

PROFESSOR: OK, Let's use the last 10 minutes to discuss an application. So that's our Fermi golden rule there. Let's leave it in the blackboard.

So the example I will discuss qualitatively will not compute the rate for this example is auto ionization, or also called Auger transitions. So we imagine of-- I think the reason this example is interesting is that there is continuum states, sometimes in cases that you would not think about it, or you wouldn't have thought about them. So let's assume we have a helium atom.

So you have two electrons, z equals 2, two protons and two electrons. And I will assume that we have a hydrogenic state. So you see the Hamiltonian of this whole , atom there's a P_1^2 squared over $2n$ plus a P_2^2 squared over $2n$, roughly for each electron, plus or minus e^2 squared over r_1 minus e^2 squared over r_2 , and then plus e^2 squared over $r_1 - r_2$, which is the Coulomb repulsion. So that's roughly the Hamiltonian you need to consider, at least to 0-th order.

And let's assume this is H_0 , and let's consider states of this Hamiltonian, H_0 . So these are hydrogenic states. We treat each electron independently. And therefore, the energy levels, E are defined by two principle quantum numbers, n_1 and n_2 , and they are minus 13.6 eV times z^2 squared.

The energy of an electron in a nucleus with z protons gets multiplied by z^2 there times $1/n^2$, and $1/n_1^2$ plus $1/n_2^2$. Those are the principle quantum numbers. And z^2 is equal to 4, because z is equal to 2. So this number comes out at 54.4. So here is E of n_1, n_2 is minus 54.4 eV, $1/n_1^2$ plus $1/n_2^2$.

OK, let's look at the spectrum of this atom. I'll erase here. So let's draw a line here, put 0 here. Ground state. n_1 is equal to n_2 is equal to 1. The ground state is at minus 108. That's a state that we call state 1, 1 for the two quantum numbers being 1 and 1.

OK, so then we go on, and look what happens. Then we have, for example, the state 2, 1. And the state 2, 1 is going to be less bound, because instead of having 1 and 1, you have $1/4$ plus 1, so it's less bound. 3, 1, 4, 1. How about infinity 1? Infinity 1.

That's n_1 , the first electron being super Rydberg atom. It has its quantum principle quantum number so high that it's ∞ , and you have the other electron here with a 1. So this is at 54.4.

The infinity 1 is here. One electron is bound in the one state. The other electron is out very far. It's almost free. You could have it free. If it goes free, we'll have some more energy.

So actually, after you go here, there's a continuum of states, a continuum of states over here in which every electron here is an infinity and an electron with some momentum, k . And these are still states that are here can be represented by energies in this range. And then, eventually, you have even this goes on here.

So there's a lot of bound states there, and then we can continue. How about the state-- so I've put here infinity. I'm sorry, not infinity, k . I will put it here like k times y , because the other state remains in the 1 s. So one electron-- here both electrons were in the 1s state. And here 2s, 3s, infinity. So one electron remains in the 1s. One electron is free with some momentum over there.

Then, for example, consider the state $E_{2,2}$. This happens to be a 2 and a 2 is a quarter, a quarter, $1/2$. So this is at minus 27 eV. It's right here in the middle. This is the state 2, 2. After the state 2, 2 sets in, there is a state 3, 2, 4, 2, 5, 2, infinity 2.

So one term is infinity. The other is 2. So it's one quarter. And infinity 2 is that the hydrogen level, 13.6. And after that infinity 2, there is a new continuum of states that has some momentum and are still in 2. And there's going to be an infinite number of continuums arising here when you consider one electron and the other one.

So what is our other transition? This was at minus 27.2. The other transition we're typically interested in is this one in which you have this discrete state overlapped by the continuum of states in which one electron is gone out, and one electron has remained in the-- has gone to the 1s state. So I'll write it down here. So what is the other transition?

We have precisely this situation with indicate that here in which you have one state surrounded by a continuum. So the other transition, so the line 2, 2, at minus 27 eV is in the middle of the infinity 1 continuum. So we'll have a transition into 2s squared-- that's this state, that's the name of it-- goes to 1s times 3. The $E_{2,2}$ is going to be E_1 infinity plus E_3 .

And the $E_{2,2}$ is minus 27.2. This is minus 54.4, and this is E_3 . So E_3 goes to 27.2 eV for the free electron. So how do we visualize this? If you wanted to compute it, and it's kind of a fun thing, but we will not do it, you think of the initial 2s state. Have the atom. The one is the two electrons here. And you think of them as having an initial wave function, hydrogenic, and then

suddenly, because maybe something happened. This 1s electron got excited, and suddenly, for an instant of time, the two are there in the 2s.

And then this interaction turns on, and this is the v that makes that transition from this 2s squared state to 1s 3. That has good matrix elements, and this is going to happen. And this is the dominant transition, because a radiative transition in which one goes down here is fairly suppressed. It doesn't happen. So this is our example of an application of Fermi's golden rule, and we'll explore more next time.