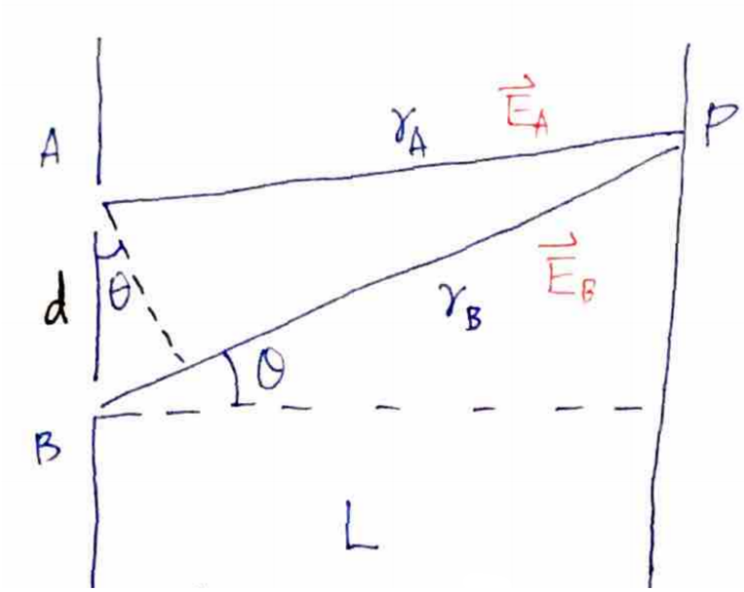
8.03 Lecture 21

Last lecture:
*Thin film interference: We explained why soap bubbles are colorful.
*We will learn about:

1. Interference phenomenon with double-slit experiment: laser, water ripple
2. How phased radar works (radio waves 3 kHz – 300 GHz)
3. Connection to quantum mechanics

*Reminder: Huygens Principle:
All points on a wave front become a source of a spherical waves.

That works for odd spatial dimension and can be derived from Maxwell's equations.
Last time: Double-Slit experiment:



$$\text{where } L \gg d$$

Optical path length difference:

$$r_B - r_A = d\sin\theta$$

Phase difference:

$$\delta = \frac{d\sin\theta}{\lambda}2\pi = (d\sin\theta)k$$

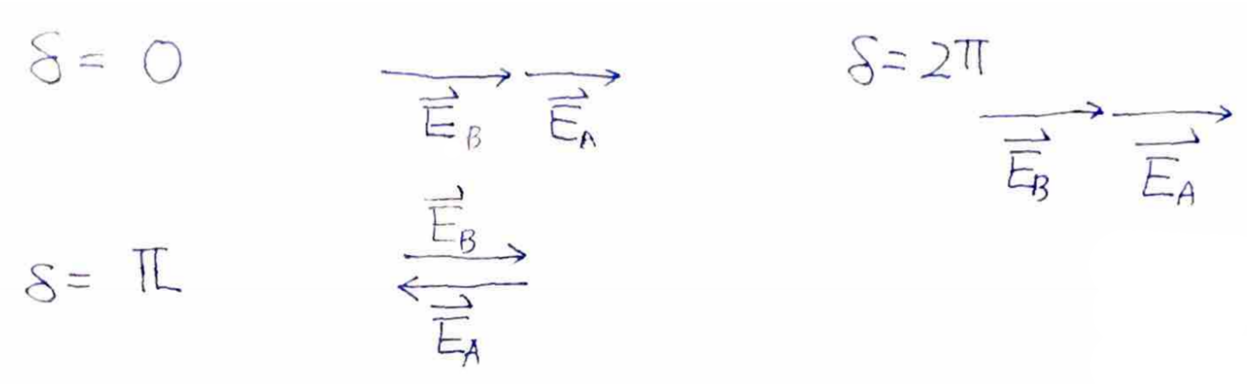
What is the resulting intensity?
First: write down the electric field in complex notation.

