which literally turn gravity on its head and cause gravity to become repulsive. In terms of the details which we'll be learning about more later, what it takes to produce gravitational repulsion is a negative pressure.

According to general relativity, it turns out-- and we'll be talking more about this later-- both pressures and energy densities can produce gravitational fields. Unlike Newtonian physics, where it's only mass densities that produce gravitational fields.

And the positive pressure produces an attractive gravitational field, which is what you would probably guess if somebody just asked you to guess. Positive pressures are sort of normal pressures, and attractive gravity is normal gravity. So normal pressures produce normal gravity. But it is possible to have negative pressures, and negative pressures produce repulsive gravity. That's the secret of what makes inflation possible.

So inflation proposes that at least a small patch of this repulsive gravity material existed in the early universe. We don't really know exactly when in the history of the universe inflation happened, which is another way of saying we don't know exactly at what energy scale inflation happened. But a very logical, plausible choice-- I don't know if logical is the right word, but plausible is a good word-- very plausible choice

for when inflation might have happened, would be when the energy scales of the universe where at the scale of grand unified theories.

Grand unified theories-- we'll talk about a little bit later-- are theories which unify the weak, strong, and electromagnetic interactions into a single unified interaction. And that unification occurs at a typical energy of about 10 to the 16 GeV, where GeV, in case you don't know, is about the mass-- or the energy equivalent of the mass-- of a proton. So we're talking about energies that are about 10 to the 16 times the equivalent energy of a proton mass. And at those energies we think that these states that create repulsive gravity are very likely to exist.

And if that happened at that scale, the initial patch would only have to be the ridiculously small size of about 10 to the minus 28 centimeters across to be able to lead ultimately to the creation of everything that we see on the vast scale at which we see it. The patch certainly does not have to be the entire universe. And it could in fact be incredibly rare, because one thinks that outside of that patch essentially nothing will happen interesting. So we expect that the universe that we observe today would be entirely the consequence of such a patch.

The gravitational repulsion created by this small patch of repulsive gravity material would be then the driving force of the Big Bang, and it would cause the region to undergo exponential expansion. And by exponential expansion, as you probably know, it means that there's a certain doubling time, and if you wait the same amount of time it doubles again. If you wait the same amount of time, it doubles again.

And because these doublings build up so dramatically, it doesn't take very much time to build the whole universe. In about 100 doublings, this tiny patch of 10 to the minus 28 centimeters can become large enough not to be the universe, but to be a small marble size region, which will then ultimately become the observed universe as it continues to coast outward after inflation ends.

So the doubling time would be incredibly small if this was all happening at the grand unified theory set of numbers-- 10 to the minus 37 seconds, which is pretty fast. The patch would expand exponentially by at least a factor of about 10 to the 28, which

as I mentioned takes only about 100 doublings, and could expand to be much more. There's no cut off there. If there's more expansion than we need to produce our universe, it just means that the patch of universe that we're living in is larger than we see. But that's OK. Everything that we see looks uniform as far as we can see, and how much there is beyond that we really just have no way of knowing. So larger amounts of inflation are perfectly consistent with what we see.

The amount of time it would take would only be about 10 to the minus 35 seconds, which is just 100 times 10 to the minus 37 if you can do that complicated arithmetic in your head. And the region that's destined to become our presently observed universe at the end of inflation would have been only about the size of a marble-- about one centimeter or so across.

Now what ends inflation is the fact that this repulsive gravity material is unstable. So it decays, using the word decay in the same sense that a radioactive substance decays. It doesn't necessarily mean exactly that it rots like an apple decays, but it means that it turns into other kinds of material. And in particular, it turned into material which is no longer gravitationally repulsive. So the gravitational repulsion ends, and in fact the particles produced by this energy that's released at the end of inflation become the hot soup of the conventional Big Bang.

And this is where the prequel ends, and the main feature begins-- the conventional Big Bang theory. The role of inflation is just to set up the initial conditions for the conventional Big Bang theory. Now there's a little caveat here. Inflation ends because the material is unstable, but it only ends almost everywhere, not quite everywhere. And this is basically the way exponentials work. And we'll come back to this when we talk about the late time behavior and the idea of eternal inflation.

This repulsive gravity material decays, but it decays like a radioactive substance-- which is also an exponential-- as a half life. But no matter how many half lives you wait, there's still a tiny little bit, a tiny fraction that remains. And that turns out to be important for the idea that in many cases inflation never completely ends. We'll come back to that.

So I want to talk more about what goes on during this exponential expansion phase. There's a very peculiar feature of this inflation-- this exponential expansion driven by repulsive gravity-- which is that while it's happening, the mass density or energy density of the inflating material-- this repulsive gravity material-- does not decrease. You would think that if something doubled in radius, it would multiply by a factor of eight in volume. You would think the energy density would go down by a factor of eight.

