**PROFESSOR:** OK, so, local picture. It's all about getting insight into how the way function looks. That's what we'll need to get. These comments now will be pretty useful. For this equation you have one over psi, d second psi, d x squared is minus 2 m over h squared, E minus v of x. Look how I wrote it, I put the psi back here, and that's useful.

Now, there's a whole lot of discussion-- many textbooks-- about how the way function looks, and they say concave or convex, but it depends. Let's try to make it very clear how the wave function looks. For this we need two regions. So, the first case, A, is when the energy minus v of x is less than 0. The energy is less than v of x, that's a forbidden region-- as you can see there-- so it's a classically forbidden. Not quantum mechanically forbidden, but classically forbidden.

What is the main thing about this classically forbidden region is that the right hand side of this equation is positive. Now, this gives you two possibilities. It may be that psi at some point is positive, in which case the second psi must also be positive, because psi and the second psi appear here. If both are positive, this is positive. Or, it may be case two, that psi is negative, and the second psi-- the x squared-- it's also negative.

Well, how do we plot this? Well, you're at some point x, and here it is, a positive wave function seems to be one type of convexity, another type of convexity for a negative, that's why people get a little confused about this. There's a way to see in a way that there's is no confusion. Look at this, it's positive, second derivative positive. When you think of a second derivative positive, I think personally of a parabola going up. So, that's how it could look.

The wave function is positive, up, it's all real. We're using the thing we proved at the beginning of this lecture: you can work with real things, all real. So, the wave from here is x, and here negative. And the negative opening parabola, that's something they got. So nice. So, the wave function at any point could look like this if it's positive, or, it could look like this if it's negative. So, it doesn't look like both, it's not double value. So, either one or the other. But, this is easy to say in words, it is a shape that is convex towards the axis. From the axis it's convex here and convex there. So, convex towards the axis.

Now, there's another possibility I want to just make sure you visualize this. Sometimes this looks funny-- doesn't mean actually the way function can look like that-- but, it's funny because of the following reason. It's funny because if you imagine it going forever, it doesn't make sense because you're in a classically the forbidden region. And the way function's becoming bigger and bigger is going to blow up.

So, eventually something has to happen. But, it can look like this. So, actually what happens is that when you're going to minus infinity-- here is x and we use minus infinity-- it can look like this. This is an example of this piece that is asymptotic, and it's positive, and the second derivative is positive. Or, negative and the second derivative is negative. So that's a left asymptote.

Or, you could have a right asymptote, and it looks like this. Again, second derivative positive, positive wave function. Second derivative negative, negative wave function. So, you may find this at the middle of the potential, but then eventually something has to take over. Or, you may find this behavior, or this behavior, at plus minus infinity. But, in any case you are in a classically forbidden, you're convex towards the axis. That's the thing you should remember.

On the other hand, we can be on the classically allowed region. So, let's think of that. Any questions about the classically forbidden? Classically allowed, B. E minus v of x greater than 0, classically allowed. On the right hand side of the equation is negative. So, you can have, one, a psi that is positive, and a second derivative that is negative. Or, two, a psi that is negative, and a second derivative that this positive.

So, how does that look? Well, positive and second derivative negative, I think of some wave function as positive, and negative is parabolic like that. And then, negative and second derivative positive, it's possible to have this. The wave function there it's negative, but the second derivative is positive. These things are not very good-- they're not very usable asymptotically, because eventually if you are like this, you will cross these points. And then, if you're still in the allowed region you have to shift.

But, this is done nicely in a sense if you put it together you can have this. Suppose all of this is classically allowed. Then you can have the wave function being positive, the second derivative being negative, matching nicely with the other half. The second derivative positive, the wave function negative, and that's what the psi function is. It just goes one after another. So, that's what typically things look in the classically allowed region. So, in this case, we say that it's concave towards the axis. That's probably worth remembering.

So, one more case. The case C, when E is equal to E minus v of x not is equal to 0. So, we have the negative, the positive, 0. How about when you have the situation where the potential at some point is equal to the energy? Well, that's the turning points there-- those were our turning points. So, this is how x 0 is a turning point.

And, something else happens, see, the right hand side is 0. We have that one over psi, the second psi, the x squared is equal to 0. And, if psi is different from 0, then you have the second derivative must be 0 at x not. And, the second derivative being 0 is an inflection point.

So, if you have a wave function that has an inflection point, you have a sign that you've reached a turning point. An inflection point in a wave function could be anything like that. Second derivative is positive here-- I'm sorry-- is negative here, second derivative is positive, this is an inflection point. It's a point where the second derivative vanishes. So, that's an inflection point.

And, it should be remarked that from that differential equation, you also get that the second psi, the x squared, is equal to E minus v times psi, which is constant. And, therefore, when psi vanishes, you also get inflection points automatically because the second derivative vanishes. So, inflection points also at the nodes.

Turning point is an inflection point where you have this situation. Look here, you have negative second derivative, positive second derivative, the point where the wave function vanishes and joins them is an inflection point as well. Is not the turning point-- turning point are more interesting-- but inflection points are more generic.