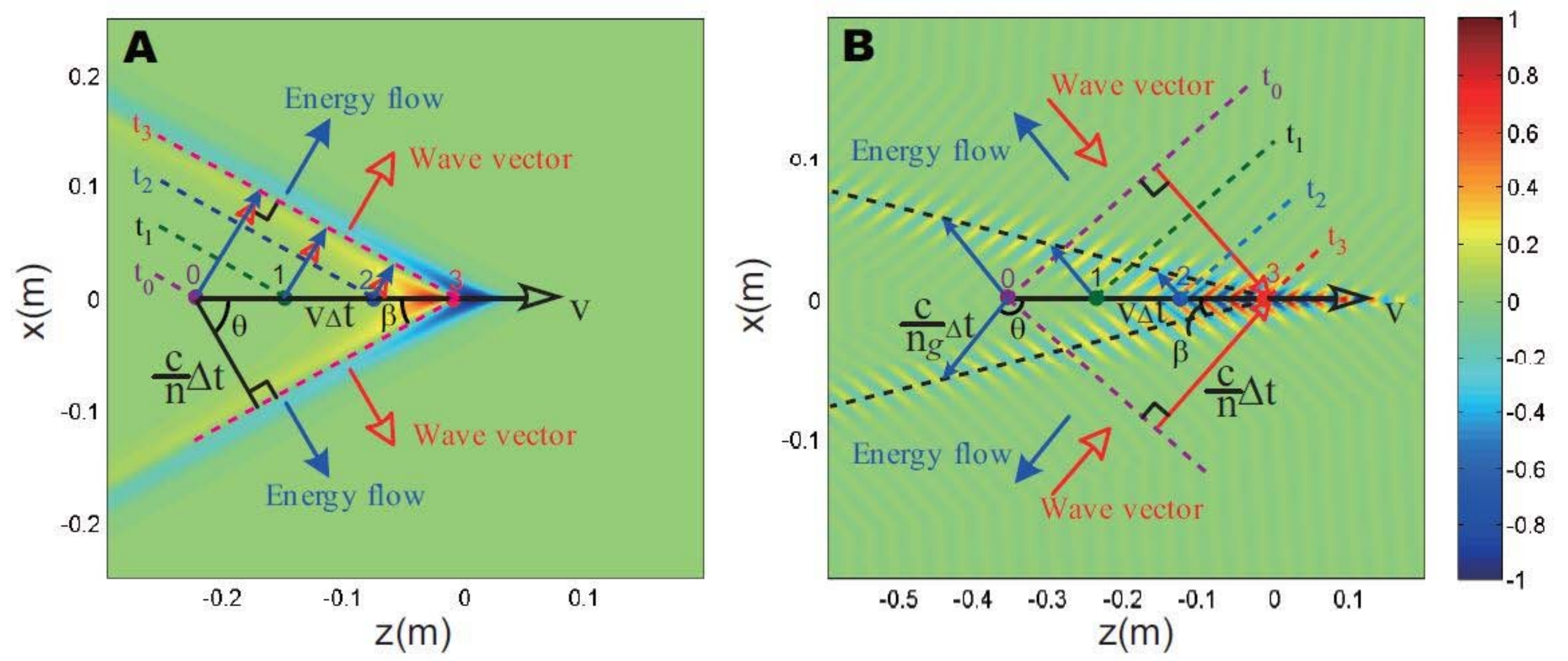
Reversed Cherenkov Radiation in Left-Handed Meta-material



8.07 Lecture, Nov 21, 2012

Prof. Min Chen

8.07 is not just an abstract theory;
it is a tool
which can be applied to change the world around you.

Example: ***Left-Handed Meta-material***

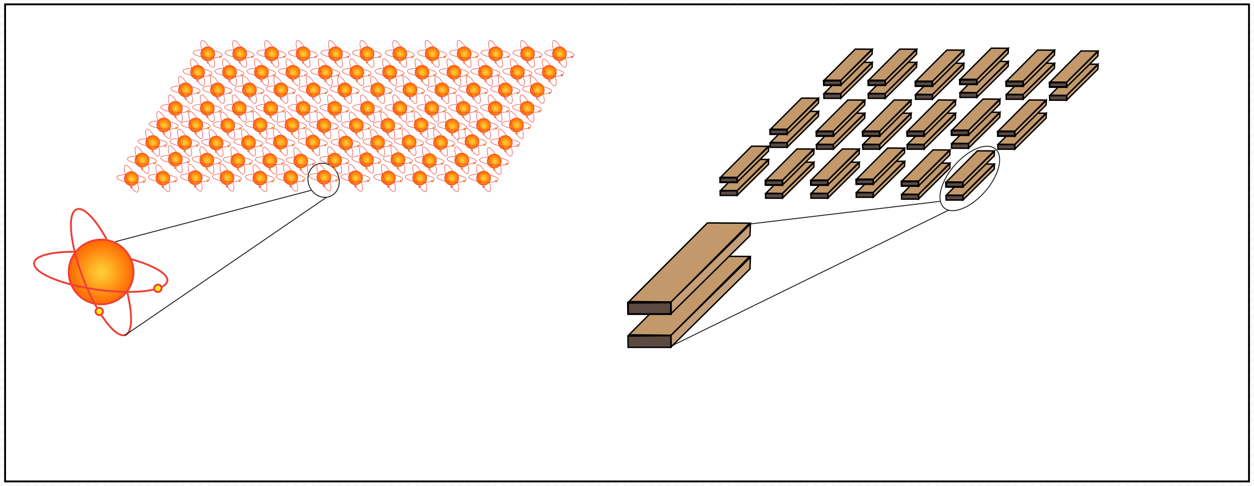
1. Introduction

What are Metamaterials?

Engineered (at the atomic level) materials that have unique properties not found in nature due to the arrangement and design of their constituents.

NOTE: THE PROPERTIES ARE THAT OF THE ENTIRE ARRANGEMENT AND NOT THE CONSTITUENTS THEMSELVES

"Meta" = above, superior, beyond



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1. Introduction

Overview of materials

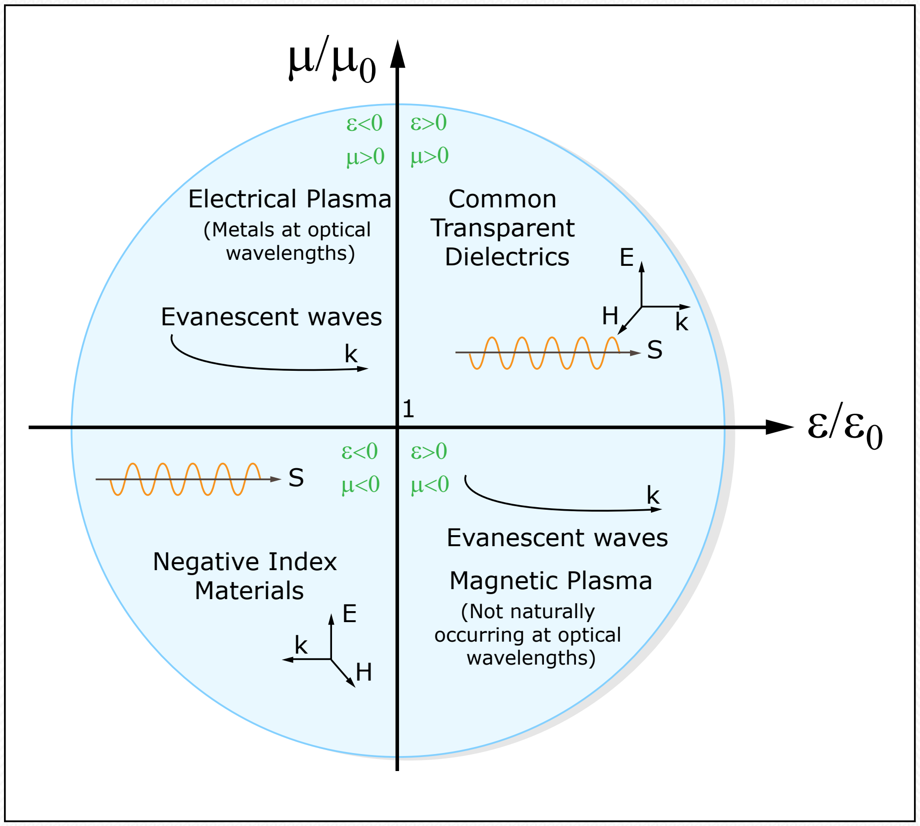

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FIG. 3. The material parameter space.

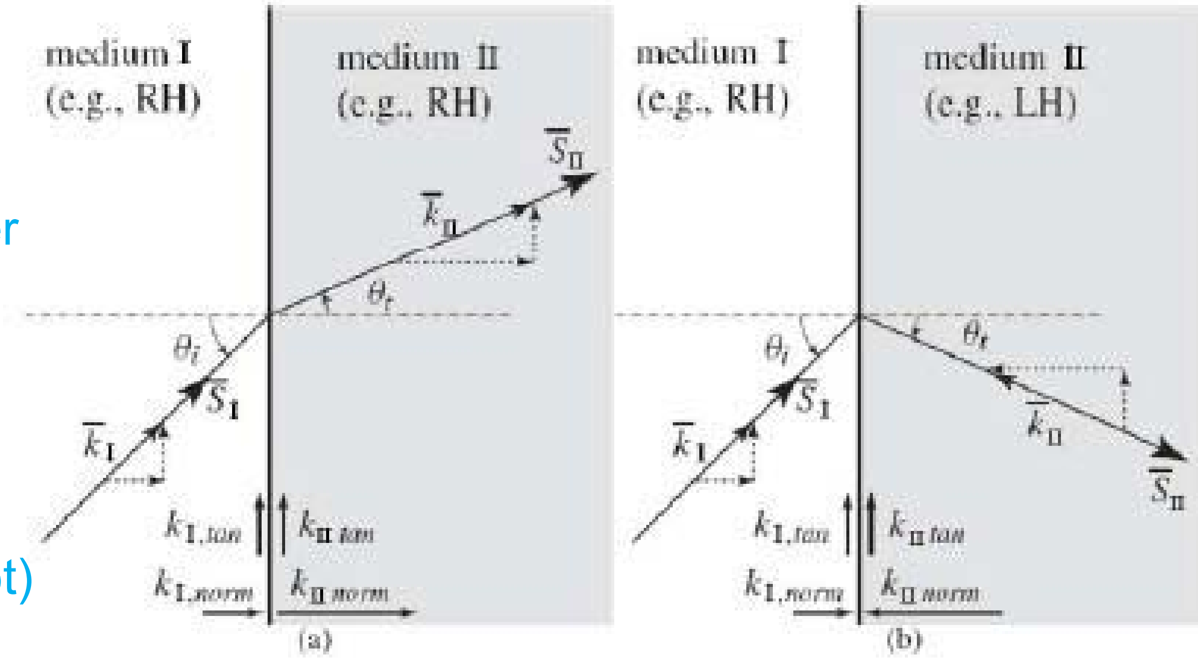
What is LH material?

Refraction of RH and LH material

k = 2 π / λ
Wave number

c/n = ω/k = (ω/k_0)/n

$e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$



$\sin\vartheta_t/\sin\vartheta_i = n$

Figure 1.3: Refraction of an electromagnetic wave at the interface between two different media. (a) Case of two media of same handedness (either RHM or LHM): positive refraction. (b) Case of two media of different handedness (one RHM and the other one LHM): negative refraction.

LH e Refraction

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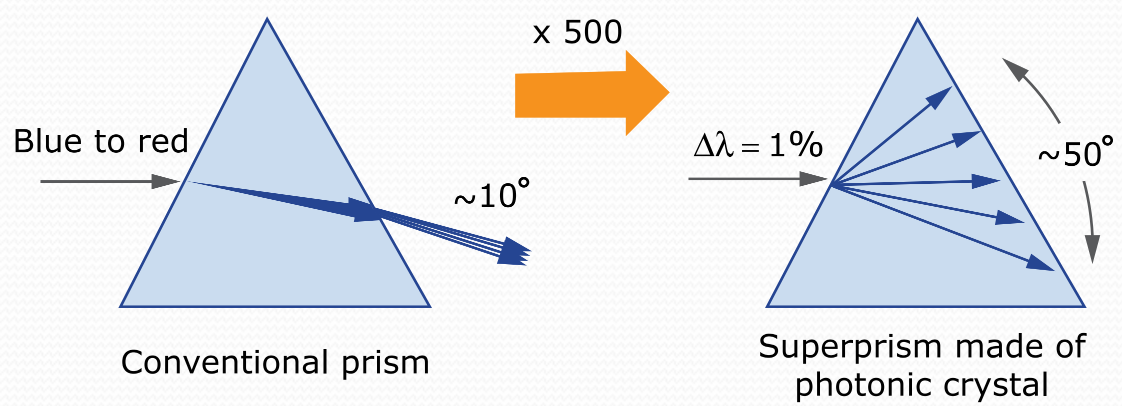
Application example 2 of LH material

Superprism



Schematic illustration of superprism phenomena. The wavelength dispersion in a superprism made of PhCs is approximately 500 times stronger than in a prism made of conventional crystals.

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Application example 3 of LH material

Flat Lens

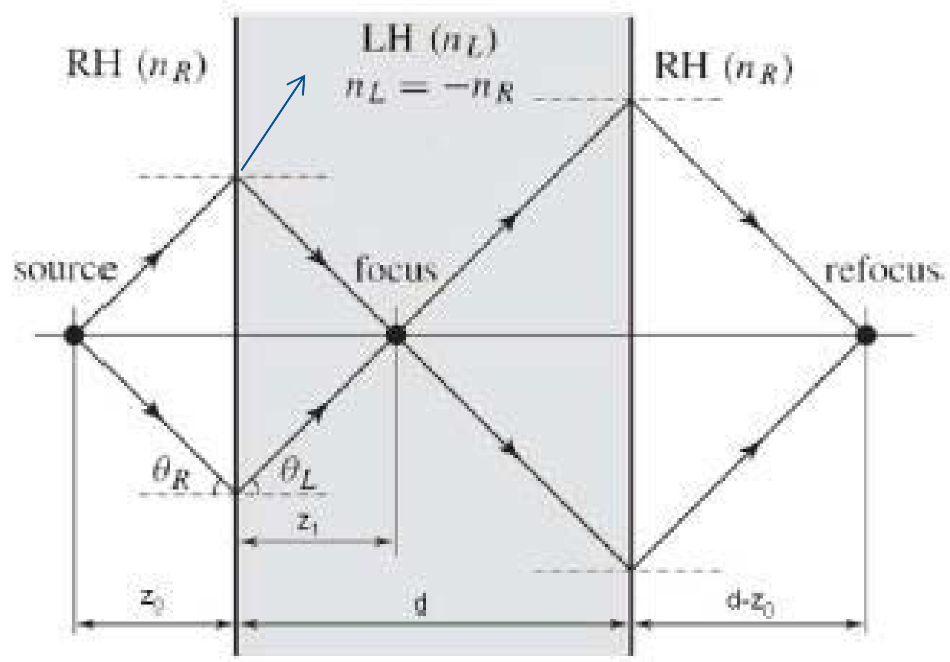

Figure 1.4: Double focusing effect in a "flat lens", which is a LHM slab of thickness d and refractive index n_L sandwiched between two RH media of refractive index n_R with $n_L = -n_R$.

