PROFESSOR: So let's write the Hamiltonian again in terms of v and v dagger. So for this equation, v dagger v, from this equation, is equal to 2 h bar over m omega, a dagger a. And immediately above equation v dagger v's, this-- we substitute into the Hamiltonian. And Hamiltonian becomes the nice object h bar omega, a dagger a plus 1/2 if you want.

All right. We did this hard work of factorization. We have to show what's good for. Well, in fact, we're going to be able to solve the harmonic oscillator without ever talking about differential-- almost ever talking about differential equations. In fact, we will not talk about the second order differential equation. Thanks to our great work here, we will have to talk about a first order differential equation, and a much simpler one. And only one, not for and n 1, 2, 3, infitinty, infinity number of polynomials.

It's a great simplification. Other Hamiltonians admit factorization. In fact, there's whole books of factorizable Hamiltonians, because those are the nicest Hamiltonians to solve. Let's see why, though. We haven't said why yet.

Here is the leading thing that we can do. Remember we recalled 5 psi the integral dx phi star of x psi of x. This is just notation. So the expectation value, calculate the expectation value of the Hamiltonian in some state psi. Could be the general state sum.

So what are you supposed to do? You're supposed to do psi h psi. This is normalized state, the expectation value is the integral of psi star h psi. That's what this is. But now, let's put in this information. And the expectation value of this would be psi h-- or let me do it this way-- h bar ome-- well, let's go slow. Psi h omega a dagger a a psi plus h omega over 2 psi.

So I just calculated h on psi, and I wrote what it is-- h omega this, this term. So this is two terms-- h omega psi a dagger a psi plus h omega over 2 psi psi. OK. So what did I gain with the factorization? So far, it looks like nothing. But here we go-- this term is equal to 1, because the wave function is normalized.

And here I can do one thing-- I can remember my definition of a Hermitian conjugate. I can move an operator and put its Hermitian conjugate on the other side. So think of this operator, a dagger-- a dagger is acting on this wave function. What is a dagger? It's this. And p is h bar over idex.

So this-- you know how to act. But if a dagger is here, I can put it on the first wave function, but I must put the dagger of this operator, and the dagger of a dagger is a. So this is h omega, a psi a psi plus h omega over 2.

Now here comes the next thing. If this is an inner product, any phi phi is greater or equal than 0, because you would have phi star phi, and that's positive. So any of that [INAUDIBLE] is greater or equal than 0. Note-- here you have some function, but here the same function. It is this case. That thing is greater or equal than 0. That is the great benefit of the factorized Hamiltonian-- if h has a v dagger v, you can flip the v dagger here and it becomes v psi v psi, and it's positive. And you've learned something very important, and you can get positive energies.

In fact, from here, since this is positive, this must be greater or equal than h omega over 2. Because this is greater than equal than 0. So the expectation value of the Hamiltonian-- if you would be thinking now of energy eigenstates, the energy eigenvalue is the expectation value of the Hamiltonian in an energy eigenstate must be greater than h bar omega over 2.

And in some blackboard that has been erased, we remember that the lowest energy state had energy-- there it is. The lowest energy state has energy h omega over 2. So look at this, and you say, OK, this shows that any eigenstate must have energy greater than h omega over 2. But could there be one state for which the energies exactly h omega over 2.

Yes, if this inner product is 0. But for an inner product of two things to be 0, each function must be 0. So from this, we conclude that if there is a ground state, it's a state for which a phi-- or a psi is equal to 0. So this is a very nice conclusion. So if the lower bound is realized, so that you get a state with energy equal h bar over 2, then it must be true that a psi is equal to 0.

And a psi equal to 0 means x plus ip over m omega on psi is equal to 0, or x plus p is h bar over i ddx, so this is h bar over m omega d/dx on psi of x is equal to 0. And that was the promised fact. We have turned the second order differential equation into a first order differential equation.

Think of that magic that has happened to do that. You had a second order differential equation because the Hamiltonian has x squared b squared. By factorizing, you go two first order differential operators. And by Hermeticity, you were led to the condition that the lowest energy state had to be killed by a. That's why a is called the annihilation operator. It should be killed. And now you have to solve a first order differential equation, which is a game. An easy game compared with a second order differential equation.

So let's, of course, solve it. It doesn't take any time. Let's call this the ground state. If it exists. And this gives you d psi 0 v x is equal to minus m omega over h bar x psi . 0. This can be degraded easily or you can guess the answer. It's an exponential. Anything that differentiates that you should extend the same function as an exponential-- e to the minus m omega 2 h squared x squared is the solution.

Psi 0 of x is equal to some number times that. This was-- the number is the Hermit polynomials sub 0, and that exponential, this exponential, we wrote a few blackboards ago. It's a good exponential. It's a perfect Gaussian. It's our ground state. And 0, if you want to normalize it, m 0 is equal to m omega over phi h bar to the 1/4.

And that is the ground state. And it has energy, h omega over 2. You could see what the energy is by doing this very simple calculation. Look, get accustomed to these things. H hat psi 0. What is h? Is h omega a dagger a plus 1/2 acting on psi 0. The a acting on psi 0 already kills it. Because that's the defining equation. Well that's 0. And you get 1/2 h bar omega, confirming that you did get this thing to be correct.

So this is only the beginning of the story. We found the ground state, and now we have to find the excited states. Let me say a couple of words to set up this discussion for next time. The excited states appear in a very nice way as well. So first a tiny bit of language, of h bar. This equal h omega, a dagger a is usually called the number operator. We'll explain more on that next time.

So n number operator is a dagger a. It's a permission operator, and it's pretty much the Hamiltonian. It's the number, it's called. Why is it called the number is what we have to figure out. It is a counting operator-- it just looks at the state and counts things. So what does this give us? Well, we also know that the number operator kills phi 0, because a kills psi 0. A kills it. So that's what we have.

So we did say that a was a destruction operator, annihilation operator, because it annihilates the ground state. So if a annihilates the ground state, a dagger cannot annihilate the ground state. Why? Because a dagger with a computator is equal to 1. Look at this. This is a a dagger minus a dagger a.

Act on the ground state. That's it. Now this term kills it. But this term better not kill it, because it has to give you back the ground state if this is true. And this is true. So a dagger doesn't kill the ground state. Since it doesn't kill it, it's called a creation operator.

So you have this state, but now there's also this state a dagger acting on the vacuum. And there's a state a dagger a dagger acting on the vacuum. And all those. And what we will figure out next time is that, yes, this is the ground state. And this is the first excited state. And this is the second excited state. And goes on forever. So we'll have a very compact formula for the excited states of the harmonic oscillator. They're just creation operators acting on the ground state or the [INAUDIBLE].