And that certainly happens for ordinary particles. It's certainly what would happen if you had a gas, an ordinary gas, that you just allowed to expand by a factor of two in radius-- the density would go down by a factor of eight with volumes of cubes of distances. But this peculiar repulsive gravity material actually expands at a constant density. Now that sounds like it must violate conservation of energy, because it really does mean that the total amount of energy inside this expanding volume is increasing. The energy per volume is remaining constant, and the volume is getting bigger and bigger exponentially.

So the claim is that I've not gone crazy, that this actually is consistent with the laws of physics as we know them. And that it is consistent with conservation of energy. Conservation of energy really is a sacred principle of physics. We don't know of anything in nature that violates this principle of conservation of energy, that energy ultimately cannot be either made or destroyed, that the total amount of energy is basically fixed. So it sounds like there's a contradiction here. How do we get out of it? What's the resolution?

Well, this requires my second miracle of physics. Energy-- it really is exactly conserved. I'm not going to tell you about any miracles which changed that. But the catch here is that energies are not necessarily positive. There are things that have negative energies. And in particular, the gravitational field has a negative energy. This statement by the way is true both in Newtonian physics and in general relativity. We'll prove it later.

I might just say quickly if some of you have learned in an E&M course how to talk

about and calculate the energy density of an electrostatic field-- probably many of you have, maybe all of you have-- the energy density of an electrostatic field is a constant times the square of the electric field strength. And you can prove that energy is exactly the energy that you need to put into a system to create an electric field of a given configuration. If you think about Newton's law of gravity and compare it with Coulomb's law, you realize that it really is the same law, except they have a different constant in front of them.

They're both inverse square laws in proportion to the two charges, where in the case of gravity it's the masses that play the role of charges. But they have opposite signs. Two positive charges, as we all know we tell each other, two positive masses attract each other.

So in fact the very same argument which allows you to calculate the energy density of a Coulomb field can allow you to calculate the energy density of a Newtonian gravitational field-- still sticking to Newtonian physics-- and this change in sign of the force just carries through. It changes the signs of all the work that's being done, and you get the negative answer that is the correct answer for Newtonian gravity. The energy density of a Newtonian gravitational field is negative. And the same is true in general relativity in a more subtle way.

So what that means in terms of conservation of energy is that we can have more and more matter, more and more energy building up in the form of ordinary matter-- which is what happens during inflation-- as long as there's a compensating amount of negative energy that's created in the gravitational field which is filling this ever larger region of space. And that's exactly what happens in inflation.

The positive energy of this repulsive gravity material which is growing and growing in volume is precisely canceled by the negative energy of the gravitational field that's filling the region. So the total energy does remain constant, as it must, and there's certainly a good possibility that the total energy is exactly zero. Because everything that we know of is at least consistent with the possibility that these two numbers are exactly equal to each other or something very close.

Schematically, the picture is that if one thinks about the total energy of the universe, it consists of a huge positive amount in the form of matter and radiation-- the stuff that we see, the stuff that we normally identify the energy of-- but there's also a huge negative amount of energy in the gravitational field that fills the universe. And as far as we can tell, the sum is at least consistent with being 0.

In any case, what happens during inflation is the black bar goes up and the red bar goes down. And they go up and down by equal amounts. So certainly what happens during inflation conserves energy, as anything consistent with the laws of physics that we know of must conserve energy.

I just remembered I was planning to turn out these blackboard lights. It probably makes it a little more comfortable to watch the screen. OK. So, onward.

I want to talk some about the evidence for inflation. So far I've described what inflation is-- and I'm sort of done describing what inflation is for today. As I said, we'll be coming back and talking about all this during the coming semester. Now let's move on to discuss some of the reasons why we think that our universe may very likely have actually undergone this process called inflation I was just telling you about. So there are three things I want to talk about.

The first of which is the large scale uniformity of the universe. Which is related to what I told you at the beginning, that if you look out in different directions in the universe, it really looks the same in all directions. And the object that can be measured with the most precision in terms of how things vary with angle, is the cosmic background radiation-- because we can measure it from all directions, and it's essentially a uniform background.

And when that's been done, what's been found is that the radiation is uniform to the incredible accuracy of about one part in 100,000-- which really is a rather spectacular level of uniformity. So it means the universe really is rather incredibly uniform.

I might mention one proviso here just to be completely accurate. When one actually

just goes out and measures the radiation, one finds something-- one finds an asymmetry that's larger than what I just said. One finds an asymmetry of about 1 part in 1,000, with one direction being hotter than the opposite direction. But that 1 part in 1,000 effect we interpret as our motion through the cosmic background radiation, which makes it look hotter in one direction and colder in the opposite direction. And the effect of our motion has a very definite angular pattern.