Electron lens, prism and splitter

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8.07 lecture on natural magnetized material: from m_{micro} to M_{macro} to J

Microscopically, $\vec{M}_{micro}(\vec{r}) = \sum_n \vec{m}_n \delta^3(\vec{r}-\vec{r}_n)$

$$\vec{M}_{macro}(\vec{r}) = \vec{M}(\vec{r}) = \langle \vec{M}_{micro}(\vec{r}) \rangle$$

$\langle \ \rangle$ = average over region large compared to atoms, but small compared to the scale over which macroscopic quantities vary

$\vec{r}$ is center of averaging region

Then $\vec{J}_{micro}(\vec{r}) = -\sum_n \vec{m}_n \times \vec{\nabla}_{\vec{r}} \delta^3(\vec{r}-\vec{r}_n)$

$$\vec{J}_{micro}(\vec{r}) = \vec{\nabla}_{\vec{r}} \times \sum_n \vec{m}_n \delta^3(\vec{r}-\vec{r}_n)$$

$$= \vec{\nabla} \times \vec{M}_{micro}(\vec{r})$$

Then $\langle \vec{J}_{micro}(\vec{r}) \rangle = -\vec{\nabla} \times \langle \vec{M}_{micro}(\vec{r}) \rangle$

$$\vec{J}_{bound}(\vec{r}) = \vec{\nabla} \times \vec{M}(\vec{r})$$

curl B = $\mu_0 J_b$

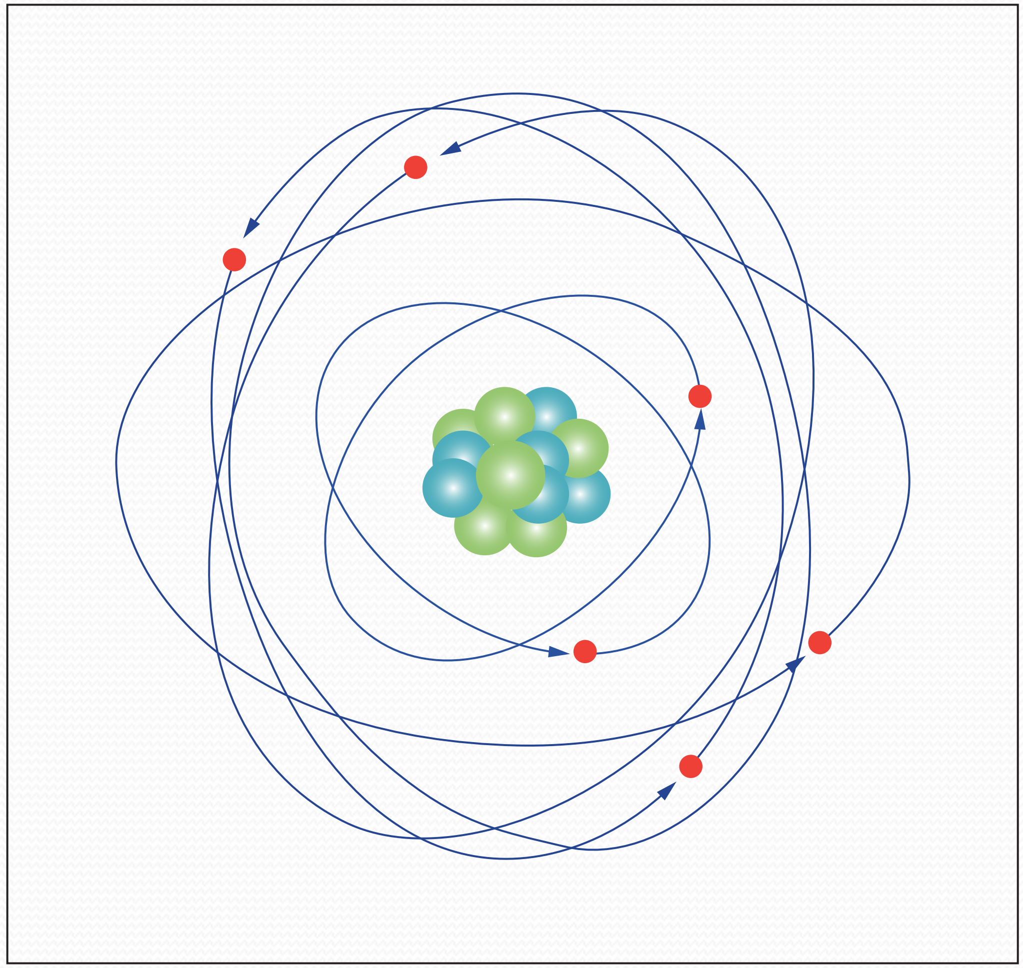
Index of refraction n
Polarize Atoms to make dipoles:

Wave speed = c/n' = $\sqrt{}\,(\varepsilon_r \mu_r)$

$(\varepsilon_{eff} - \varepsilon_o) \quad E = \mathbf{P}$

real part of n

$n' = \sqrt{\mu_\perp \epsilon_\parallel} > 1$



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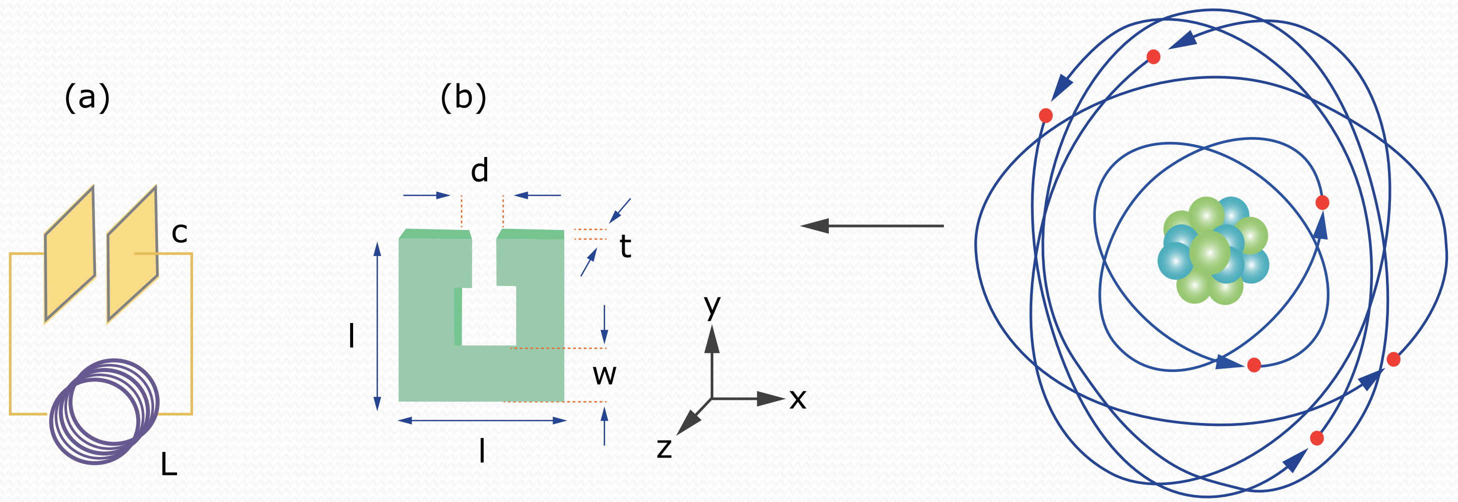
Man made atomic dipoles



(a) IIlustration of the analogy between a usual LC circuit, consisting of a C and inductance L, and (b) a split-ring resonator (SRR).

Image by MIT OpenCourseWare.

To make artificial material with $n < 0$

- Make new atoms using driven-resonance LRC- circuits
- Calculate inductance L and capacitance C
- Calculate induced complex resonance current I_{micro}
- Calculate $m_{micro} = IA$ to obtain M_{macro}
- and $B = \mu_o (H + M)$
- Obtain permeability $B/H = \mu = \mu_r + i\,\mu_i$
- Similarly permittivity $D/E = \varepsilon = \varepsilon_r + i\,\varepsilon_i$
- Pick regions with real negative permittivity and negative permeability, i.e. $\varepsilon_r < 0$ and $\mu_r < 0$; note $\varepsilon_i > 0$ and $\mu_i > 0$
- Obtain negative Index of refraction $n^2 = \varepsilon_r \mu_r$, $n = -\sqrt{(\varepsilon_r \mu_r)}$

Maxwell Eqs. In matter

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

Then, with $\rho = \rho_f + \rho_b$, $\vec{J} = \vec{J}_f + \vec{J}_b$,

$$\vec{\nabla} \cdot \vec{D} = \rho_f \quad =0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$\vec{J}_f = 0$

No free charge or current

For linear materials, $\vec{D} = \epsilon \vec{E}$, $\vec{H} = \frac{1}{\mu} \vec{B}$

ϵ = dielectric constant

μ = relative permeability.

$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$ = displacement current.

How to make LH material?

Maxwell Eqs. In material free of q, j

$$\varepsilon_0 \nabla\cdot\vec{\mathbf{E}} = \rho = -\nabla\cdot\vec{\mathbf{P}}$$

$$\mu_0 \nabla\cdot\vec{\mathbf{H}} = \rho_H = -\nabla\cdot \mu_0\vec{\mathbf{M}}$$

$$\nabla\times\vec{\mathbf{E}} + \mu_0 \frac{\partial \vec{\mathbf{H}}}{\partial t} = -\mathbf{J}_M = -\mu_0 \frac{\partial \vec{\mathbf{M}}}{\partial t}$$

$$\nabla\times\vec{\mathbf{H}} - \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \mathbf{J} = \frac{\partial \vec{\mathbf{P}}}{\partial t}$$

$\mu_{eff} = \langle B\rangle/\langle H\rangle$

$(\varepsilon_{eff} - \varepsilon_o)\, E = \mathbf{P} = \mathbf{J}/i\omega$

real part of n

$n' = -\sqrt{\mu_\perp \epsilon_\parallel}$

Left Handed Meta-material:

Use L, R, C devices smaller than wavelengths to make new ‘***molecules***’ , with novel properties of **P** and **M**

Energy and Momentum flow

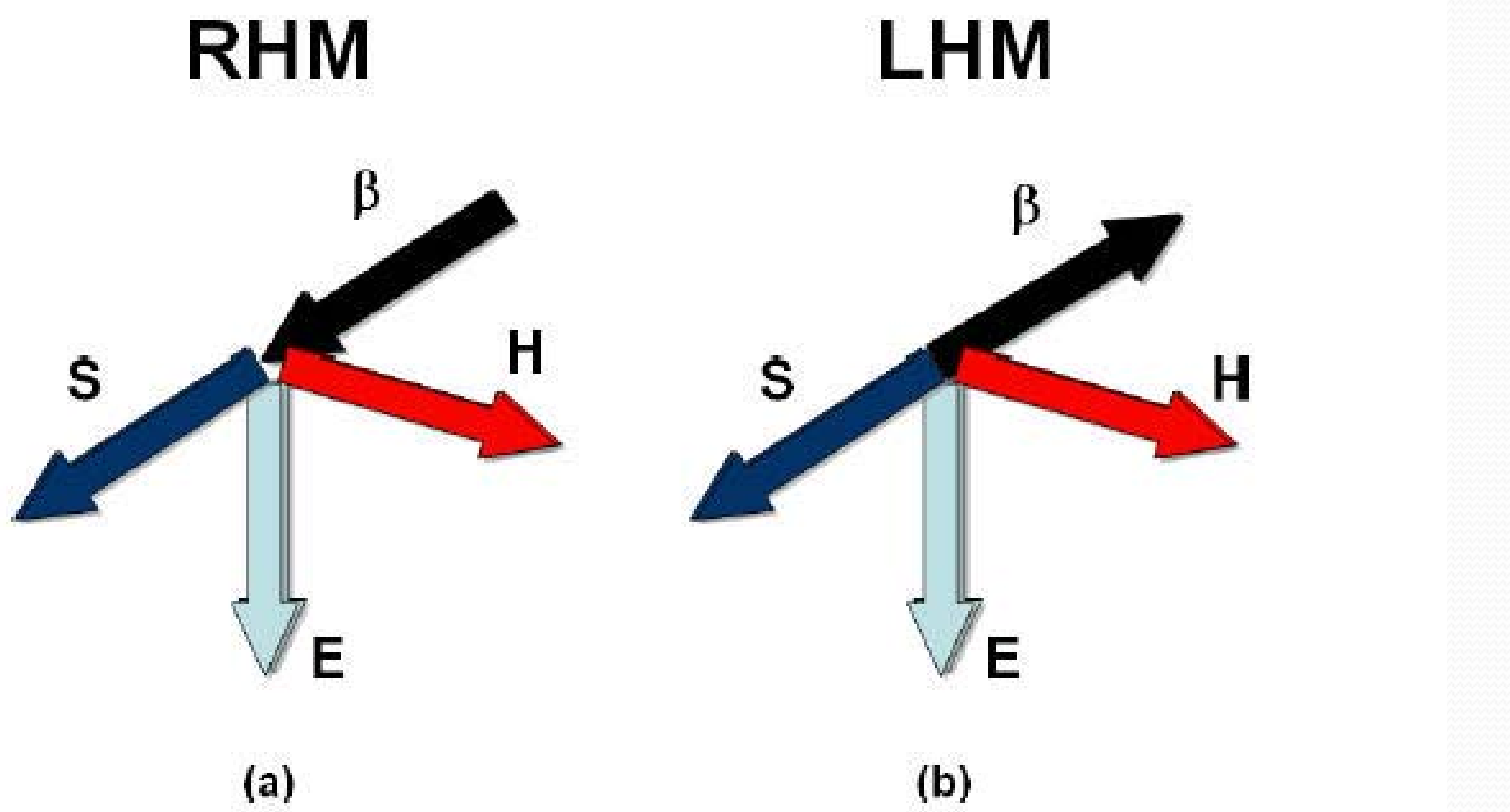


Figure 1.1: Orientation of field quantities $\vec{E}$, $\vec{H}$, Poynting vector $\vec{S}$, and wavevector number $\vec{\beta}$ in right-handed media (RHM) and left-handed media (LHM).

