PROFESSOR: I'll begin by reviewing quickly what we did last time. We considered what are called finite range potentials, in which over a distance R, in the x-axis, there's a non-zero potential. So the potential is some v of x for x between capital R and 0, is equal to 0 for x larger than capital R, and it's infinity for x negative. So there's a wall at x equals 0. And there can be some potential, but this is called a finite range potential, because nothing happens after distance R.

As usual, we considered scattering solutions, solutions that are unnormalizable with energies, h squared k squared over 2m, for a particle with mass m. And if we had no potential, we wrote the solution phi of x, the wave function, which was sine of kx. And we also wrote it as a superposition of an incoming wave. Now, an incoming wave in this set up is a wave that propagates from plus infinity towards 0. And a reflected wave is a wave that bounces back and propagates towards more positive x.

So here we'll write this as minus e to the minus ikx over 2i, plus e to the ikx over 2i. This is the sine function rewritten in terms of exponential in such a way that here is the incoming wave. Remember the time dependence is minus iet over h bar. So this wave combined with a time is a wave that is moving towards the origin. This wave is moving outwards.

Then we said that there would be, in general, with potential. With a potential, you would have a solution psi effects, which we wrote after some tinkering in the farm i delta sine of kx plus delta. And if you look at the part of the phase that has the minus ikx would have a minus delta and a delta here. So they would cancel. So this solution has the same incoming wave as the no potential solution.

On the other hand here, you would have e to the 2i delta, e to the ikx over 2i, and this solution is only valid for x greater than R. You see, this is just a plane wave after all. There's nothing more than a plane wave and a phase shift. The phase shift, of course, doesn't make the solution any more complicated or subtle, but what it does is, by depending on the energy, this phase shift delta depends on the energy, and we're on k. Then, it produces interesting phenomena when you send in wave packets.

So if we write psi, we usually write psi is equal to the phi plus psi s, where psi s is called the scattered wave. You see, the full wave that you get, for x greater than R, we would have to solve and work very carefully to figure out what is the wave function in the region 0 to R. But for x greater than R is simple, and for x greater than R the wave function psi is the free wave function, in the case of no potential, plus the scattered wave.

Quick calculation with this, things [? give to ?] you the scatter wave is e to the i delta sine delta e to the ikx is an outgoing wave. And this coefficient is called the scattering amplitude. It's the amplitude of the scattered wave. This is a wave that is going out, and this is its amplitude. So it has something to do with the strength of the scattering, because if there was no scattering, the wave function would just behave like the no potential wave function.

But due to the potential, there is an extra piece, and that represents an outgoing wave beyond what you get outgoing with a free no potential wave function. So it's the scattering amplitude, and therefore sometimes we are interested in as squared, which is just sine squared delta. Anyway, those are the things we did last time. And we can connect to some ideas that we were talking about in the past, having to do with time delays, by constructing a wave packet. That's what's usually done.

Consider the process of time delay, which is a phenomenon that we've observed happens in several circumstances. If you have an incident wave, how do you construct an incident wave? Well, it has to be a superposition of e to the minus ikx, for sure. So we'll put the function in front, we'll integrate over k, and we'll go from 0 to infinity. I will actually add the time dependence as well.

So let's do phi of x and t. Then, we would have e to the minus i, e of k, t over h bar, and this would be valid for x greater than R. Again, as a solution of the Schrodinger equation. You see, it's a free wave. There's nothing extra from what you know from the de Broglie waves we started a long time ago.

So if this is your incident wave, you have to now realize that you have this equation over here telling you about the general solution of the Schrodinger equation. The general solution of the Schrodinger equation, in this simple region, the outside region, is of this form, and it depends on this delta that must be calculated. This is the incoming wave, this is the reflected wave, and this is a solution. So by superposition, I construct the reflected wave of x and t.

So for each e to the minus ikx wave, I must put down one e to the ikx, but I must also put an e to the 2i delta of the energy, or delta of k. And I must put an extra minus sign, because these two have opposite signs, so I should put a minus 0 to infinity dk f of k. And we'll have the e to the minus i, e of k, e over h bar. And just for reference, f of k is some real function that picks at some value k naught.