We have no other way of knowing what our velocity is relative to the cosmic background radiation. So we just measure it from this asymmetry, but we're restricted. We can't let it account for everything. Because it has a very different angular form, we only get to determine one velocity. And once we determine that, that determines one asymmetry and you can subtract that out. And then the residual asymmetries, the asymmetries that we cannot account for by saying that the Earth has a certain velocity relative to the cosmic background radiation, those asymmetries are at the level of 1 part in 100,000. And this is 1 part in 100,000 that we attribute to the universe and not to the motion of the earth.

OK. So to understand the implications of this incredible degree of uniformity, we need to say a little bit about what we think the history of this cosmic background radiation was. And what our theories tell us-- and we'll be learning about this in detail-- is that in the early period-- Yes.

AUDIENCE: I'm sorry. I'm curious. When they released WMAP and stuff, did they already subtract out the relativistic effect?

PROFESSOR: Well, the answer is that they analyze things according to angular patterns and how they fit different angular patterns. So in fact, I think they don't even report it with WMAP, but it would be what they would call L equals 1, the dipole term. They analyze the dipole, quadrupole, octupole, et cetera. So it really does not contribute at all to anything except that L equals 1 term, which is one out of 1,800 things that they measure. So basically, I think they don't even bother reporting that one number, and therefore it's subtracted out.

OK. Do feel free to ask questions, by the way. I think it's certainly a small enough

class that we can do that.

OK. So what I was about to say is that this radiation during the early period of the universe, when the universe was a plasma, the radiation was essentially locked to the matter. The photons were moving at the speed of light, but in the plasma there's a very large cross section for the photons to scatter off of the free electrons in a plasma. Which basically means that the photons move with the matter-- because when they're moving on their own, they just move a very short distance and then scatter, and then move in a different direction. So relative to the matter, the photons go nowhere during the first 400,000 years of the history of the universe.

But then at about 400,000 years the universe cools enough-- this is all according to our calculations-- the universe cools enough so that the plasma neutralizes. And when the plasma neutralizes, it becomes a neutral gas like the air in this room. And the air in this room seems completely transparent to us, and it turns out that actually does extrapolate to the universe.

The gas that filled the universe after it neutralized really was transparent, and it means that a typical photon that we see today in the cosmic background radiation really has been traveling on a straight line since about 400,000 years after the Big Bang. Which in turn means that when we look at the cosmic background radiation, we're essentially seeing an image of what the universe looked like at 400,000 years after the Big Bang. Just as the light traveling from my face to your eyes gives you an image of what I look like.

So that's what we're seeing-- a picture of the universe at the age of 400,000 years, and it's bland-- uniform to 1 part in 100,000. So the question then is, can we explain how the universe to be so uniform? And it turns out that if you-- Well, I should say first of all that if you're willing to just assume that the universe started out perfectly uniform to better than one part in 100,000, that's OK. Nobody could stop you from doing that. But if you want to try to explain this uniformity without assuming that it was there from the beginning, then within the context of the conventional Big Bang theory, it's just not possible.

And the reason is that within the evolution equations of the conventional Big Bang theory, you can calculate-- and we will calculate later in the course-- that in order to smooth things out in time for it to look smooth in the cosmic background radiation, you have to be able to move around matter and energy at about 100 times the speed of light. Or else you just couldn't do it. And we don't know of anything in physics that happens faster than the speed of light. So within physics as we know it, and within the conventional Big Bang theory, there's no way to explain this uniformity except to just assume that maybe it was there from the very beginning. For reasons that we don't know about.

On the other hand, inflation takes care of this very nicely. What inflation does is it adds this spurt of exponential expansion to the history of the universe. And the fact that this exponential expansion was so humongous means that if you look at our picture of the universe before the inflation happened, the universe would have been vastly smaller than in conventional cosmology which would not have this exponential spurt of expansion.

So in the inflationary model there would've been plenty of time for the observed part of the universe to become uniform before inflation started-- when it was incredibly tiny. And then would become uniform just like the air in the room here tends to spread out and produce a uniform distribution of air rather than having all the air collected in one corner. Once that uniformity is established on this tiny region, inflation would then take over and stretch this region to become large enough to include everything that we now see, thereby explaining why everything that we see looks so uniform. It's a very simple explanation, and it's only possible with inflation and not within the conventional Big Bang theory.

So, the inflationary solution. In inflationary models the universe begins so small that uniformity is easily established. Just like the air in the lecture hall-- same analogy I used-- spreads uniformly to fill the lecture hall. Then inflation stretches the region to become large enough to include everything that we now observe. OK. So that's the first of my three pieces of evidence for inflation.

The second one is something called the flatness problem. And the question is, why was the early universe so flat? And the first question maybe is, what am I talking about when I say the early universe was flat? One misconception I sometimes find people getting is that flat often means two dimensional. That's not what I mean. It's not flat like a two dimensional pancake. It's three dimensional. The flat in this context means Euclidean-- obeying the axioms of Euclidean geometry-- as opposed to the non-Euclidean options that are offered by general relativity.