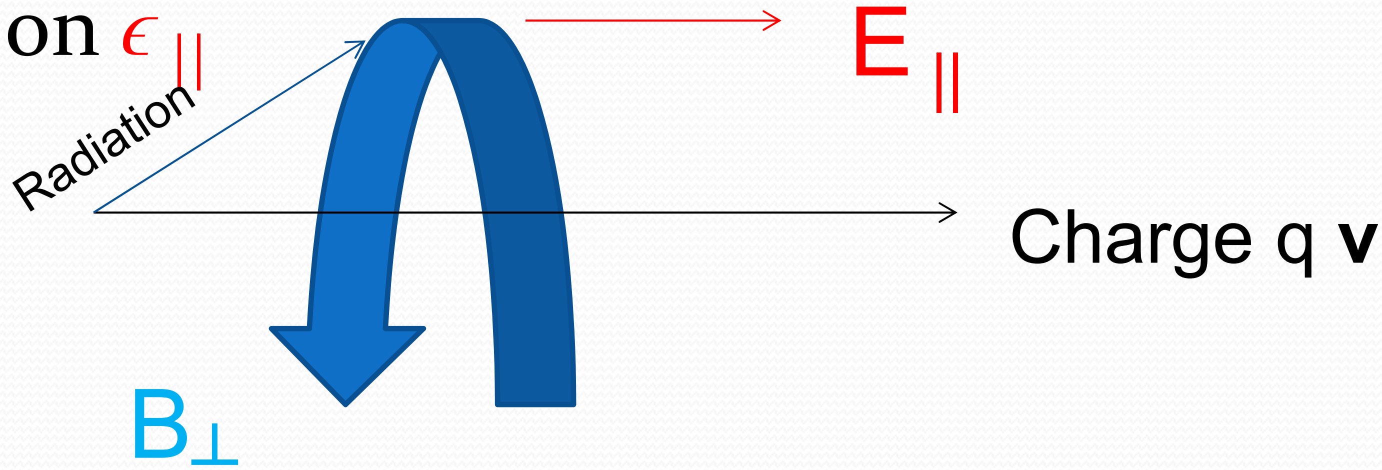
Maxwell Eqs

Separate into ⊥ *and* // components

- 3×3 complex matrix μ and ϵ diagonized

$$\mu = diag[\mu_{/\!/} \;\; \mu_{\perp} \;\; \mu_{/\!/}]$$

- Transverse $B_\perp$ depends on only $\mu_\perp$
- $E_{||}$ on $\epsilon_{||}$



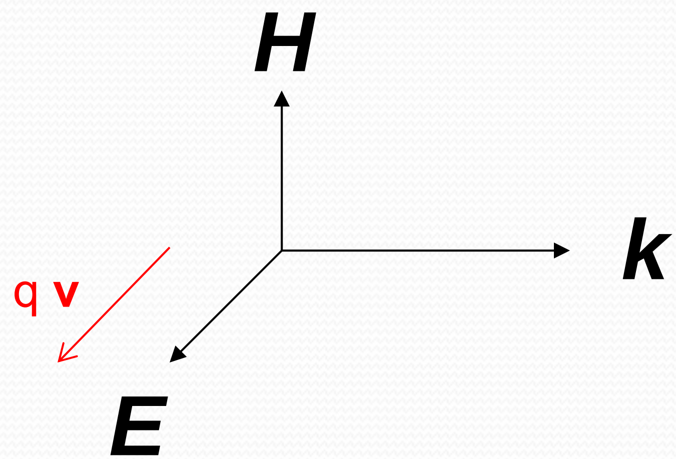
Maxwell Eqs
in Cylindrical symmetric geometry

Separate into ⊥ *and* ∥ components

$$\mathbf{k_s} \times \mathbf{H}_{\perp} = -\omega \epsilon_{\parallel} \mathbf{E}_{\parallel}$$

and

$$\mathbf{k_s} \times \mathbf{E}_{\parallel} = \omega \mu_{\perp} \mathbf{H}_{\perp}$$



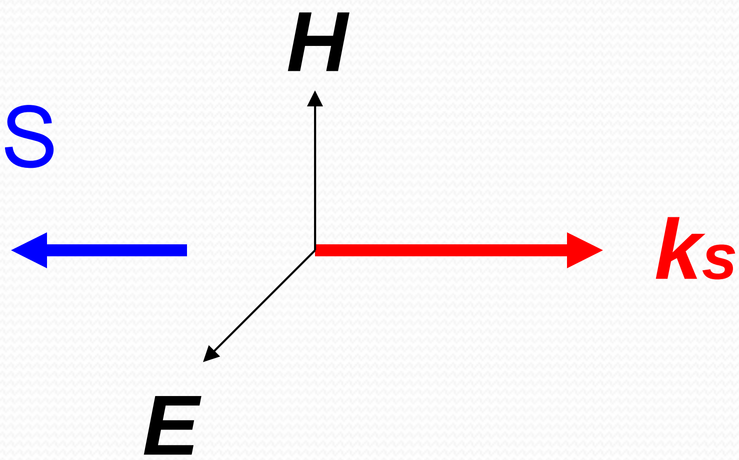
$\mu_{\perp}$, $\epsilon_{\parallel}$ are negative

$\mathbf{E}_{\parallel}$, $\mathbf{H}_{\perp}$, and $\mathbf{k}_s$ form a left-handed triad

Poynting & wave vector

in ⊥ *and* // components

Poynting vector S



Poynting vector

$\langle \boldsymbol{S} \rangle = \langle \boldsymbol{E}_{/\!/} \times \boldsymbol{H}_{\perp}{}^{*} \rangle = |E_s|^2 \, 2\omega\mu_{\perp} \boldsymbol{k}_s$

opposite to the wave vector $\boldsymbol{k}_s$ for a negative $\mu_{\perp}$, representing a backward propagating wave

Negative index of refraction

The Helmholtz wave equation gives,

$$k_s = \frac{\omega n}{c}$$

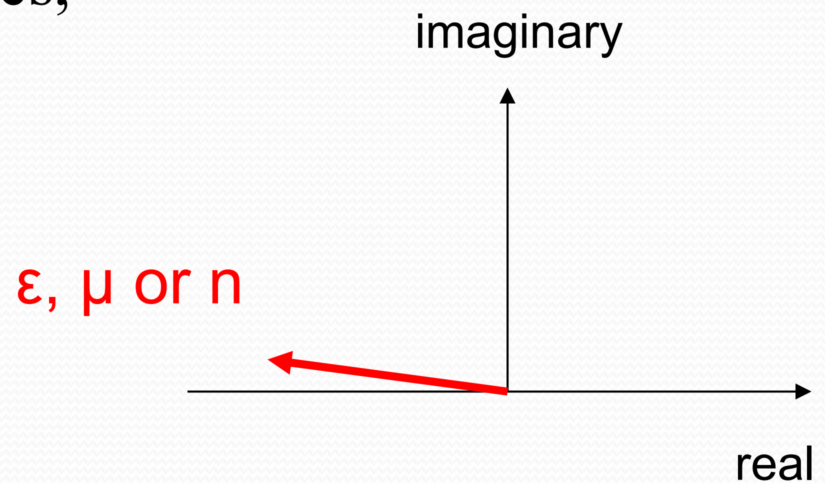
where the real refractive index

$$n = \pm\sqrt{\frac{\mu_\perp \varepsilon_{||}}{\mu_0 \varepsilon_0}}$$

For passive media

The imaginary μ and ϵ and n > 0,

Thus - sign for n.



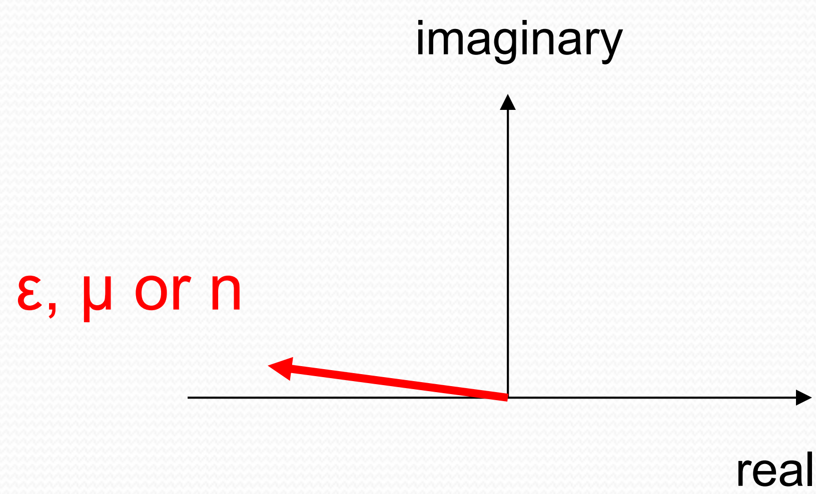
$$e^{ik\,x} = e^{(i\,n\_r - n\_i)\,\omega x/c}$$

$(-1+ i a) (-1 + ib) \sim 1 - i (a + b)$

Taking square root

---> $\sim$ - {1 -i (a+b)/2}

= { -1+i (a+b)/2}



$$n = \pm\sqrt{\frac{\mu_{\perp}\varepsilon_{||}}{\mu_0\varepsilon_0}}$$

For passive media

The imaginary μ and ϵ and n > 0,

Thus - sign for n.

Cherenkov Radiation

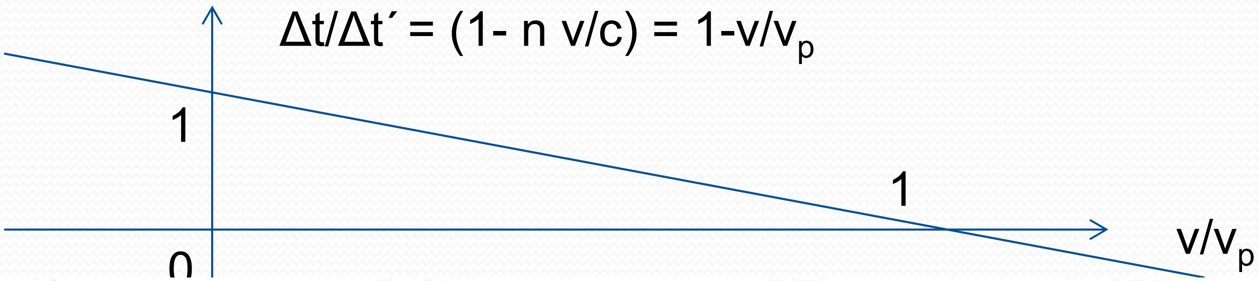
Generated by objects moving faster than the wave speed in the medium,

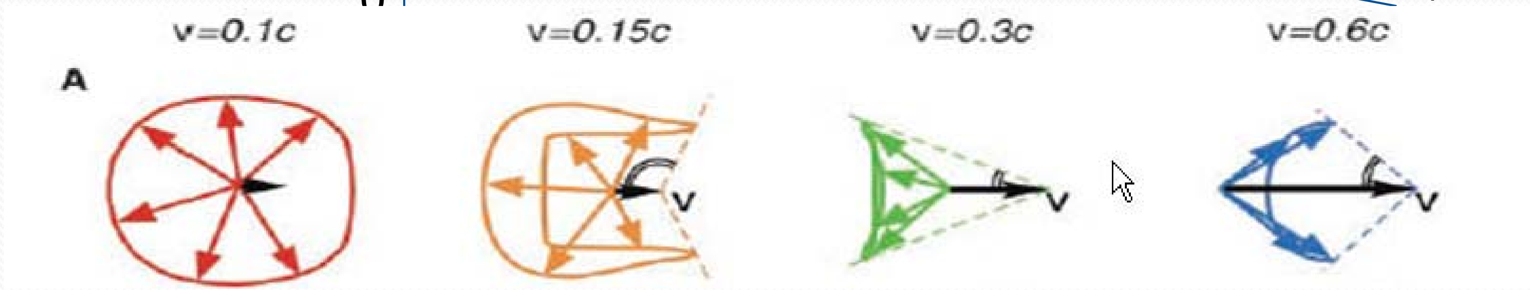
$v > c/n = \omega/k = (\omega/k_0)/n$

Examples:

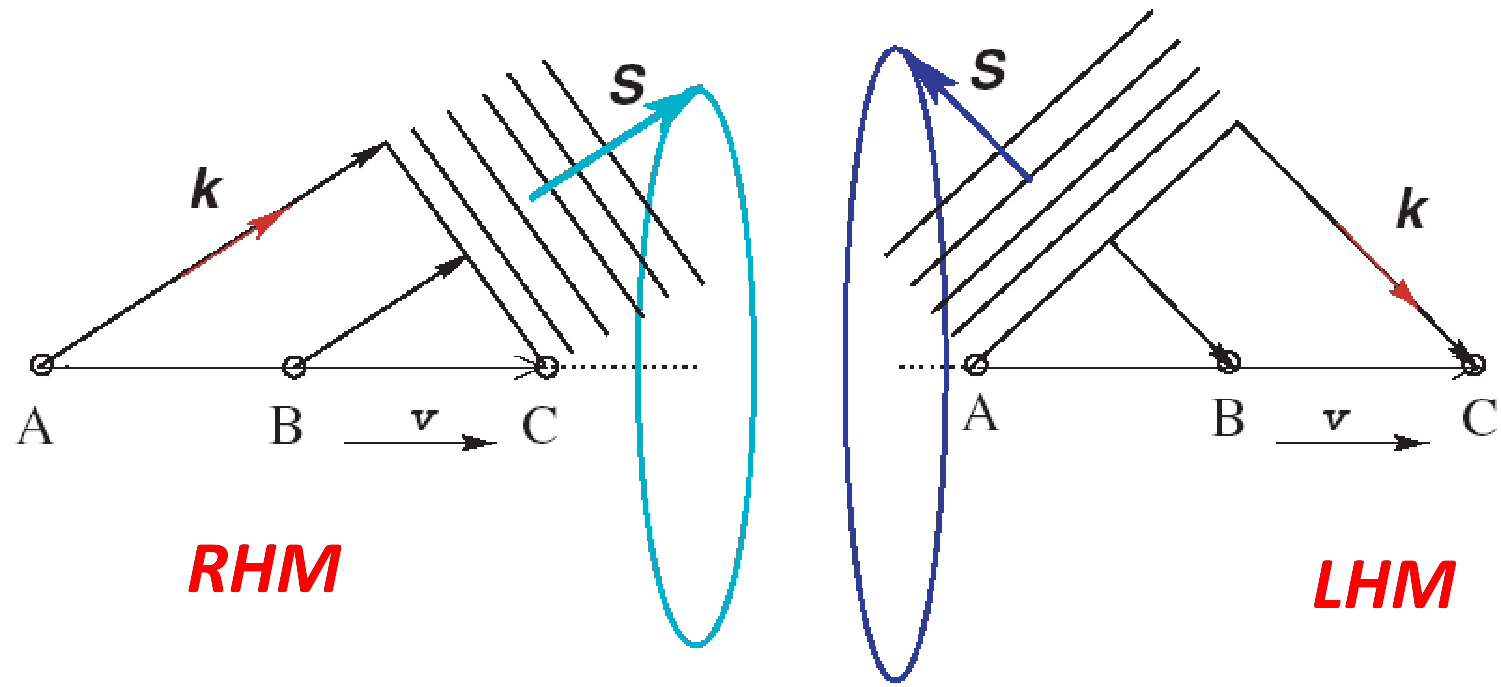
- Sonic boom generated by a supersonic jet
- Wakes from a speedy boat
- Blue light when comic rays going through closed eyes