So you see, just like what we did in the case of the step potential, in which we had an incident wave, a reflected wave packet, a transmitted wave packet, the wave packets go along with the basic solution. The basic solution had coefficients A, B, and C, and you knew what B was in terms of A and C. Therefore, you constructed the incoming wave with A e to ikx, and then the reflected wave with B e to the minus ikx. The same thing we're doing here inspired by this solution, the psi affects we superpose many of those, and that's what we've done here.

Now of course, we can do the stationary phase calculations that we've done several times to figure out how the peak of the wave packet moves. So a stationary phase at k equal k naught. As you remember, the only contribution can really come when k is near k naught, and at that point, you want the phase to be stationary as a function of k. I will not do here the computation again for psi incident. You've done this computation a few times already.

For psi incident, you find the relation between x and t, and I will just write it. It's simple. You find that x is equal to minus h bar k naught over mt, or minus some v velocity, group velocity, times t. That is the condition for a peak to exist. The peak satisfies that equation, and this makes sense when t is negative. This solution for psi incident only makes sense for x positive if in fact x greater than R. So this solution needs x positive. So it needs t negative [? indeed. ?] This is a wave that is coming from plus infinity, x equal plus infinity, at time minus infinity, and it's going in with this velocity.

For psi reflected, the derivative now has to take the derivative of delta, with respect to e, and then the derivative of e with respect to the energy. And the answer, in this case-- you've done this before-- it's v group times t minus 2 h bar delta prime of E. So yes, in the reflected wave, x grows as t grows and it's positive. t must now be positive, but in fact, if you would have a just x equal v group t, this would correspond to a particle that seems to start at the origin at time equals 0 and goes out. But this actually there is an extra term subtracted. So only for t greater than this number the particle begins to appear.

So this is a delay, t minus some t naught, the packet gets delayed by this potential. Now, this delay can really get delayed. Sometimes it might even accelerated, but in general, the delay is given by this quantity So I'll write it here. The delay, delta t, is 2 h bar delta prime of E. And let's write it in a way that you can see maybe the units better and get a little intuition about what this computation gives. For that, let's differentiate this with respect to k, and then k with respect to energy.

So v delta with respect to k, and dk with respect to energy. This is 2 over 1 over h bar dE with respect to k. I do a little rearrangement of this derivative is one function of one variable k and neither is a single relation. So you can just invert it. This is more dangerous when you have partial derivatives. This is not necessarily true but for this ordinary derivatives is true, and then you have this 2 to the left here.

The h bar went all the way down, and I have d delta dk. And here, we recognize that this is 2, and this is nothing else than the group velocity we were talking before. The E, the energy, is h squared k squared over 2m. You differentiate, divide by h bar, and it gives you the group velocity hk naught over m. Because these derivatives all have to be evaluated at k naught. So this derivative is really evaluated at k naught. This is also evaluated at k naught.

So this is the group velocity, d delta, dk, and finally, let me rewrite it in a slightly different way. I multiply by 1 over R. Why? Because d delta dk, k has units of 1 over length. So if I multiply by 1 over R, this will have no units. So I claim that one over R d delta dk is equal to delta t, and you'll have 2 over vg and R. So I did a few steps. I moved the 2 over vg down to the left, and I multiplied by 1 over R, and now we have a nice expression.

This is the delay. Delta t is the delay, but you now have divided it by 2R divided by the velocity, which is the time it takes the particle with the group velocity to travel back and forth in the finite range potential. So that gives you an idea. So if you compute the time delay, again, it will have units of microseconds, and you may not know if that's little or much. But here, by computing this quantity, not exactly delta prime of v but this quantity. You get an [? insight, ?] because this is the delay divided by the free transit time. It's kind of a nice quantity. You're dividing your delay and comparing it with the time that it takes a particle, with a velocity that is coming in, to do the bouncing across the finite range potential.