General relativity allows three dimensional space to be curved. And if we only consider uniform curvature, which is-- we don't see any curvature, actually, but-- We know with better accuracy that the universe is uniform than we do that it's flat. So imagine in terms of discussion of cosmology three possible curvatures for the universe, all of which would be taken to be uniform. Three dimensional curved spaces are not easy to visualize, but all three of these are closely analogous to two dimensional curved spaces, which are easy to think about.

One is the closed geometry of the surface of a sphere. Now the analogy is that the three dimensional universe would be analogous to the two dimensional surface of a sphere. The analogy changes the number of dimensions. But important things get capped. Like for example on the surface of a sphere, you can easily visualize-- and there's even a picture to show you-- that if you put a triangle on the surface of a sphere, the sum of the three angles at the vertices would be more than 180 degrees. Unlike the Euclidean case, where it's always 180 degrees. Question?

AUDIENCE: Yeah. Is the 3D curving happening in a fourth dimension? Just like these 2D models assume another dimension?

PROFESSOR: Good question. The question was, is the 3D curvature happening in a fourth dimension just like this 2D curvature is happening in a third dimension? The answer I guess is yes. But I should maybe clarify the "just like" part. The third dimension here from a strictly mathematical point of view allows us to visualize the sphere in an easy way, but the geometry of the sphere from the point of view of people who study differential geometry is a perfectly well defined two dimensional space without

any need for the third dimension to be there.

The third dimension is really just a crutch for us to visualize it. But that same crutch does work in going from three to four. And in fact when we study the three dimensional curved space of the closed universe, we will in fact do it exactly that way. We'll introduce the same crutch, imagine it in four dimensions, and it will be very closely analogous to the two dimensional picture that you're looking at.

OK. So one of the possibilities is a closed geometry where the sum of the three angles of a triangle is always bigger than 180 degrees. Another possibility is something that's usually described as saddle shaped, or a space of negative curvature. And in that case the sum of the three angles-- they get pinched, and the sum of the angles is less than 180 degrees. And only for the flat case is the sum of the three angles exactly 180 degrees, which is the case of Euclidean geometry.

The geometries on the surfaces of these objects is non-Euclidean, even though if you think of the three dimensional geometry of the objects embedded in three dimensional space, that's still Euclidean. But the restricted geometry to the two dimensional surfaces are non-Euclidean there and there, but Euclidean there.

And that's exactly the way it works in general relativity. There are closed universes with positive curvature and the sum of angles being more than 180 degrees. And there are open universes where the sum of three angles is always less than 180 degrees. And there's the flat case-- which is just on the borderline of those two-- where Euclidean geometry works. And the point is that in our universe, Euclidean geometry does work very well. That's why we all learned it in high school. And in fact we have very good evidence that the early universe was rather extraordinarily close to this flat case of Euclidean geometry. And that's what we're trying to understand and explain.

According to general relativity this flatness of the geometry is determined by the mass density. There's a certain value of the mass density called the critical density-- which depends on the expansion rate, by the way, it's not a universal constant of any kind. But for a given expansion rate one can calculate a critical density, and that

critical density is the density which makes the universe flat. And cosmologists define a number called omega-- capital omega-- which is just the ratio of the actual mass density to the critical mass density. So omega equals 1 says the actual density is the critical density, which means the universe would be flat. Omega bigger than 1 would be a closed universe, and omega less than 1 would be an open universe.

What's peculiar about the evolution of this omega quantity is that omega equals 1 as the universe evolves in conventional cosmology behaves very much like a pencil balancing on its tip. It's an unstable equilibrium point. So in other words, if omega was exactly equal to 1 in the early universe, it would remain exactly equal to 1. Just like a pencil that's perfectly balanced on its tip would not know which way to fall and would in principle stay there forever. At least with classical mechanics. We won't include quantum mechanics for our pencil. Classical pencil that we're using for the analogy.

But if the pencil leans just a tiny bit in any direction, it will rapidly start to fall over in that direction. And similarly if omega in the early universe was just slightly greater than 1, it would rapidly rise towards infinity. And this is a closed universe. Infinity really means the universe has reached its maximum size, and it turns around and collapses. And if omega was slightly less than 1, it would rapidly dribble off to 0, and the universe would just become empty as it rapidly expands.

So the only way for omega to be close to 1 today-- and as far as we can tell, omega is consistent with 1 today-- the only way that can happen is if omega started out unbelievably close to 1. Unless it's this pencil that's been standing there for 14 billion years and hasn't fallen over yet. Numerically, for omega to be somewhere in the allowed range today, which is very close to 1, it means that omega at one second after the Big Bang had to be equal to 1 to the incredible accuracy of 15 decimal places. Which makes the value of the mass density of the universe at one second after the Big Bang probably the most accurate number that we know in physics. Since we really know it to 15 decimal places. So if it wasn't in that range, it wouldn't be in the [? lab manuals ?] today. We have this amplification effect of the evolution of the universe.