Cherenkov Radiation for n=2 and $v_p = \omega/k = c/2$





Cherenkov Radiation

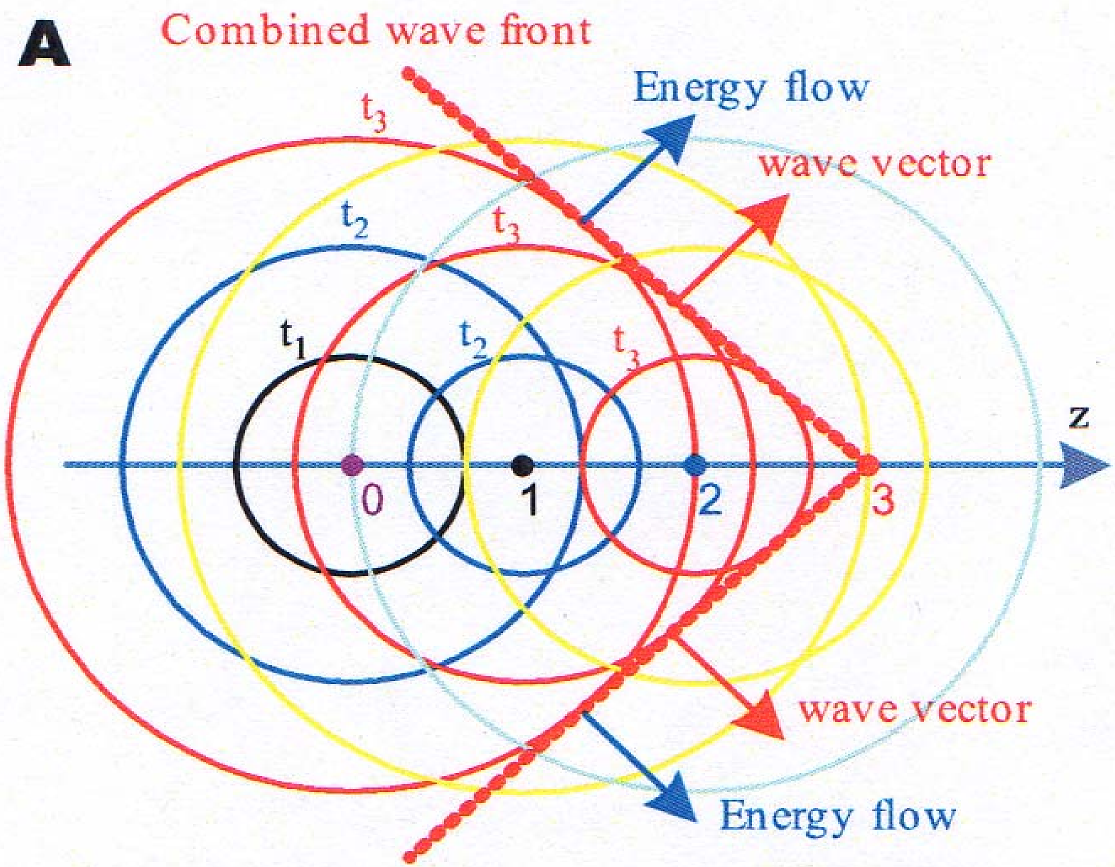


$\theta = \cos^{-1}[c/(nv)]$ with n>1 $\theta = \cos^{-1}[c/(nv)]$ with n<-1

V. G. Veselago, *Sov. Phys. Usp.* 10, 509 (1968).

Cerenkov Radiation in Materials with Negative Permittivity and Permeability
J. Lu, T. Grzegorczyk, Y. Zhang, J. Pacheco Jr., B.-I. Wu, J. A. Kong, and M. Chen
Optics Express 11, 723-734 (2003)

Forward Cherenkov Radiation in RH material

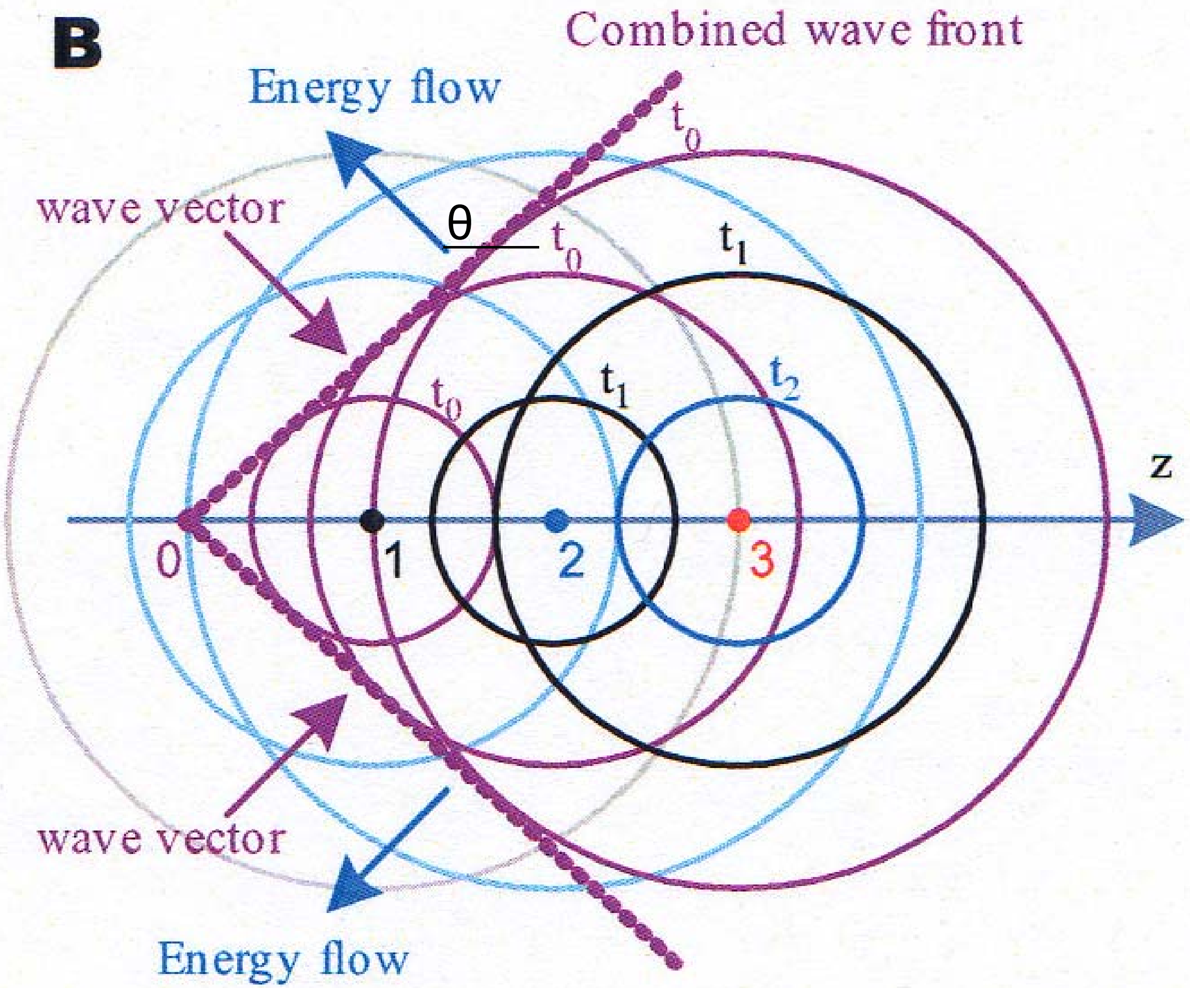


V

Cos θ = c/ nv with n > 1

Wave front ⊥ **V of Energy flow** **&** **wave vector k**

Reversed Cherenkov Radiation in LHM



$\cos\theta = c/nv$ with $n < -1$

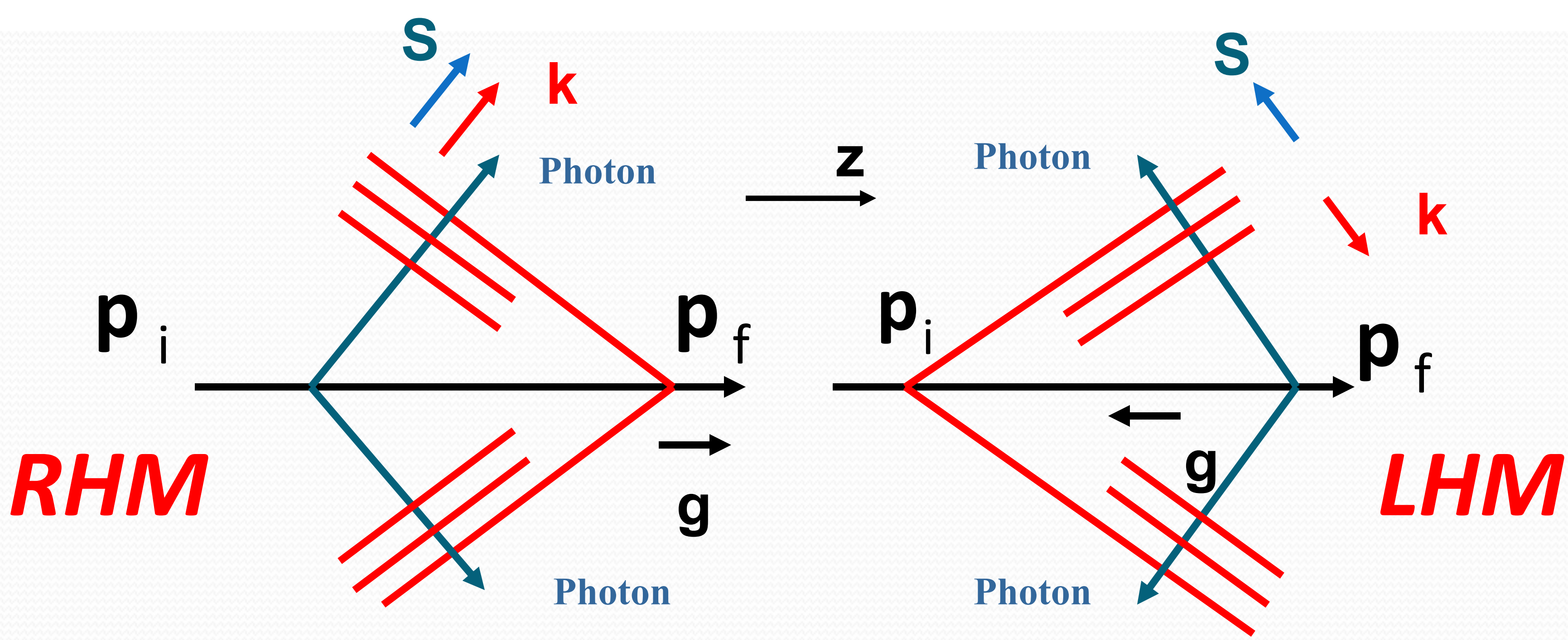
Reversed Cherenkov Radiation in LHM

The Electromagnetics Academy
at Zhejiang University

Two puzzling issues:
Apparent Violation of

- ***Energy-momentum conservation***
- ***Causality***

Momentum & energy conservation?