$$\begin{aligned}
\vec{E} = \vec{E}_A + \vec{E}_B &= \left(E_0 e^{i(\omega t - kr_A)} + E_0 e^{i(\omega t - kr_B)}\right)\hat{z} \\
&= E_0 e^{i(\omega t - kr_A)}\left[1 + e^{-i\delta}\right]\hat{z} \\
&= E_0 e^{i(\omega t - kr_A)} e^{-i\delta/2}\Big[\underbrace{e^{+i\delta/2} + e^{-i\delta/2}}_{=2\cos(\delta/2)}\Big]\hat{z}
\end{aligned}$$

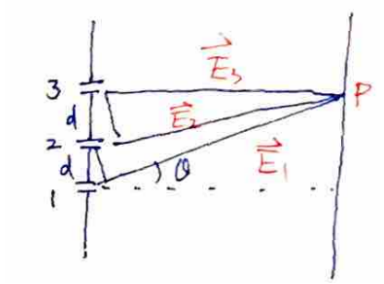
$$\begin{aligned}
\langle I \rangle &\propto |\vec{E}|^2 = E \cdot E^* \propto \cos^2\left(\frac{\delta}{2}\right) \\
\langle I \rangle &= A\cos^2\left(\frac{\delta}{2}\right)
\end{aligned}$$

Where A is the intensity at $\delta = 0$

$$\sin\theta = \frac{\lambda}{2\pi d}\delta$$

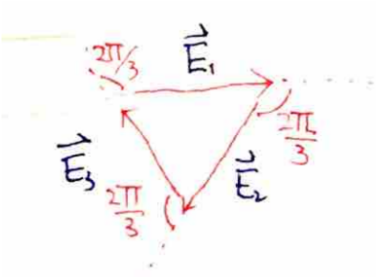


Now we have the knowledge we need to understand how radars work!! Consider a triple-slit interference experiment:



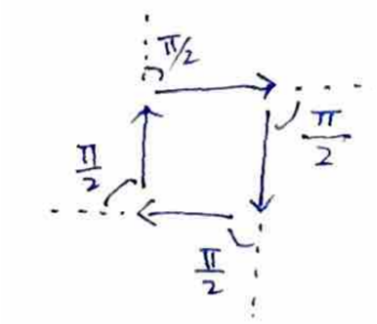
$$\delta_{12} = \delta_{23} = d \sin\theta = \delta$$

What is the required minimum δ to have destructive interference?



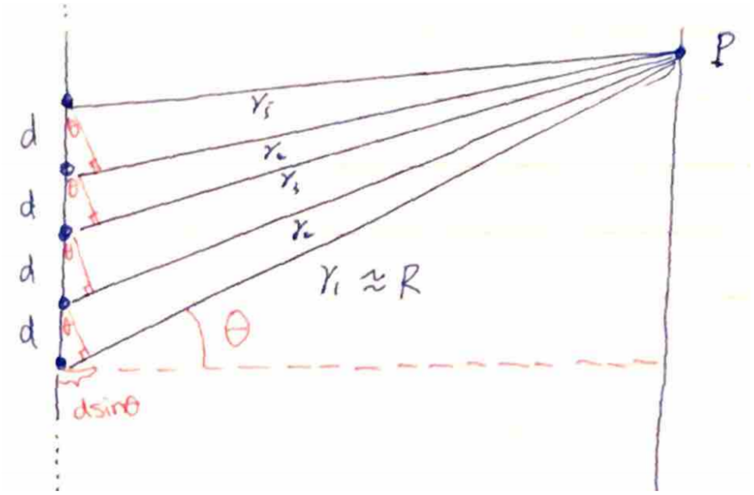
$$\Rightarrow \delta = \frac{2\pi}{3}$$

How about 4-slit?