So the question is, how did this happen? In conventional Big Bang theory, the initial value of omega could have been anything, logically. To be consistent with what we now observe it has to be within this incredibly narrow range, but there's nothing in the theory which causes it to be in that narrow range. So the question is, why did omega start out so incredibly close to 1? Like the earlier problem about homogeneity, if you want to just assume that it started out-- exactly like, it had to be-- at omega equals 1, you could do that. But if you want to have any dynamical explanation for how it got to be that way, there's really nothing in conventional cosmology which does it. But in fact, inflation does.

In the inflationary model we've changed the evolution of omega because we've turned gravity into a repulsive force instead of an attractive force, and that changes the way omega evolves. And it turns out that during inflation, omega is not driven away from 1-- as it is during the entire rest of the history of the universe-- but rather during inflation omega is driven rapidly towards 1, exponentially fast, even. So with the amount of inflation that we talked about-- inflation by a factor of 10 to the 28 or so-- that's enough so that the value of omega before inflation is not very much constrained. Omega could have started out before inflation not being 1, but being 2 or 10 or 1/10 or 100 or 1/100.

The further away you start omega from 1, the more inflation you need to drive it to 1 sufficiently. But you don't need much more inflation to make it significantly far away from 1 because of this fact the inflation drives omega to 1 exponentially. Which really means it's a very powerful force driving omega to 1. And giving us a very simple, therefore, explanation for why omega in the early universe appears to have been extraordinarily close to 1.

So I think that's-- Oh, I have a few more things to say.

There's actually a prediction that comes out of this, because this tendency of inflation to drive omega to 1 is so strong, that you expect that omega really should be 1 today. Or to within measurable accuracy. You could arrange inflationary models where it's say, 0.2-- which is what people used to think it was-- but in order

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to do that, you have to arrange for inflation to end at just the right time before it makes it closer. Because every e-fold drives it another factor of 10 closer. So it's very rapid effect. So if you don't fine tune things very carefully, most any inflationary model will drive omega so close to 1 that today we would see it as 1.

That did not used to appear to be the case. Before 1998 astronomers were pretty sure that omega was only 0.2 or 0.3, while inflation seemed to have a pretty clear prediction that omega should be 1. This personally I found rather uncomfortable, because it meant whenever I had dinner with astronomers, they would always sort of snidely talk about how inflation was a pretty theory, but it couldn't be right because omega was 0.2, and inflation was predicting omega is 1. And it just didn't fit.

Things changed a lot in 1998, and now the best number we have-- which comes from the Planck satellite combined with a few other measurements, actually-- is that now the observational number for omega is 1.0010, plus or minus 0.0065. So the 0.0065 is the important thing. This is very, very close to 1, but the error bars are bigger than this difference. So it really means to about a half a percent or maybe 1%, we know today that omega is 1, which is what inflation would predict. That it should essentially be exactly 1 today.

The new ingredient that made all this possible, that drove-- changed the measurement of omega from 0.2 to 1 is a new ingredient to the energy budget of the universe, the discovery of what we call dark energy. And we'll be learning a lot about dark energy during the course of the term. The real discovery in 1998 was that the universe is not slowing down under the influence of gravity as had been expected until that time, but rather the universe actually is accelerating.

And this acceleration has to be attributed to something. The stuff that it's attributed to is called the dark energy. And even though there's considerable ignorance of what exactly this dark energy is, we can still calculate how much of it there's got to be in order to produce the acceleration that's seen. And when all that is put together, you get this number, which is so much nicer for inflation than the previous

number. Yes.

AUDIENCE: So, was the accelerating universe like the missing factor which they-- gave a wrong assumption which made them think that omega was 0.2 or 0.3?

PROFESSOR: Yeah, that's right. It was entirely because they did not know about the acceleration at that time. They in fact were accurately measuring the stuff that they were looking at. And that does only add up to 0.2 or 0.3. And this new ingredient, the dark energy, which we only know about through the acceleration, is what makes the difference. Yes.

AUDIENCE: And that data that they were measuring is really just sort of the integrated stuff in the universe that we see through telescopes? Very straight-forward in that way?

PROFESSOR: That's right. Including dark matter. So it's not everything that we actually see. There's also-- not going into it here, but we will later in the course-- there is also stuff called dark matter, which is different from dark energy. Even though matter and energy are supposed to be the same, they are different in this context. And dark matter is matter that we infer exists due to its effect on other matter. So by looking, for example, at how fast galaxies rotate, you can figure out how much matter there must be inside those galaxies to allow those orbits to be stable. And that's significantly more matter than we actually see. And that unseen matter is called the dark matter, and that was added into the 0.2 or 0.3. The visible matter is only about 0.04.