$P_i = mV_i \quad P_f = mV_f$

$\mathbf{p}_f = \mathbf{p}_i - \mathbf{g}$

$\mathbf{En}_f = \mathrm{En}_i - \mathrm{En}$

$\mathbf{g} = \mathbf{D} \times \mathbf{B} = \varepsilon_r \mu_r \mathbf{E} \times \mathbf{H}$

$=$ **Poynting vector*n²?**

Energy Density and Flux

Poynting's theorem in material:

$$\nabla \cdot \boldsymbol{E} \times \boldsymbol{H} = - \frac{\partial}{\partial t}(1/2\{\epsilon_0 E^2 + \mu_0 H^2)\}$$

$$- E \cdot \frac{\partial}{\partial t} P - H \cdot \frac{\partial}{\partial t} M$$

To get time averaged EM energy density

$$\langle W \rangle = 1/2\{ \frac{\partial(\omega\epsilon)}{\partial\omega} E^2 + \frac{\partial(\omega\mu)}{\partial\omega} H^2\}$$

Complex EM energy density

$$W = 1/4\{ \frac{\partial(\omega\epsilon)}{\partial\omega} E \cdot E^* + \frac{\partial(\omega\mu)}{\partial\omega} H \cdot H^*\}$$

Momentum and Poynting vectors in a dispersive medium

In isotropic LHM, average momentum density

$$\langle \overline{G} \rangle = \frac{1}{2}\mathbf{Re}\{\overline{D} \times \overline{B}^* + \frac{\overline{k}}{2}(\frac{\partial \bar{\bar{\epsilon}}}{\partial \omega}\overline{E} \cdot \overline{E}^* + \frac{\partial \bar{\bar{\mu}}}{\partial \omega}\overline{H} \cdot \overline{H}^*)\}$$

$$= \frac{1}{2}\mathbf{Re}\{\frac{1}{2}\frac{\overline{k}}{\omega}(\overline{D}\cdot\overline{E}^*) + \frac{1}{2}\frac{\overline{k}}{\omega}(\overline{B}\cdot\overline{H}^*) + \frac{\overline{k}}{2}(\frac{\partial \bar{\bar{\epsilon}}}{\partial \omega}\overline{E} \cdot \overline{E}^* + \frac{\partial \bar{\bar{\mu}}}{\partial \omega}\overline{H} \cdot \overline{H}^*)\}$$

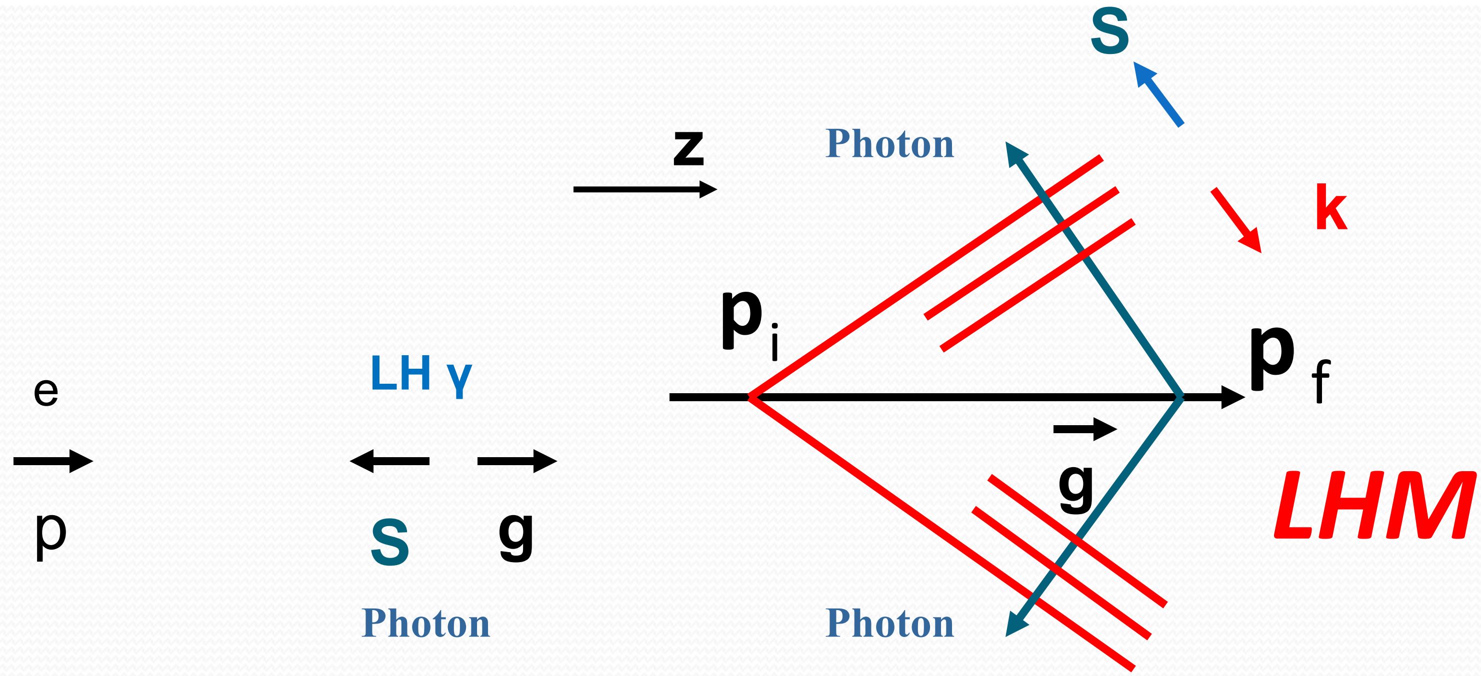
$$= \frac{1}{4}\mathbf{Re}\{\frac{\overline{k}}{\omega}\left[(\bar{\bar{\epsilon}} + \omega\frac{\partial \bar{\bar{\epsilon}}}{\partial \omega})\overline{E} \cdot \overline{E}^* + (\bar{\bar{\mu}} + \omega\frac{\partial \bar{\bar{\mu}}}{\partial \omega})\overline{H} \cdot \overline{H}^*\right]\}$$

$$= \frac{W}{\omega}\overline{k} = \hbar\overline{k} \text{ N}$$

<G> is along the k direction and opposite to the Poynting vector.

T. Musha, *Proceedings of the IEEE* 60, 12 (1972).

LH-Photon momentum
anti-parallel to energy flow



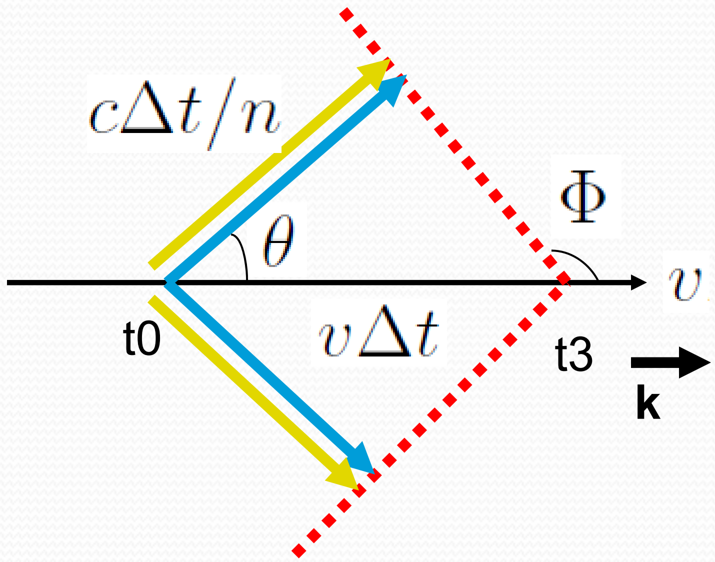
LHγ-e head-on collisions gain energy

Tail collisions lose energy

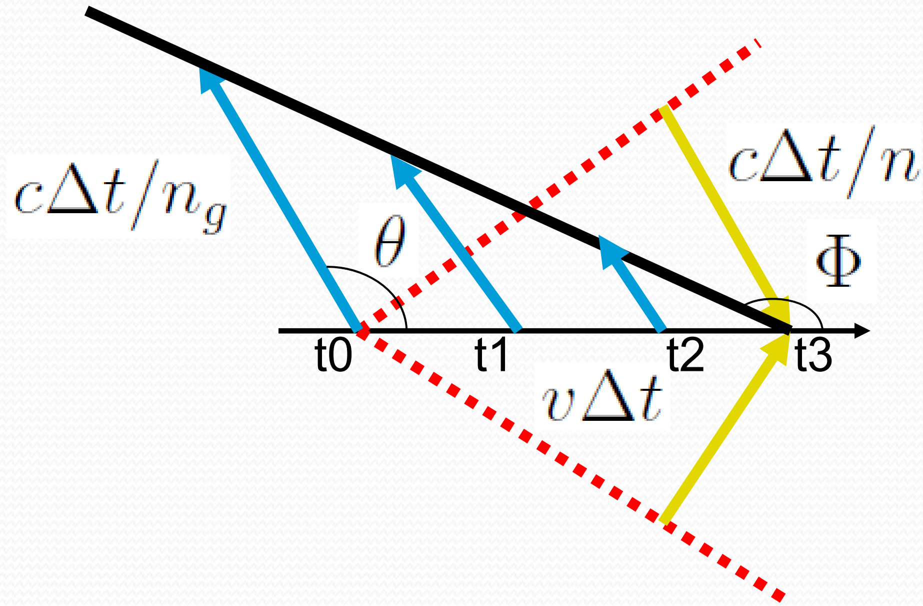
Reverse Doppler effect

Causality: Cherenkov radiation

Energy flow, Wave vector, Phase front, Wake front, for charge at t3



RHM forward



LHM backward

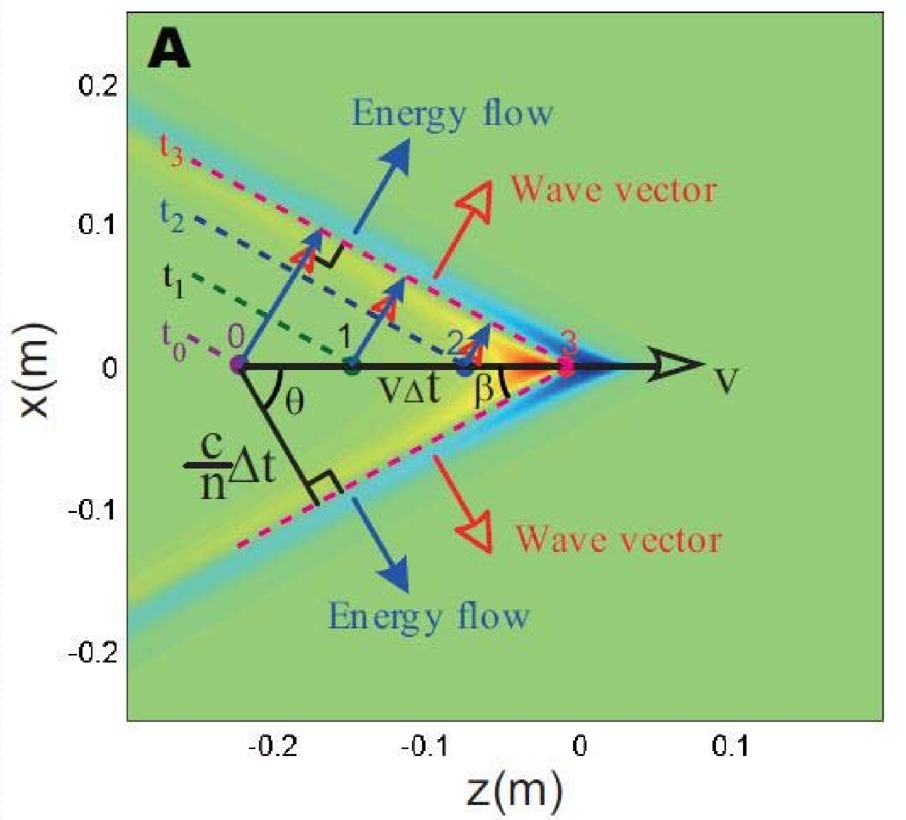
$$\theta = \cos^{-1}[c/(nv)]$$ with n>1

Wake front ⊥ Wave vector

$$\theta = \cos^{-1}[c/(nv)]$$ with n<-1

$$\Phi = 180^\circ - \tan^{-1}\left[\frac{(n/n_g)\sin\theta\cos\theta}{1-(n/n_g)\cos^2\theta}\right]$$

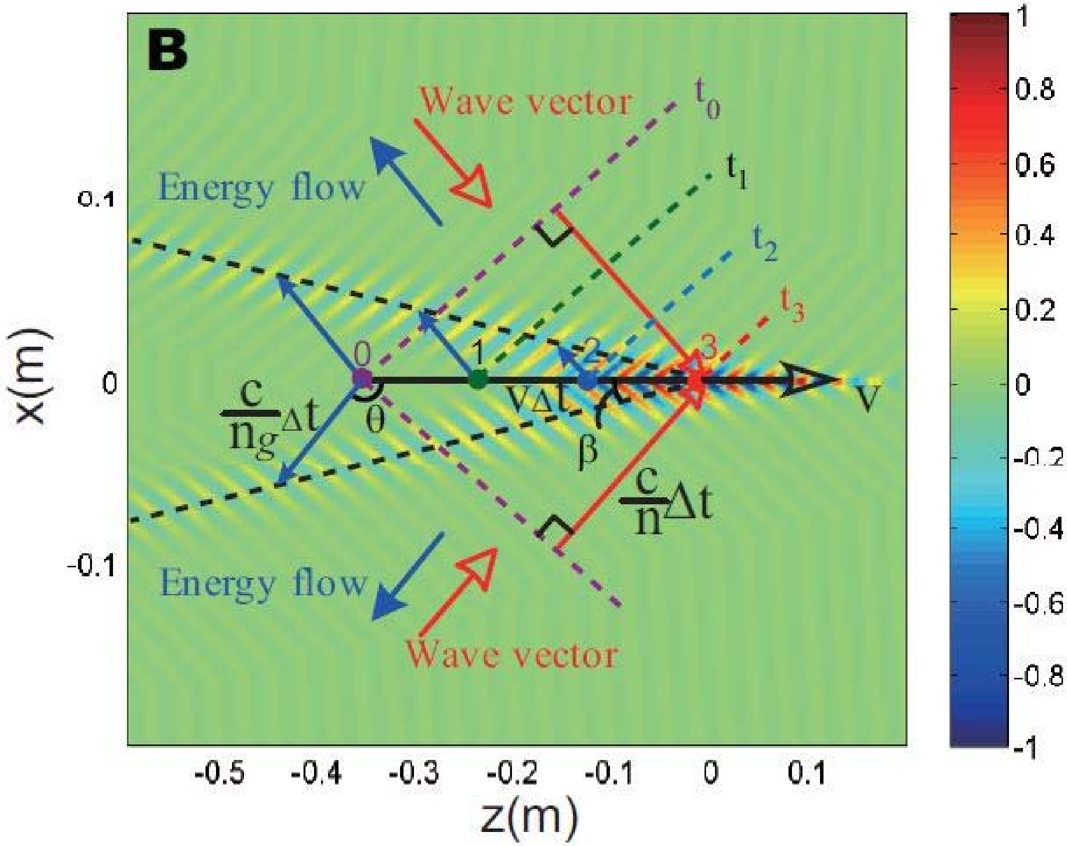
Forward Cherenkov Radiation in RH material obeys causality



