PROFESSOR: This is the answer. Tan of 2 k a plus delta. It's a little messy, no wonder. Just plotted this functions. [INAUDIBLE] Sin k plus a cosh kappa a [INAUDIBLE] plus a prime over kappa cosh a prime a sinh kappa a. You have sin a prime a sinh. The sinh causes outer [INAUDIBLE] the same but this one changes. Centered here, k prime over kappa cos k prime a cosh kappa a.

So it's a well-defined expression, if you know the number of kappa a, you can calculate everything. Let me just make sure you can see that. If you know-- if you said, for example, ka, this is a natural variable. ka is just-- the k is related to the energy direction. So ka has no limits, call it u.

Then you have two parameters of the square well [INAUDIBLE], a depth we don't and a height we want. So there's natural to define just [INAUDIBLE] would have z 0 squared which is 2m v 0 a squared over h squared. You also define z 1 squared [INAUDIBLE] as 2m v1 a squared over h squared.

And then the energy-- we can do another thing, we're going to find the energy divided say, by v 1 to be little e. And that's reasonable because these energies is compared always with v 1, and we're solving-- we've solved this problem in the domain when the energy is less than v 1, and that's why we'll have kappas here, and if energy was bigger than v 1, you would have trigonometric functions everywhere.

Now you can switch from trigonometric to hyperbolic by analytic continuation, letting and angle become imaginary, a trigonometric function becomes a hyperbolic function. Most of us are rather comfortable doing this at the beginning, because of the matter of sine if you mess up a sine, it's a big problem.

But at the end of the day, it actually saves a lot of work. So a little of that in the homework you will see. But at any rate, this is valued for e in this range. And therefore this ratio is h squared k squared. You can put an a squared over 2m v 1 a squared. And you can see a u squared here. And-- I'm sorry, not there. A u squared here, and the h squared divided here gives you a z 1. So this is u squared over z 1 squared.

A couple of extra things-- I'm just putting it here because if you ever have the curiosity of doing this plots, this may help. k prime squared is related to the energy, therefore u squared and v 0 z 1 squared. This is-- c 0 squared, [INAUDIBLE] 0. And for the other one, kappa a squared is equal to z 1 squared minus u squared.

So everything becomes a function of u Wherever you have a kappa prime a you have a square root of u. Now, somebody has to give you the values of z 0 and z 1. Those are the data of the potential-- z 0 and z 1 are numbers you know then. Therefore you know kappa a is a function of u, kappa-- k prime a is a function of u, and ka, which is u. Therefore this equation becomes a function of u [INAUDIBLE] this arc tangent here, and so for delta.

It's messy. There's no-- remember I mentioned the other day the fact that you could do trigonometric identities, and so for tan delta here. But then this I don't think simplifies when you solve that. It becomes just a bigger mess. You did solve for tan delta, it's very messy.

So anyway, let's leave it there. Are there any questions in solving this problem? So in principle, we solved it. I didn't plot anything, so you still don't have any insight as to what's happening. But you've learned in principle how to solve it. Any questions?

OK, so let's plot this then. Maybe I should start here. OK. I'll start here. Oh, if I'm going to plot, I have to choose things. So we'll choose z 0 squared equals to 1, or z 0 equals to 1. And z 1 squared equals to 5. So we put a big barrier there.

And now let's go delta as a function of ka or u. Now, from this equation, we see that u and the little e must be at most 1 for our formulas to be correct. So u must be up to square-- up to z 1, because you must keep this ratio less than 1. So we can plot u up to square root of 5. 5, here's 2. And that's it.

And here we're going to plot delta of u. And here is minus pi over 2 minus pi. And so what does it do, the phase? There no way you can guess, I think, from this formula. I could not guess from this formula. So we could try to imagine-- it's possible to guess, actually, after you've solved this week's homework.

You probably will have a good guess that the phase shift begins with minus 2ka, as if with very little energy you reflect back here. So there will be a shift that is calculable without doing any work. So it begins linearly. And it represents a time advance. So it goes linearly and negative. That's how it begins.

Because for very little energy, it's going to bounce back. And you know that the delay is proportional to this derivative. So must be negative like that. So then what does it do? It crosses this point, which is almost pi, and here is what something quite remarkable happens. That some value you star, which is about 1.8523, I think. That's what I calculated. Let's see [INAUDIBLE].

The face that is almost minus pi suddenly jumps very, very fast, crosses pi over 2, and then about [INAUDIBLE] something like this. I don't know what else it does, because we haven't calculated it. But it jumps very fast. Almost a value of pi. Now, let me-- this is quite interesting.

If you think of what we used to call the scattering amplitude as squared was sin squared delta. The amplitude of the scattered wave, sin squared delta, the amplitude of the scattered wave is going to be quite large here, it's going to be 1, which is the maximum, as squared is here. So I want to keep these two blackboards aligned.

So here it goes like this. It's going to go up like this, and do this, and broadly go down, and then very sharply go up. More sharply, at least. Go up and then something like that. So here it is. This point over here has a strong scattering amplitude, but there's nothing too dramatic happening here.

This sine corresponds to time advanced, the derivative is negative. And time advanced cannot be too large, as you know. On the other hand, here is time delayed, and apparently they can't be very large. So we think this must be the resonance, and this is not a resonance, even though the scattering amplitude is bigger.

So we continue here, and plot the time-- interestingly, the amplitude inside the well. So here it is. How does the wave function become, through this constant, inside the well? And indeed, the this confirms that nothing very special is happening here. What happens now is some sort of behavior like this, and a big jump here, in which the amplitude apparently-- I have not quite confirmed this number, it's at least a value 3. [INAUDIBLE] very large and short.

I should have room for one more plot [INAUDIBLE] the star plot of this thing is-- I'll do it compressed here. The total delay, 1 over a d delta dk. Well, it begins negative, and remember that when we did the 1 over a, d delta dk, this is a pure number. It expresses the delay in terms of the time that it would take to travel the inside region.

So how many-- if you would get a 1, or a minus 1, it's just a delay of the size of the time needed to travel back and forth. So actually this goes a little negative at the beginning-- we know the derivative is like that-- and when you plot this, you see that it's very sharp, and it's a value of about fourteen. Fourteen times gets delayed from what you would expect naturally of the time that it should have spent travelling back and forth A gigantic time delay. A peak in the time delay. Peak in time delay. Peak in the amplitude inside the well.