OK. So, so much for the flatness problem. Next item I want talk about is the small scale nonuniformity of the universe. On the largest scale, the universe is incredibly uniform-- one part in 100,000-- but on smaller scales, the universe today is incredibly lumpy. The earth is a big lump in the mass density distribution of the universe. The earth is in fact about 10 to the 30 times denser than the average matter density in the universe. It's an unbelievably significant lump. And the question is, how did these lumps form? Where did they come from?

We are confident that these lumps evolved from the very minor perturbations that

we see in the early universe, that we see most clearly through the cosmic background radiation. The early universe we believe was uniform in its mass density to about one part in 100,000. But at the level of one part in 100,000, we actually see in the cosmic background radiation that there are nonuniformities.

And things like the Earth form because these small nonuniformities in the mass density are gravitationally unstable. In regions where there's a slight excess of matter, that excess of matter produces a gravitational field pulling more matter into those regions, producing a still stronger gravitational field pulling in more matter. And the system is unstable, and it forms complicated lumps which are galaxies, stars, planets, et cetera.

And that's a complicated story. But it all starts from these very faint nonuniformities that existed, we believe, shortly after the Big Bang. And we see these nonuniformities in the cosmic background radiation, and measuring them tells us a lot about the conditions of the universe then, and allows us to build theories of how the universe got to be that way. And that's what these satellites like COBE, WMAP, and Planck are all about-- measuring these nonuniformities to rather extraordinary accuracy.

Inflation has an answer to the riddle of where the nonuniformities came from. In the conventional Big Bang theory, there was really just no explanation. People just assumed they were there and put them in by hand, but there was no theory of what might have created them. In the context of inflationary models where all the matter really is being created by the inflation, the nonuniformities are also controlled by that inflation, and where nonuniformities come from is quantum effects.

It's a little hard to believe that quantum effects could be important for the large scale structure of the universe. The Andromeda galaxy doesn't look like it's something that should be thought of as a quantum fluctuation. But when one pursues this theory quantitatively, it actually does work very well. The theory is that the ripples that we see in the cosmic background radiation really were purely the consequence of quantum theory-- basically the uncertainty principle of quantum theory, which

says that it's just impossible to have something that's completely uniform. It's not consistent with the uncertainty principle.

And when one puts in the basic ideas of quantum mechanics, we can actually calculate properties of these ripples. It turns out that we would need to know more about the physics of very high energy-- the physics that was relevant during the period of inflation-- to be able to predict the actual amplitude of these ripples. So we cannot predict the amplitude. In principle, inflation would allow you to if you knew enough about the underlying particle physics, but we don't know that much. So in practice we cannot predict the amplitude.

But inflationary models make a very clear prediction for the spectrum of these fluctuations. And by that I mean how the intensity varies with wavelength. So the spectrum really means the same thing as it would mean for sound, except you should think about wavelength rather than frequency because these waves don't really oscillate. But they do have wavelengths just like sound waves have wavelengths, and if you talk about the intensity versus wavelengths, this idea of a spectrum is really the same as what you'll be talking about with sound.

And you can measure it. This is not quite the latest measurements, but it's the latest measurements that I have graphed. The red line is the theoretical prediction. The black dots are the measurements. This goes through the seven year WMAP data. We have a little Eureka guy to tell you how happy I am about this curve.

And I also have graphs of what other ideas would predict. For a while, for example, people took very seriously the idea that the randomness that we see in the universe-- these fluctuations-- may have been caused by the random formation of things called cosmic strings that would form in phase transitions in the early universe. That was certainly a viable idea in its day, but once this curve got measured, the cosmic strings were predicting something that looked like that, which is nothing at all like that curve. And they have since been therefore excluded as being the source of density fluctuations in the universe. And various other models are shown here. I don't think I'll take the time to go into, because there are other

things I want to talk about.

But anyway, marvelous success. This is actually the latest data. This is the Planck data that was released last March. I don't have it plotted on the same scale, but again you see a theoretical curve based on inflation and dots that show the data with little tiny error bars. But absolutely gorgeous fit. Yes.

AUDIENCE: What happened to your theory of inflation after they discovered dark energy? Did it change significantly?

PROFESSOR: Did the theory change?

AUDIENCE: Or like, in the last graph there was a different curve.

PROFESSOR: Well it's plotted on a different scale, but this actually is pretty much the same curve as that curve. Although you can't tell.

AUDIENCE: Sorry.

PROFESSOR: Oh. Oh, inflation without dark energy, for example. I think it's not so much that the theory of inflation changed between these two curves, but the curve you actually see today is the result of what things looked like immediately after inflation combined with the evolution that took place since then. And it's really the evolution that took place since then that makes a big difference between this inflationary curve and the other inflationary curve. So inflation did not have to change very much at all. It really did not. But of course it looks a lot better after dark energy was discovered because the mass density came out right, and gradually we also got more and more data about these fluctuations which just fit beautifully with what inflation predicts.

