MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.231, Physics of Solids I — Due on Wed., Dec. 8

Problem set #10

1. **Landau level for mass-less Dirac fermions**

   We have seen that the electron in the Graphene has a linear dispersion relation $\epsilon(\boldsymbol{k}) = v|\boldsymbol{k}|$. Use the semi-classical approach to calculate the energy ϵ_n of the n^{th} Landau level in the uniform magnetic field B. Assume $v = 1\text{eV} \times 1\text{Å}/\hbar$, find $\epsilon_1 - \epsilon_0$ in eV for a magnetic field of 30 Tesla. Can we see quantum Hall effect in Graphene at room temperature?

2. **Hall conductance of electrons in a lattice**

   Consider a 2D spin-less non-interacting electron gas. The electron density is n. A uniform magnetic field $\boldsymbol{B} = B\boldsymbol{z}$ is applied.

   (a) Assume that the electrons have a dispersion $\epsilon(\boldsymbol{k}) = \frac{\hbar^2\boldsymbol{k}^2}{2m}$. Use a classical approach to show that $\rho_{xy} = +\frac{B}{enc}$ and $\rho_{xx} = 0$, or $\sigma_{xy} = -\frac{enc}{B}$ and $\sigma_{xx} = 0$. (Be careful about the signs and we assume the electron mean-free path to be $l = \infty$.) Let S be the area enclosed by the Fermi surface. Show that

   $$\sigma_{xy} = -\frac{eSc}{4\pi^2 B}$$

   The above formula is more general and works for an arbitrary dispersion $\epsilon(\boldsymbol{k})$, as long as the Fermi surface forms a closed loop that encloses the occupied $\boldsymbol{k}$-points.

   (b) We like to apply the above classical result $\sigma_{xy} = -\frac{eSc}{4\pi^2 B}$ for electrons in square lattice with a lattice constant a. The dispersion $\epsilon(\boldsymbol{k})$ is given by

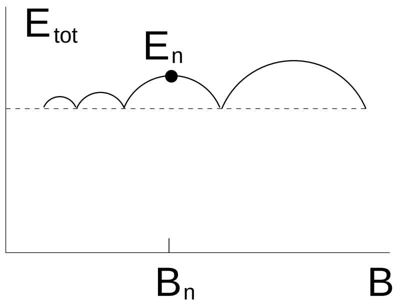
   $$\epsilon(\boldsymbol{k}) = -2t[\cos(k_x a) + \cos(k_y a)]$$

   Find σ_{xy} as a function of electron density n. (Hints: (a) The maximum density corresponds to a filled band. (b) σ_{xy} is known to be zero for a filled band. (c) $\sigma_{xy} = -\frac{eSc}{4\pi^2 B}$ is valid only if the Fermi surface forms a closed loop that encloses the occupied $\boldsymbol{k}$-points. (d) For a nearly filled band, we may view the system as a system of holes.)

   (c) (Optional) Guess how the above classical result should be modified if we include the quantum effect and impurity effect. Sketch the modified σ_{xy} as function of electron density n in the weak B field limit.

3. **Diamagnetism – a simple way**:

   Consider a 2D spin-less non-interacting electron gas. The electron mass is m and the density is n. Let $E_{tot}(B)$ be the total ground state energy of the electrons.



(a) Find the values of $E_{tot}(B)$ when the magnetic field B is such that the filling fraction ν is an integer.

(b) Find the values of $E_n = E_{tot}(B_n)$ when the magnetic field B_n is such that the n^{th} Landau level is half filled. Show that $E_n = c_0 + c_1 B_n^2$ and find the values of c_0 and c_1.

(c) Assume at $T \neq 0$, $\langle E_{tot} \rangle = c_0 + c_1 B^2$, find the corresponding magnetic susceptibility χ.

4. Prob. 5 on page 319 of Kittle.

5. Prob. 6 on page 320 of Kittle.