Cos θ = c/ nv with n =2, v=0.99c

Wake front || Wave front ⊥ V of Energy flow & wave vector k

Reversed Cherenkov Radiation in Left-handed medium also



$$v = 0.99c.$$

$$\epsilon_r = 1 - \frac{\omega_{ep}^2}{\omega^2 + i\omega\gamma_e}$$

$$f_{ep} = 20 \text{ GHz}$$

$$\mu_r = 1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_0^2 + i\omega\gamma_m}$$

$$f_{mp} = 10 \text{ GHz}$$

$$f_0 = 4 \text{ GHz}$$

$$W(f) = e^{-(f-f_c)^2/(2\sigma^2)}$$

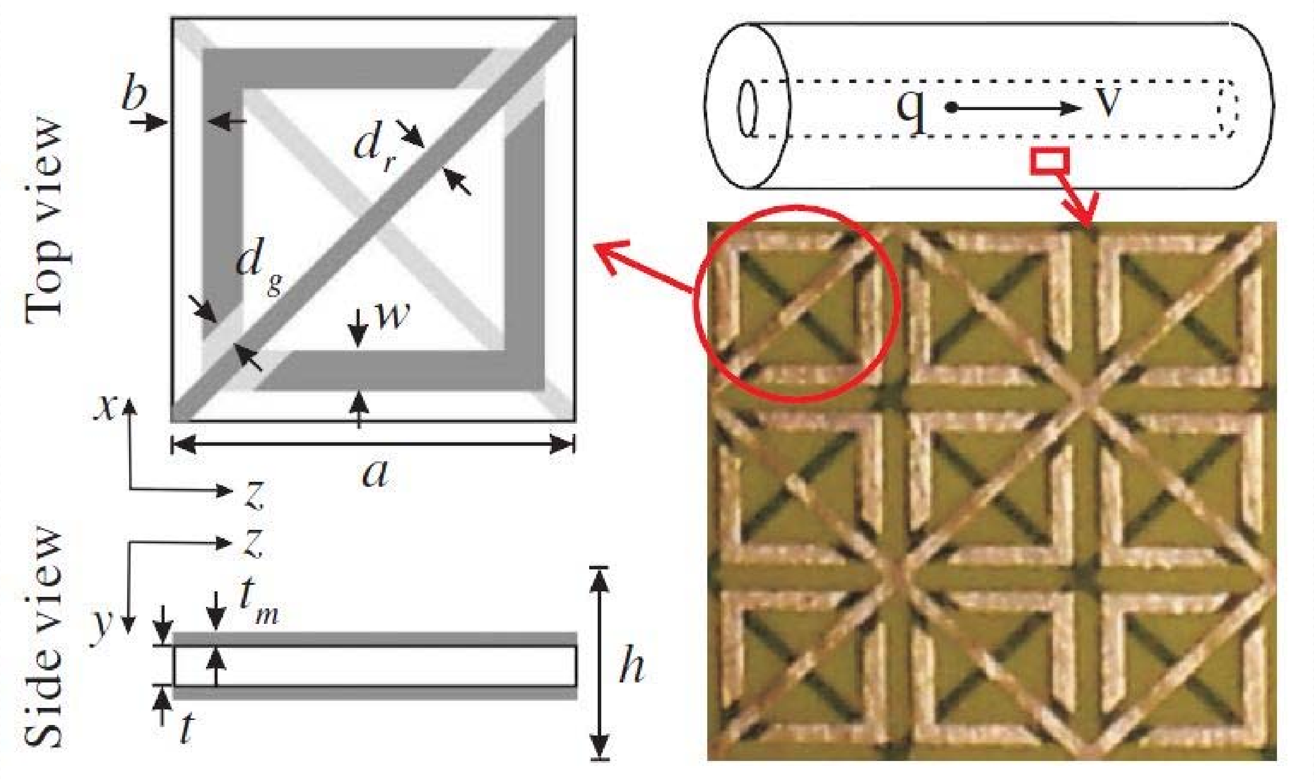
$$f_c = 8 \text{ GHz}$$

$$\sigma = 0.5 \text{ GHz}$$

$$n = -2.385 \qquad n_g = c/(d\omega/dk) = 6.569$$

θ = acos[c/ nv]=115º   Φ = 172.6º

New '***molecules***' for μ_φ<0, & ϵ_r , ϵ_z<0

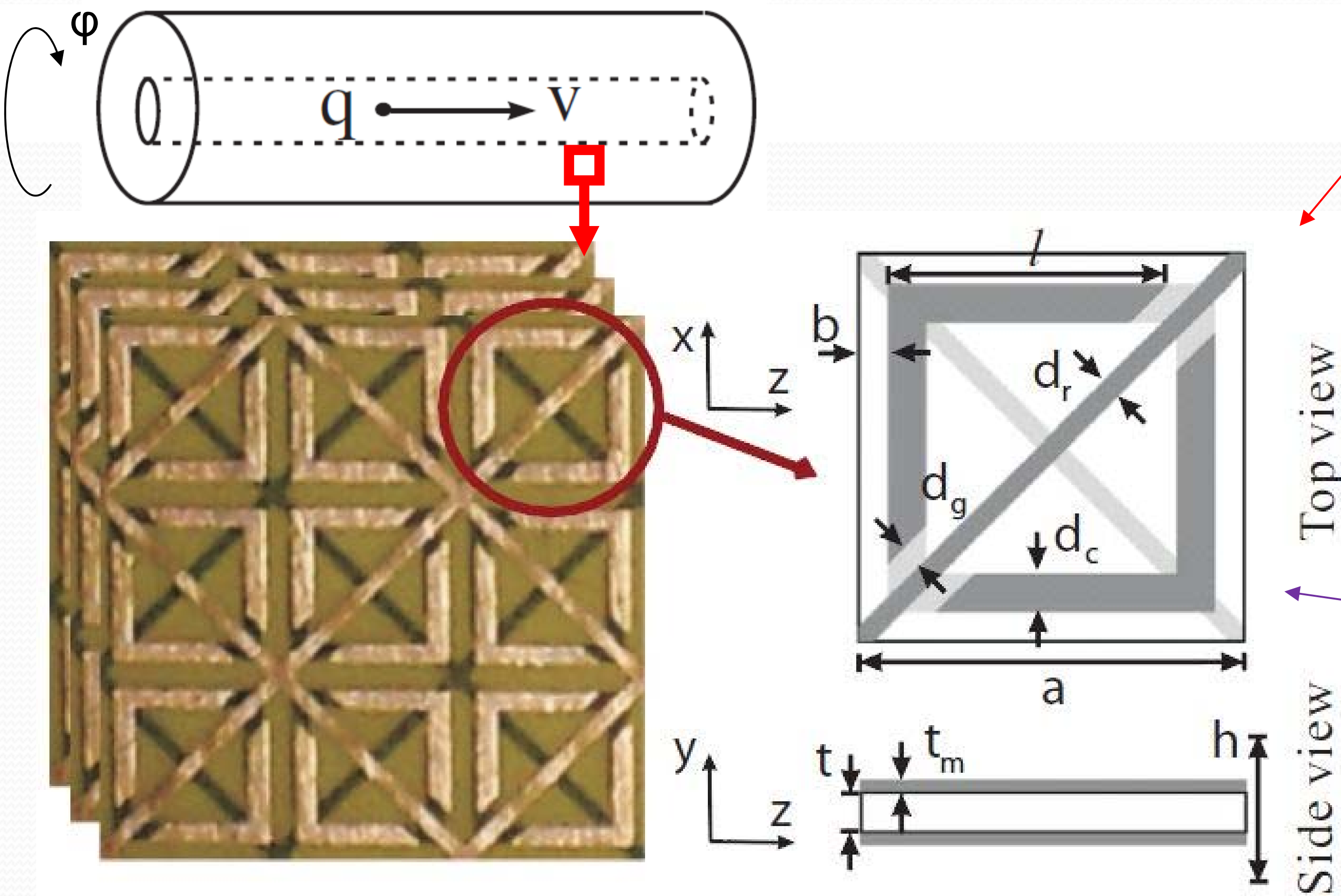


Configuration of the TM-LHM for experiment. In the top view, the dark (light) gray trips represent the copper printed on the top (bottom) of the substrate

Split Resonant rings SRR

- External B_o $_{or}$ H_o penetrates metal rings to induce I
- I produces H_i = FJ to enhance or oppose H_o, dipolar.
- Resonant λ_o >> d
- Small gaps between the rings -> large C -> lower ω_o
- Low loss, and high quality
- At $\omega > \omega_o$, real $\mu_{eff} = \langle B \rangle / (\langle H \rangle * \mu_o) = H_o / H_{ext} < -1$
- Used with the negative ϵ_r of split orthogonal wires to produce negative refractive index.

New '***molecules***' for $\mu_\varphi<0$, & ϵ_ρ, $\epsilon_z<0$



wires
Provide isotropic negative permittivity in *xz* plane

split-ring

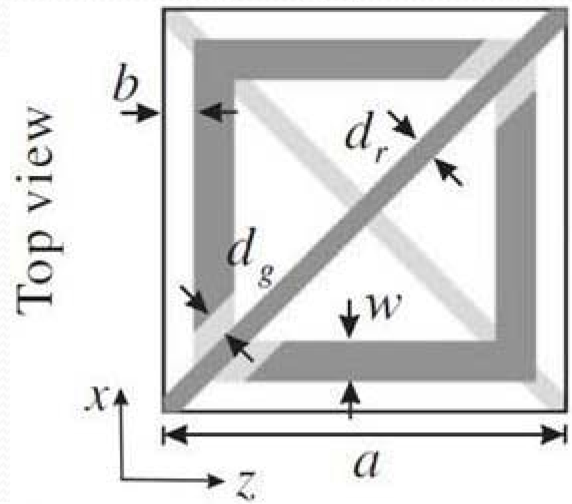
L, C

Resonators providing a negative permeability in *y* direction.

TM: B is in φ → v

Configuration of the TM-LHM for experiment. In the top view, the dark (light) gray strips represent the copper printed on the top (bottom) of the substrate

Magnetic response



H_0: incident magnetic field
J: induced current per unit length
Fields inside and outside of the loop:

$$H_{in} = H_0 + J - FJ, \text{ and}$$

$$H_{ext} = H_0 - FJ,$$

F = fraction of area inside loop

Total flux is constant J

M = F**J**

$$\mu_{eff} = \langle B \rangle / \langle H \rangle = \mu_0 H_0 / H_{ext}$$

$$\mu_{eff} = \mu_0 \left(1 - \frac{\omega^2 F L_y / (L_y + L_i)}{\omega^2 - \omega_0{}^2 \quad + i\omega\Gamma} \right)$$

Magnetic response

$$\frac{\mu_{eff}}{\mu_0} = 1 - \frac{\omega^2 F L_g/(L_g + L_i)}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

For perfect conductor, $\Gamma = 0$, real $\mu_{eff} < 0$ for

$$\omega_0 \le \omega \le \omega_0/\sqrt{[1 - F L_g/(L_g + L_i)]}$$

where

$\omega_0 = 1/\sqrt{(L_g + L_i)C}$ resonance frequency

$L_g = \mu_0 l^2/h$ geometric

$L_i = 4l/(\epsilon_0 d_c t_m \omega_{ip}^2)$ intrinsic

Scale the molecular dimensions by 1/1000, ω increase by ~900

Electrical response

Compute L, C to relate E and J of the wires and use

J = $\frac{\partial \mathbf{P}}{\partial t}$

$(\varepsilon_{eff} - \varepsilon_o)\, E = \mathbf{P} = \mathbf{J/i\omega}$

$\varepsilon_{eff} = \varepsilon_d \left(1 - \frac{\omega_p{}^2}{\omega^2 + i\omega\gamma} \right)$

real $\varepsilon_{eff} < 0$ for $\omega < \omega_p$

ε_d : permittivity of substrate

$$\omega_p^2 = \frac{2\sqrt{2}\pi c^2}{\epsilon_d h a \ln[h/(2t_m)]} \qquad \gamma = \frac{\sigma t_m d_c}{\ln[h/(2t_m)]}$$

Unique design → clean signals

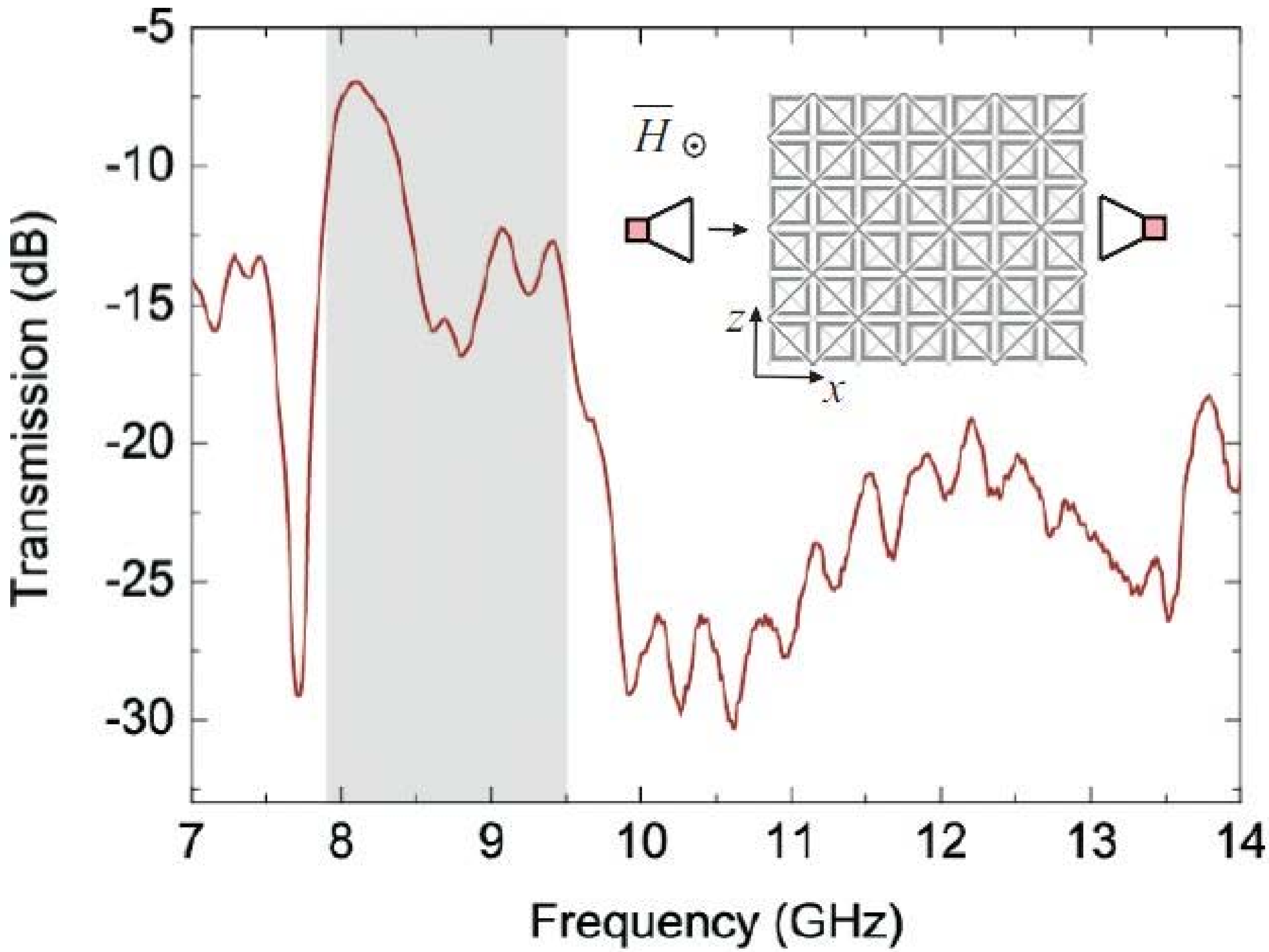
The constitutive parameter tensors

$\epsilon = diag[\epsilon_{/\!/}\ \epsilon_{\perp}\ \epsilon_{/\!/}] = diag[\epsilon_{eff}\ \ \epsilon_{d}\ \ \epsilon_{eff}]$

