PROFESSOR: Would solving this equation for some potential, and since h is Hermitian, we found the results that we mentioned last time. That is the eigenfunctions of h are going to form an orthonormal set of functions that span the space. You can expand anything on there. This is what we proved for a general condition operator to some degree.

So the eigenfunctions form an orthonormal set that spans the space. So you're going to define that psi 1 with an E1 and psi 2 with an E2, and then this continues. And this is called the spectrum of the theory because energy eigenstates are considered the gold standard. If you want to find solving a theory means finding the energy eigenstates. Because if you find the energy eigenstates, you can solve, you can write any wave function of superposition of energetic states and then just let them evolve.

And the energetic states involve easily because they are just stationary states. So the spectrum of the theory is the collection of numbers that are the allowed energies and of course, the associated eigenfunctions. So the energies may be many, maybe discrete, maybe it has a little bit of continuous partners, all kind of varieties. But your task is to find those for any problem.

So the equation that we're trying to solve is now re-written. We're going to try to solve it. So let's look at it. It's a second order differential equation with a potential in general. So we had an example there. It's there. It's boxed. So we'll write it slightly different, remove the potential to the right-hand side and get rid of the constants here.

So d x squared is equal to 2m over h squared. So this is the equation we have to solve. So whenever you have a problem, you may encounter a potential, v of x. And the question is how bad this potential can be. Well, the potential may be nice and simple, or it may be nice but then has some jumps. It may have infinite jumps, like a potential is a complete barrier, or it may have delta functions. all these are v of x equal possibles. All of them.

Many things can happen with a potential. In fact, the potential can be as strange as you're one, depending on what problems you want to solve. So it's your choice.

Now, we're going to accept, in fact, all of those potentials for our analysis. May be nice and smooth. There may have discontinuities. It may have infinite discontinuities, and worse things like delta function. But worse things than that we will ignore, and there are worse things than that. Maybe a potential discontinues at every point, or maybe a potential has delta functions and derivatives of delta functions. Or potentials that blow up and do all kinds of things.

And I'm not saying you should never consider that. I'm saying that we don't know of any very useful case where you get anything interesting with that. But a conceivable a particular time a singular potential one day could be used. So we'll look at these potentials and try to understand how to set up boundary conditions. And we're going to worry about basically psi and how does it behave.

And my first claim is that psi of x has to be continuous. So psi of x cannot jump. The wave function move along but cannot jump. And the reason is a differential equation. Look, if psi of x was not continuous, if psi of x was like this, and just had a discontinuity, psi of x equal to x, psi prime of x would contain a delta function and this is continuity. The derivative is infinite.

And psi double prime of x, the second derivative, would have a derivative of a delta function which is worse because a delta function, we think of it as a spike that is becoming thinner and higher, but the derivative of the delta function first goes to infinity and then goes to minus infinity and then comes back up. It's much worse in many ways.

And look, if you have this differential equation and psi is not continuous, well, the right-hand side is not continuous. Or if you have a delta function, then something not continuous, but left-hand side, we've had a derivative of a delta function that is nowhere on the right-hand side.

On the right-hand side, the worst that could exist is a delta function in v of x. But the derivative of a delta function doesn't exist. So you cannot afford to have a psi that is discontinuous. Psi has to be continuous.

There's other ways to argue this. You might put them in your notes, but I'll leave it like that. Now how about the next case? I will say the following happens too. Sine prime of x is continuous unless v of x has a delta function. You see, potentials of delta functions are nice, they are interesting. We will consider that. Delta functions potentials can be attractive potentials, repulsive potentials of [INAUDIBLE].

So I claim now that psi prime of x has to also be continuous. Why are we worrying about psi and psi prime is because you need two conditions whenever you're going to solve this differential equation at an interface, you will need to know psi is continuous and psi prime is continuous because of second-order differential equations.

So suppose psi prime is continuous. Then there is no problem. If psi prime is continuous, the worse that can happen is that the second derivative is discontinuous. And the second derivative is discontinuous could happen with a potential of this discontinuous, so one problem if psi prime is continuous.

But psi prime can fail to be continuous if the potential has a delta function. And let's see that. If psi prime is discontinuous, then psi double prime is proportional to a delta function.

If psi prime is discontinuous, double prime is proportional to a delta function. But here psi just takes some value-- there's nothing strange about it-- in order to have delta function, which is psi double prime. To be equal to the right-hand side, v of x must have a delta function. And v will have a delta function.

So it will be a somewhat similar potential, but we're going to look at them in about a week from now. But this will be our guidance to solve problems. The continuity of the wave function and the continuity of the derivative of the wave function. And for this slightly more complicated problems in which the potential has a delta function, then you will have a discontinuity in psi prime, and it will be calculable, and it's manageable, and it's all very nice.

Now, we do it a little complicated, and everything is mixed up, but you will see that it's quite doable.