OK. I want to now launch into the idea of the multiverse. And I guess I'll try to go through this quickly so that we can finish. We're not going try to understand all the details anyway, so I'll talk about fewer of them for the remaining 10 minutes of the class. But I'd like to say a little bit about how inflation leads to the idea of a multiverse. Of course we'll come back to it at the very end of the class, and it's

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certainly an exciting, I think, aspect of inflation.

The repulsive gravity material that drives the inflation is metastable, as we said. So it decays. And that means that if you sit in one place and ask where inflation is happening, and ask what's the probability that it's still happening a little bit later, that probability decreases exponentially-- drops by a factor of two every doubling, every half life. But at the same time, the volume of any region that's inflating is also growing exponentially, growing due to the inflation. And in fact in any reasonable inflationary model the growth rate is vastly faster than the decay rate. So if you look at the volume that's inflating, if you wait for a half life, indeed half of that volume will no longer be inflating-- by the definition of a half life. But the half that remains will be vastly larger than what you started with. That's the catch.

And that's a very peculiar situation because it doesn't seem to show any end. The volume that's inflating just gets bigger and bigger even while it's decaying, because the expansion is faster than the decay. And that's what leads to this phenomena of eternal inflation. The volume that is inflating increases with time, even though the inflating material is decaying. And that leads to what we call eternal inflation. The word "eternal" is being used slightly loosely because eternal really means forever. This is forever into the future, as far as we can tell, but it's not forever into the past. Inflation would still start at some finite time here, but then once it starts, it goes on forever.

And whenever a piece of this inflating region undergoes a transition and becomes normal, that locally looks like a Big Bang. And our Big Bang would be one of these local events, and the universe formed by any one of these local events where the inflating region decays would be called a pocket universe. Pocket just to suggest that there are many of them in the overall scale of this multiverse. They are in some sense small, even though they'd be as big as the universe that we live in. And our universe would be one of these pocket universes.

So instead of one universe, inflation produces an infinite number, which is what we call multiverse. I might just say the word multiverse is also used in other contexts

and another theories, but inflation, I think, is probably the most plausible way of getting a multiverse, and it's what most cosmologists are talking about when they talk about a multiverse.

OK. Now how does dark energy fit in here? It plays a very important role in our understanding. To review, in 1998 several groups-- two groups of astronomers discovered independently that the universe is now accelerating, and our understanding is that the universe has been accelerating for about the last five billion years out of the 14 billion year history of the universe. There was a period where it was decelerating until five billion years ago. An implication of this is that inflation actually is happening today. This acceleration of the universe that we see is very much like inflation, and we really interpret it according to similar kind of physics. We think it has to be caused by some kind of a negative pressure, just as inflation was caused by a negative pressure.

And this material that apparently fills space and has negative pressure is what we call dark energy. And dark energy is really just by definition the stuff, whatever it is, that's causing this acceleration. If we ask, what is the dark energy, really? I think everybody agrees there's a definite answer to that, which is something like, who knows? But there's also a most plausible candidate, even though we don't know. The most plausible candidate-- and other candidates are not that different, really, but we'll talk about the most plausible candidate-- which is simply that dark energy is vacuum energy. The energy of nothingness. Now it may be surprising that nothingness can have energy. But I'll talk about that, and it's really not so surprising. I'll come back to that question.

But if dark energy is really just the energy of the vacuum, that's completely consistent with everything we know about, what we can measure, the expansion pattern of the universe. Yes.

AUDIENCE: Why is it that only in the last five billion years has the universe started accelerating?

PROFESSOR: To start accelerating. Right. Right. OK. I'm now in a position to say that. I wasn't quite when I made the first statement, but now that I've said there's probably

vacuum energy, I can give you an answer. Which is that vacuum energy, because it is just the energy of the vacuum, does not change with time. And that's the same as what I told you about the energy density during inflation. It's just a constant. At the same time, ordinary matter thins out as the universe expands, throwing off in density like one over the cube of the volume.

So what happened was that the universe was dominated by ordinary matter until about five billion years ago, which produced attractive gravity and caused the universe to slow. But then about five billion years ago the universe thinned out enough so that the ordinary matter no longer dominated over the vacuum energy, and then the vacuum energy started causing repulsion. Vacuum energy was there all along causing repulsion, but it was overwhelmed by the attractive gravity of the ordinary matter until about five billion years ago. Does that make sense?

AUDIENCE: Yes.

PROFESSOR: OK. Good. Any other questions? OK.

So. The first thing I want to talk about here is why can nothing weigh something? Why can nothing have energy? And the answer is that actually this is something the physicists are pretty clear on these days. The quantum vacuum, unlike the classical vacuum, is a very complicated state. It's not really empty at all. It really is a complicated jumble of vacuum fluctuations. We think there's even a field called the Higgs field, which you've probably heard of, which has a nonzero value in the vacuum on average. Things like the photon field, the electromagnetic field, is constantly oscillating in the vacuum because of the uncertainty principle, basically, resulting in energy density in those fluctuations.