$\mu = diag[\mu_{/\!/}\ \mu_{\perp}\ \mu_{/\!/}] = diag[\mu_{0}\ \ \mu_{eff}\ \ \mu_{0}]$

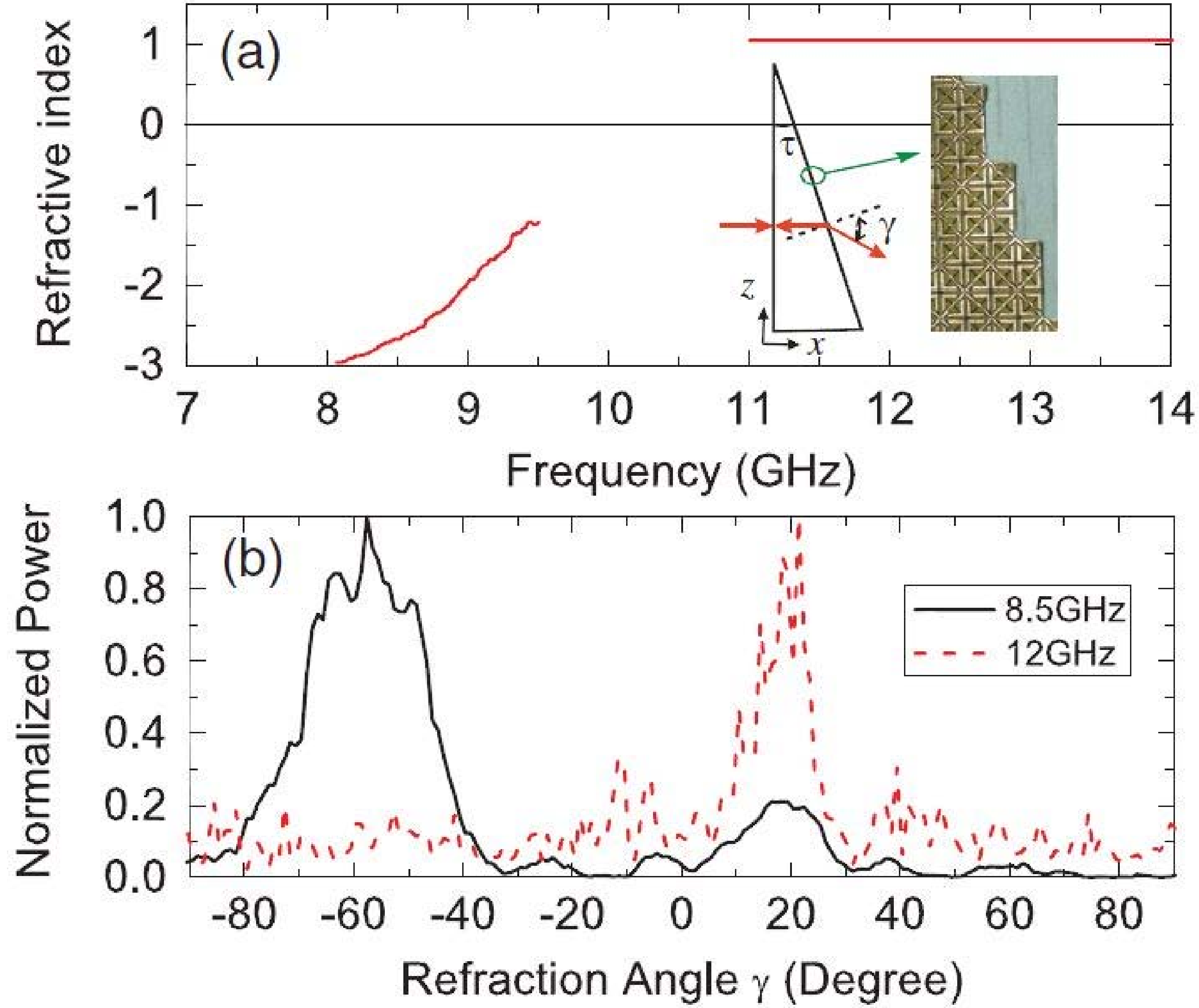
real part of n: $n' = -\sqrt{\mu_{\perp}\epsilon_{\|}}_{\epsilon_{\|}}$

Transmission experiment

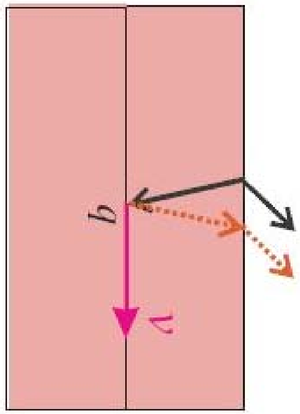


The transmission properties of a TM wave normally incident onto a 7-cell slab-like sample. The periodicity along *y-axis is h = 1.64 mm.*

Refraction experiment



(a) The refractive index of the TM-LHM calculated from the measured results. The periodicity along the y axis is h =1:64 mm. (b) The normalized far-field pattern of the prism experiment at 8.5 and 12 GHz, respectively.



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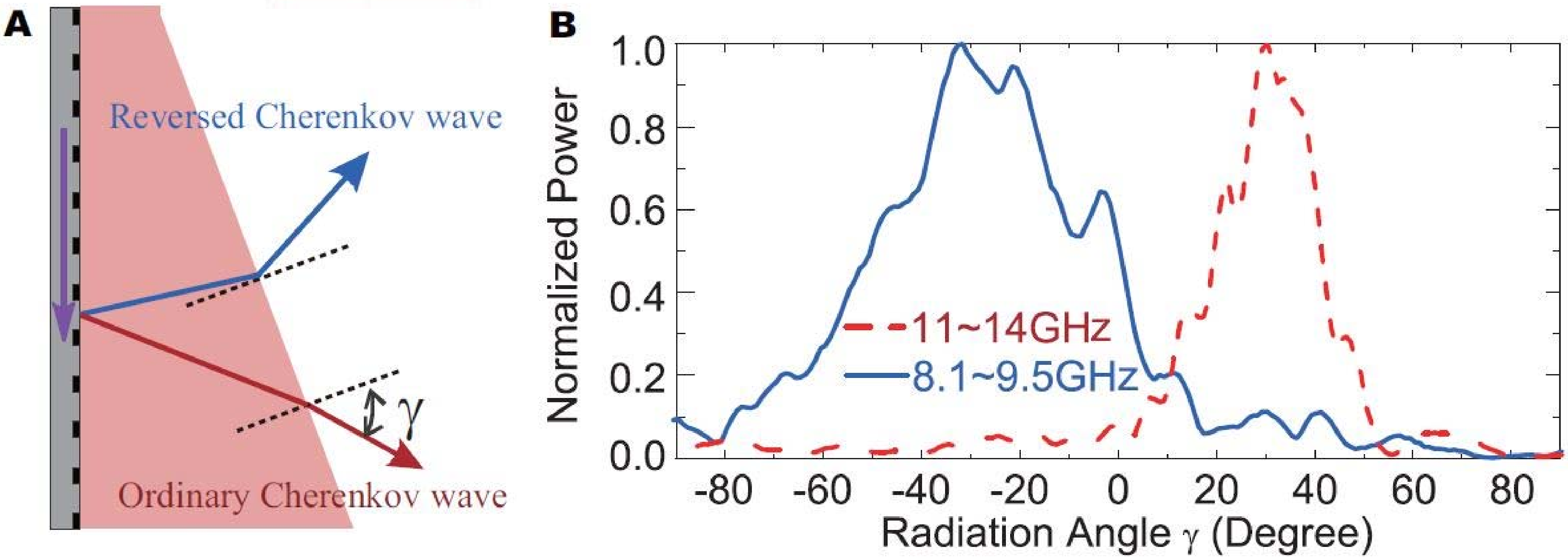


(a) Experimental setup to demonstrate reversed Cherenkov radiation. A slot waveguide is used to model a fast charged particle. The prism-like metamaterial is used to filter the Reversed Cherenkov wave.
(b) Sum of the radiation power in each angle in the negative refraction band and positive refraction band.

Application of RCR 1:

THz radiation sources
filling the gap
between
optical and electronic devices

1. Introduction

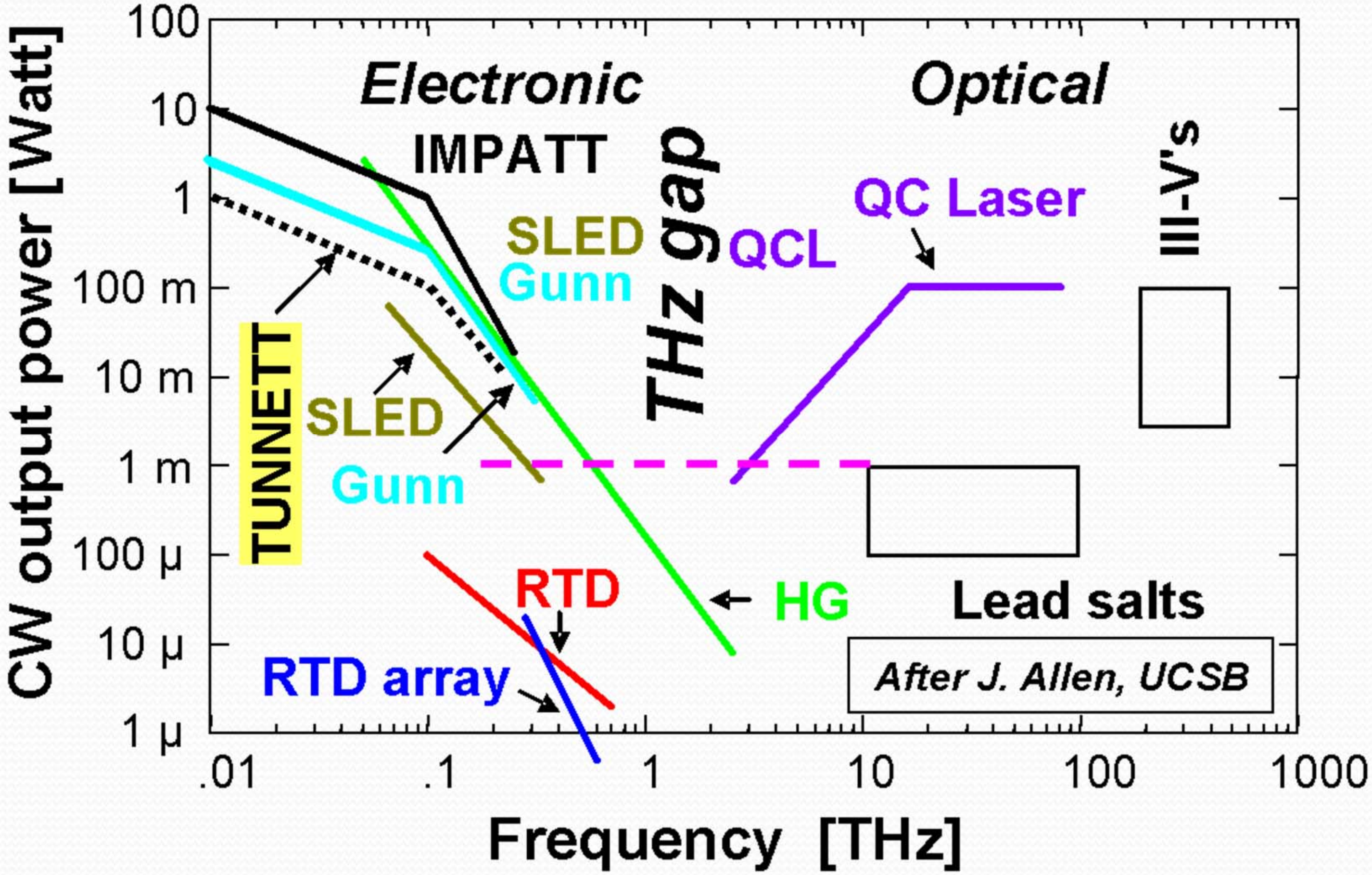
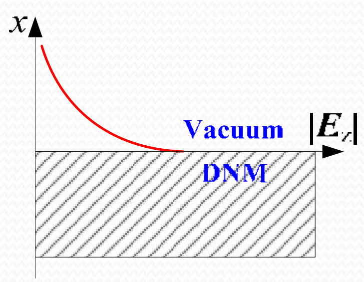


FIG. 2. The power of solid state devices and optical sources vs. frequencies.

