MITOCW | L13.1 Transition rates induced by thermal radiation

PROFESSOR: Today we're going to finish our discussion of atoms and light in interaction and finally get some of those rates. Last time, we reviewed the Einstein's argument. Einstein used his argument to discover spontaneous-- stimulated emission of radiation and how it accompanies the process of absorption all the time. There was also, of course, this process of spontaneous radiation that we found the formula for that transition rate in terms of the transition rate for stimulated radiation.

So let me review a couple of things that we had. So we have an electric field that interacts with the atom. There's a position vector that points to the charge, q. And we have two levels, level b with energy E b and level a with energy E a. We introduce a dipole moment operator, which was defined as q times r. And it's an operator because r is an operator. The position is an operator in quantum mechanics.

So we observe, and we've observed several times, that the process is b going to a of stimulated emission and the process of a going to b of absorption are the same, not equivalent, the same. The first order in perturbation theory, and we wrote, last time, the formula for it using our general formula for harmonic transitions. Not firm is golden rule, because this was a discrete transition, apparently, between these two levels.

So here is the formula. Let me remind you what it was. There was an electric field. It was of the form 2E 0 cosine omega t times the vector n. OK. All our results from the discussion of light and atoms from the previous lecture, so here we go.

The transition probability as a function of time is given by this quantity, n 0 is half the peak amplitude of the electric field. We always have this extra 2 here. d ab are the matrix elements of the operator d between the states here, a and b. So that's an inner product.

This is dotted with the vector n that refers to the polarization of the electric field. That half number is squared. And then we have the sine squared over x squared function that we've usually had.

So this is our situation. But what's going to happen, what we're interested and the situation that Einstein was considering, was when you had a box, you had thermal radiation. And those atoms were there. Here, we have a discrete process of transition from one state to another state. Neither one is a continuum.

When we had ionization, we were going from one state bound to the continuum of flying electrons. And therefore, a Fermi golden rule arose by integration. Here, what are we to integrate? Well, there is something to integrate.

The fact of the matter is that when you have this black-body radiation, you will have a spectrum, omega, of electric fields. You don't just have one electric field at one specific frequency. You will have many components of the electric field at different frequencies at different directions.

So in particular, when you have a black-body over this, you will have, in this spectrum, here is omega ba. That is the frequency for which the transitions tend to be more important. Remember, when you're omega, your frequency omega is the frequency of the external light. When your frequency omega coincides with the omega of this transition, this stimulated emission or the absorption get enhanced.

So here is omega ba. That's one of the frequencies in the black-body of radiation. But there are many, many more nearby. And they are incoherent. That is, in the black-body of radiation, each amplitude for a different frequency is independent of the other ones. They're, like, random.

So here is our sum, therefore. We must, we know, that when we have several inputs, not just one electric field but many electric fields, we will have to add the transition probability caused by the different electric fields. And in this case, we will have to sum the effects of the electric field squares of the many components of the black-body of radiation that lie near this omega ba. That is the one that would trigger a transition.

So this is how we're going to obtain something that will look like a Fermi golden rule as well here. There is a transition. The transition is between discrete states. But the trigger for the transition is a superposition of electromagnetic waves that we have to integrate over.

So one can say that the electric field, electric field is a superposition of incoherent waves with different omega i's, different frequencies, different E 0 of omega i, different amplitudes, and different vectors, unit vectors.

So each mode of this black-body radiation has a different amplitude and a different omega and a different polarization. And we have to sum over those that are near omega ba. And we know only over near ones, because this function doesn't allow you to-- or it becomes too small when you get far away from that central frequency.

So as a first step, I can rewrite this transition probability to make it a little clearer. I would say, here it is. The transition probability for spontaneous emission of radiation associated to a particular mode of the radiation field would be given by 4E 0 of omega i squared, h squared, d ab.

That is some matrix element of the dipole operator in the atom. That doesn't depend on anything having to do with the radiation. n is possibly dependent on the radiation. Sine squared of 1/2 omega ba minus omega i. The omega i is the omega of the electric field.

So I've put all these things to represent the contribution from the ith mode of the radiation field to the transition amplitude. That's what it does. So you could say, oh, the ith mode is one in which I would write 2E 0 of i.

So here would be 2E 0 of omega i cosine of omega i t times the vector n omega i. That's the ith mode of the electric field. And that ith mode of the electric field, that's this transition. And we have to sum over all these modes that are nearby here.

OK. This is one of the little non-trivial things that we have to do now. But the way to think about it is that we've seen, from the Einstein arguments and in general black-body, that there is a function u of omega, which is the energy density per unit frequency so that when you multiply by the omega, this is the energy density contained in the part of the spectrum with a range d omega. That's your energy density.

So that's an important quantity because it represents a number of photons. And that's how Einstein argued that that defines or is a part of the ingredient of constructing the rate at which transitions would be happening. So we have to relate this to energy.

So let's try to relate it to energy. So we use things from electromagnetism, some basic facts from electromagnetism. If you have-- so I'll go here. If you have a wave, in general for a wave, you have an E field and a B field. And you have energies u electric and u magnetic. These are energy densities. And there's a simple formula for them. The electric energy is equal to the magnitude of the electric field squared over 8 pi.

And in our case, the magnitude of the electric field is, the electric field in our conventions, is 2E 0 cosine omega t. So what do we get here? 4E 0 squared cosine squared omega t over 8 pi.

So this is 1 over 2 pi E 0 squared cosine squared of omega t. But nobody really cares about the fluctuation of the energy. You need the average energy at the volume. So the time dependence must be averaged.

So from here, the average over time of the electric field energy density is the average over time of cosine squared, which is 1/2. So we have 1 over 4 pi E 0 squared. So that's the average electric field energy for a wave. And that's the average over time.

But in a electromagnetic wave, the average electric energy and the average magnetic energy are the same. So u M magnetic is equal to u E. The averages are identical.

And then the total energy density, therefore, is-- therefore, total u, it's already average. Energy density would be u E plus u M, which is 2u E. And it's, therefore, 1 over 2 pi E 0 squared. So our end result is that the energy density u is 1 over 2 pi E 0 squared. OK.

Usually the thing here that is important is that there's lots of factors of 2. There's factors of 2 in defining the energy density, in averaging over time, in adding electric and magnetic contributions up. This is correct. It's important to get those factors of 2 right, otherwise your final formulas will be off.

So here is-- the end result is a nice formula that allows you to rewrite the amplitude of your oscillation in terms of its contribution to the energy density. So here it goes. We can now think of this by saying, if I have an electric field squared at some frequency, well, that's 2 pi times the energy density contributing at that frequency, at just the specialization of this to an arbitrary frequency, an omega i and an omega i here.

So with this, we can now rewrite this transition amplitude. The transition amplitude can be now expressed in terms of energy densities. So what do we have?

We have p b to a of t i would be equal to 4. That 4 remains there. And then instead of E 0 squared, we put 2 pi u of omega i over h squared. And not much changes anymore. The ab n omega i squared sine squared of 1/2 omega ba minus omega i t over omega ba minus omega i squared.