So there's no reason for the vacuum energy to be zero, as far as we can tell. But that doesn't mean that we understand its value. The real problem from the point of view of fundamental physics today is not understanding why the vacuum might have a nonzero energy density. The problem is understanding basically why it's so small. And why is smallness a problem? If you look at quantum field theory-- which we're not going to learn in any detail-- but quantum field theory says that, for example, the

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electromagnetic field is constantly fluctuating. Guaranteed so by the uncertainty principle. And these fluctuations can have all wavelengths. And every wavelength contributes to the energy density of the vacuum fluctuations.

And there is no shortest wavelength. There's a longest wavelength in any size box, but there's no shortest wavelength. So in fact, when you try to calculate the energy density of the vacuum in the quantum field theory, it diverges on the short wavelength side. It becomes literally infinite as far as the formal calculation is concerned because all wavelengths contribute, and there is no shortest wavelength.

So what does this mean about the real physics? We think it's not necessarily a problem with our understanding of quantum field theory. It really is, we think, just a limitation of the range of validity of those assumptions. They certainly-- quantum theory works extraordinarily well when it's tested in laboratory circumstances. So we think that at very short wavelengths, something must happen to cut off this infinity. And a good candidate for what happens at short wavelengths to cut off the infinity is the effects of quantum gravity, which we don't understand.

So one way of estimating the true energy density as predicted by quantum field theory is to cut things off at the Planck scale, the energy scale, length scale, associated with quantum gravity-- which is about 10 to the minus 33 centimeters. And if you do that, you can calculate the number for the energy density of the electromagnetic field of the vacuum and get a finite number. But it's too large. And too large not by a little bit, but by a lot. It's too large by 120 orders of magnitude. So we are way off in terms of understanding why the vacuum energy is what it is since our naive estimates say it should be maybe 120 orders of magnitude larger.

Now I should add that there is still a way out here. The energy that we calculate here is one contribution to the total vacuum energy. There are negative contributions as well. If you calculate the fluctuations of the electron field, that turns out to be negative in its contribution to the energy. And it's always possible that these numbers cancel-- or cancel almost exactly-- but we don't know why they should. So basically there's a big question mark theoretically on what we would

predict for the energy density of the vacuum.

Let's see. What should I do here? I am not going to be able to finish this lecture. I think it's worth finishing, however. So I think what I'll do is I'll maybe go through this slide and then we'll just stop, and we'll pick up again next time. There are just a few more slides to show. But I think it's an interesting story worth finishing. But to come to a good stopping place here-- we still have a minute and a half, I think.

I want to say a little bit about the landscape of string theory, which is going to be a possible explanation-- only possible, it's very speculative here-- but one possible explanation which combines inflation, the eternal inflation, and string theory produces a possible explanation for this very small vacuum energy that we observe. It's based on the idea that string theory does not have the unique vacuum. For many years string theorists sought to find the vacuum of string theory with no success. They just couldn't figure out what the vacuum of string theory would look like.

And then a little more than 10 years ago many string theorists began to converge around the idea that maybe they could not find a vacuum because there is no unique vacuum to string theory. Instead, what they now claim is that there's a colossal number-- they bandy around numbers like 10 to the 500th power-- a colossal number of metastable states, which are long lived, any one of which could look like a vacuum for a long period of time, even though ultimately it might decay or tunnel into one of the other metastable states. So this is called the landscape of string theory. This huge set of vacuum like states, any one of which could be the vacuum that fills a given pocket universe, for example.

When one combines this with the idea of eternal inflation, then one reaches the conclusion that eternal inflation would very likely populate all of these 10 to the 500 or more vacua. That is, different pocket universes would have different kinds of vacuum inside them, which would be determined randomly as the pocket universes nucleate, as they break off from this inflating backbone. And then we would have a multiverse which would consist of many, many-- 10 to the 500 or more-- different

kinds of vacua in different pocket universes.

Under this assumption, ultimately string theory would be the assumed laws of physics that would govern everything. But if you were living in one of these pocket universes, you actually see apparent laws of physics that would look very different from other pocket universe's. The point is that the physics that we actually see and measure is low energy physics compared to the energy scales of the string theory. So what we are seeing are just small fluctuations in the ultimate scheme of things about the vacuum that we live in.

So the very particle spectrum that we see, the fact that we see electrons and quarks, quarks that combine to form protons and neutrons-- could be peculiar to our particular pocket. And in other pockets there could be completely different kinds of particles, which are just oscillations about different kinds of vacuum. So even though the laws of physics would in principle be the same everywhere-- the laws of string theory-- in practice the observed laws of physics would be very different from one pocket to another. And in particular since there are different vacua in the different pockets, the vacuum energy density would be different in different pockets. And that variability of the vacuum energy density provides a possible answer to why we see such a small vacuum energy.

And we'll talk more about that next time on Tuesday. See you then.