2. Theoretical analysis

We describe a new method to generate intense ***terahertz (THz) surface wave (SW) and reversed Cherenkov radiation (RCR)*** *using a sheet beam bunch traveling parallel to and over a half space filled with* ***double-negative metamaterial (DNM)*****.**

***SW**:*
Surface Wave *exponentially decays in the x direction.*



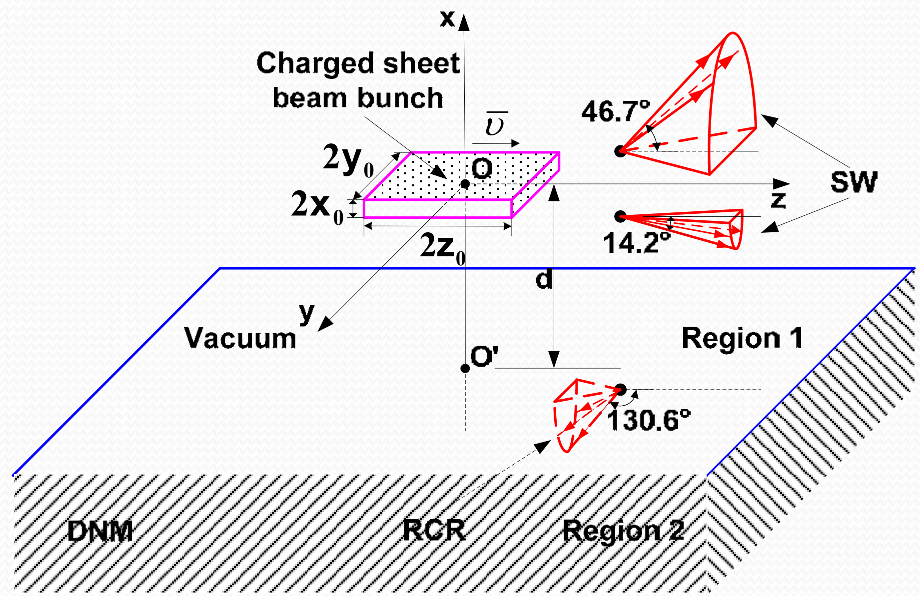
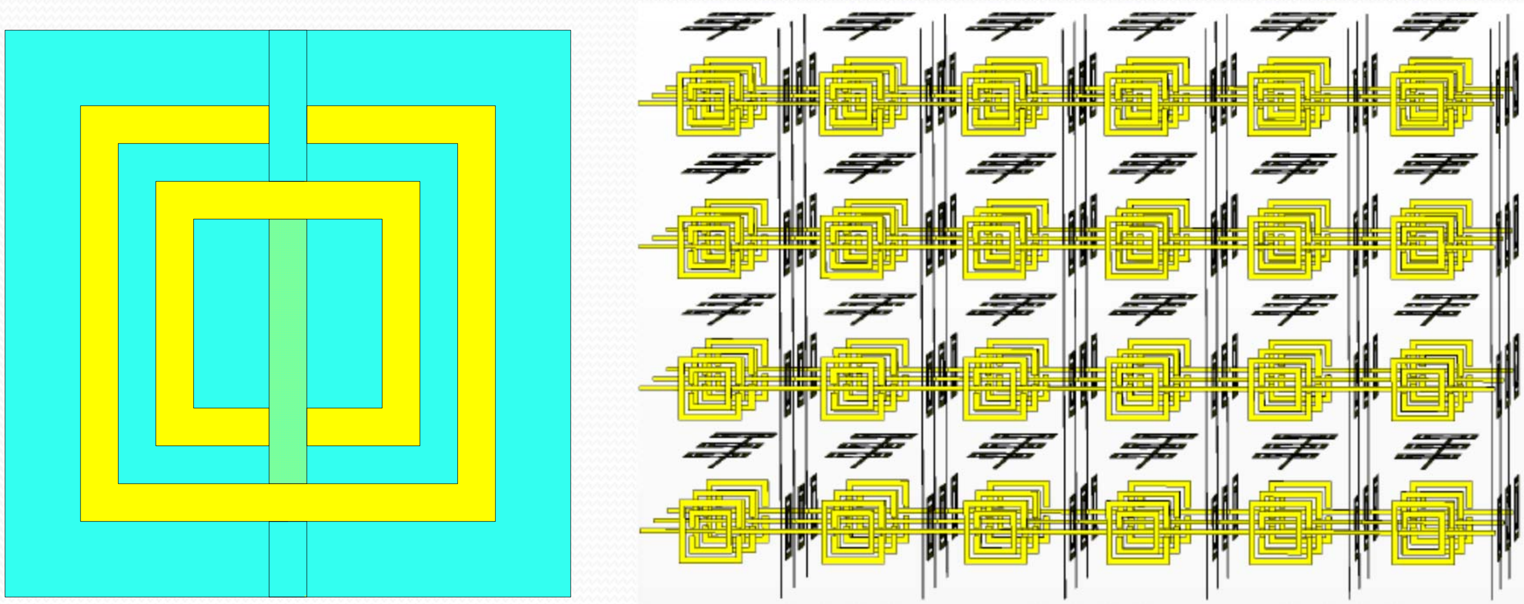


FIG. 6. The schematic of a sheet beam bunch moving with speed $\overline{\upsilon}$ in vacuum parallel to and over a half space filled with DNM, showing the resultant radiation patterns of the RCR and SW.

2. Theoretical analysis



(a) (b)

FIG. 7. (a) A sketch view of the unit cell structure formed by fixing a metal SRR and thin wire on two faces of a dielectric substrate. (b) A perspective view of an isotropic DNM formed by periodic unit cells.

3. Numerical results

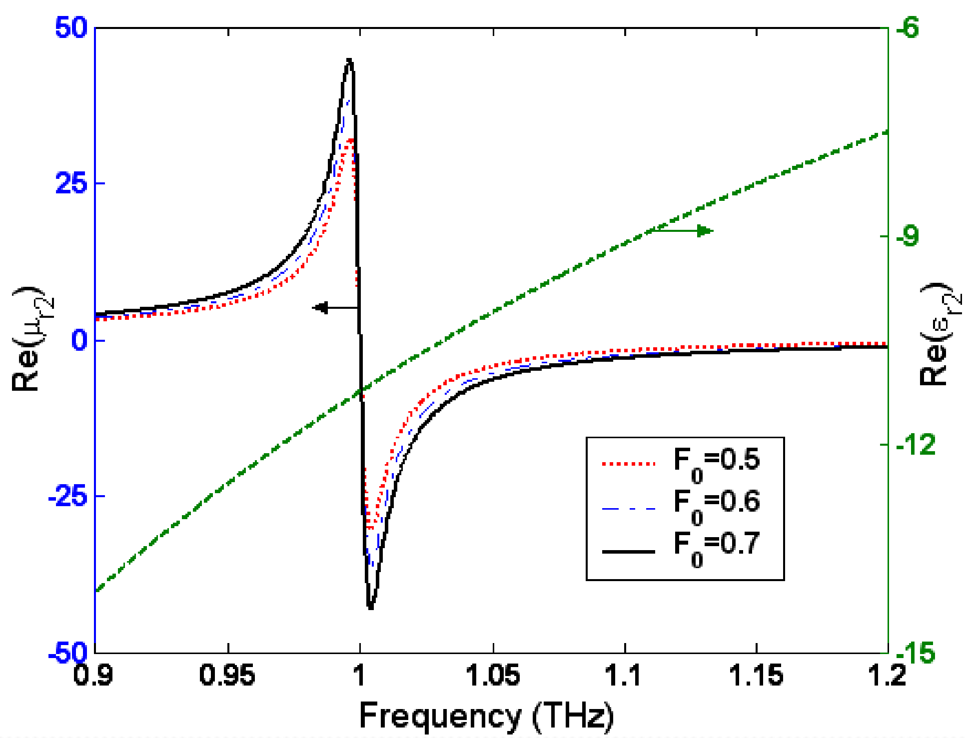
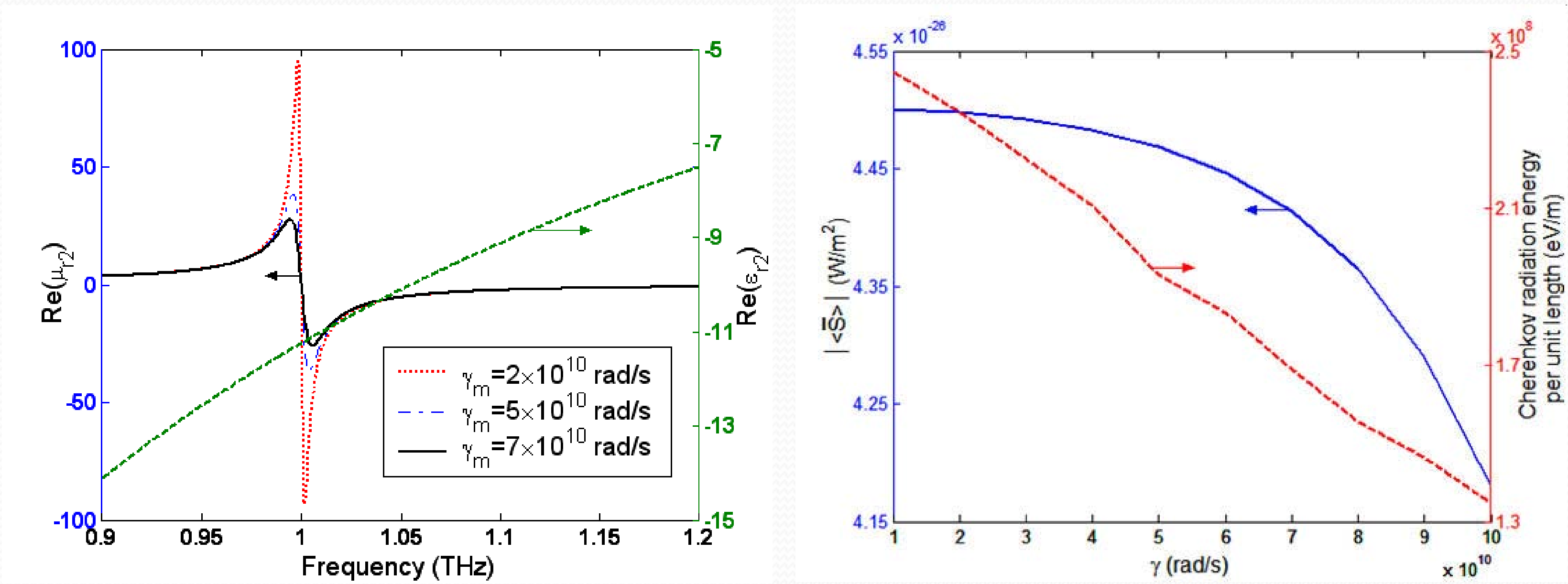


FIG. 8. (a) The relative permittivity and permeability as functions of frequency.

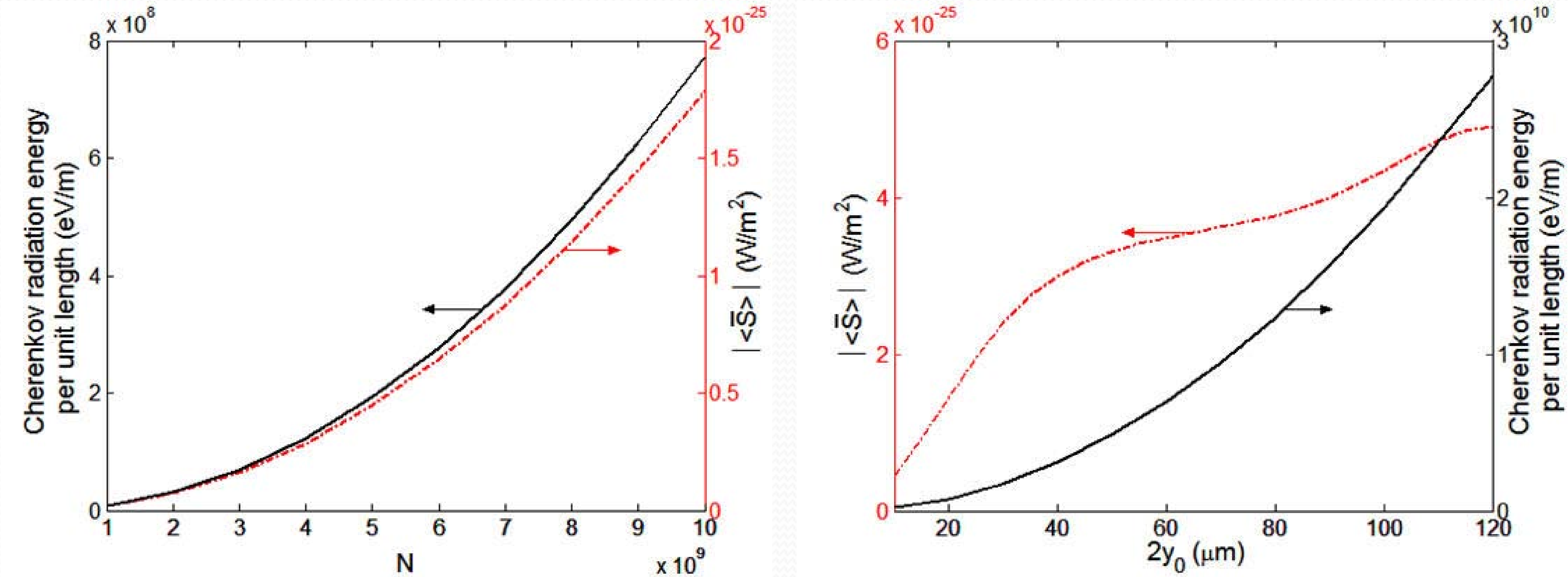
3. Numerical results



(a) (b)

FIG. 9. (a) The relative permittivity and permeability as functions of frequency for three values of the DNM loss. (b) The RCR energy and the time-averaged Poynting vector at $x = -d/2$ as functions of the DNM loss, respectively.

3. Numerical results



(a) (b)

FIG. 10. The effects of the charged particle number N (a) and the transverse dimension $2y_0$ (b) on the amplitude of the SW in region 1 and on the RCR energy in region 2, respectively.

Application of RCR 2

The only known EM process capable of detecting invisible cloaks

Invisible Cloaks made of LH light guides

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Application: Detect invisibility cloak using Cherenkov radiation

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Baile Zhang, Bae-Ian Wu, Phys. Rev. Lett. 103, 2009

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$|E_{tot}(\vec{r}, t)|$ during the radiation from a charged particle going through a spherical invisibility cloak. The dotted line represents the trajectory of the particle. The small arrow indicates the exact position of the particle’s center along its trajectory. The inner radius and outer radius of the cloak are 1 and 2 μm, respectively. The net charge corresponds to 1000 electrons. V=0.9c.

Detect a perfect invisible cloak

This image has been removed due to copyright restrictions. Please see Figure 3 on http://prl.aps.org/pdf/PRL/v103/i24/e243901.

Conclusion on

Reversed Cherenkov Radiation in LHM:

- ***Energy-momentum conservation***
- ***Causality***
- ***New molecules for TM waves***
- ***Experimental verification***
- ***Future improvements***
- ***New window of Applications***

Reversed Cherenkov radiation New window of Applications

- Particle ID: photons opposite to charged particles so interference is minimized.
- LHM can be isotropic, anisotropic, bi-anisotropic--flexible
- CR without threshold using utilizing anisotropic LHM,
  As observed in metallic grating and photonic crystals
- Strong velocity sensitivity and good radiation directionality
- Measuring intensity, detecting labeled biomolecules
- Detecting invisible cloaks
- New radiation sources

End of the lecture.

8.07 Electromagnetism II
Fall 2012