$$\Rightarrow \delta = \frac{2\pi}{4} = \frac{\pi}{2}$$

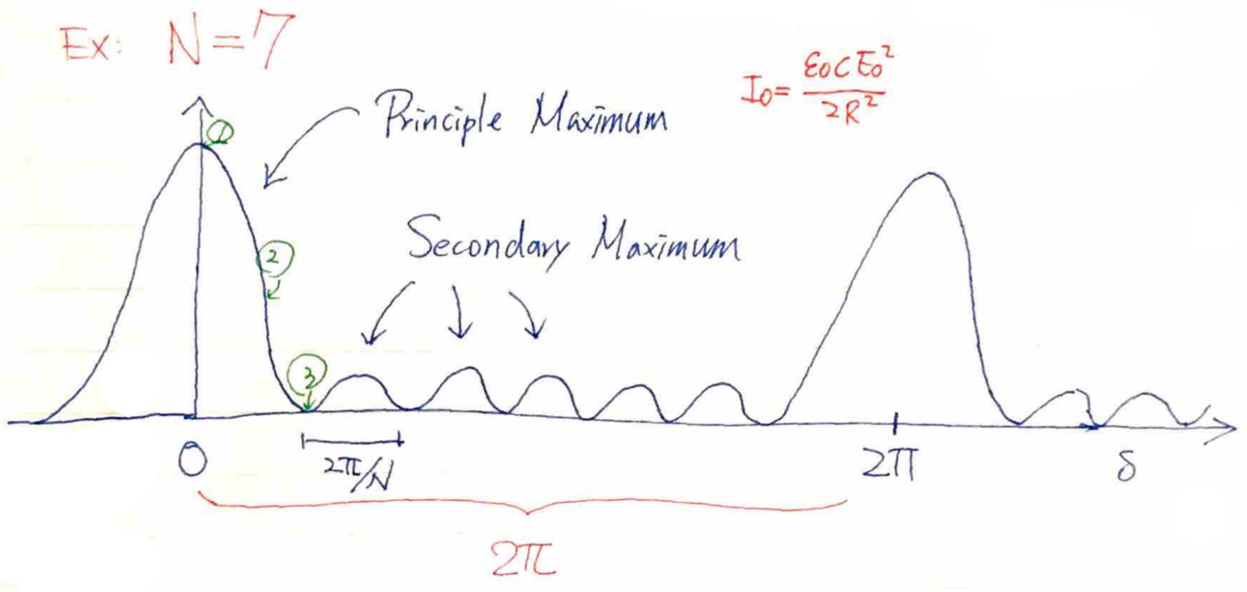
For a 5-slit experiment the minimum delta would be $2\pi/5$ and so on. You can see that the width of the intensity peak is DECREASING as we increase the number of slits!
N-slit (N source) interference:

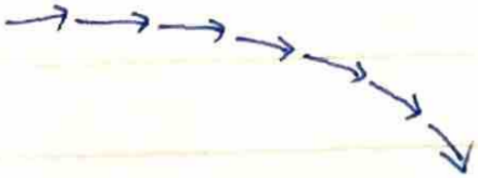
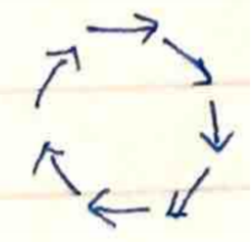


$$\delta = d\sin\theta \cdot k$$

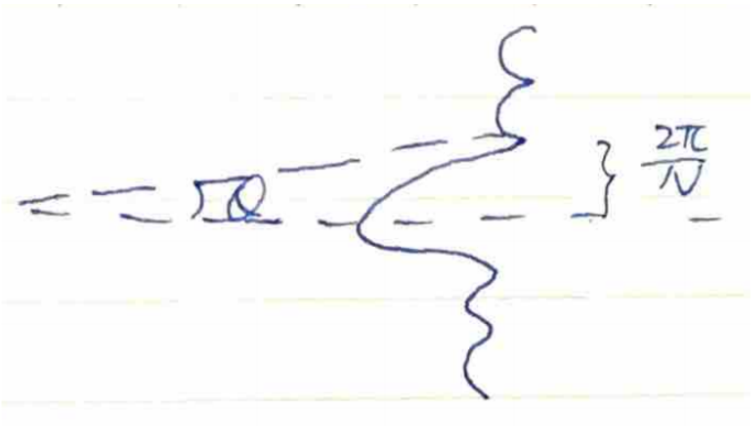
$$\begin{aligned}
E_{\text{Total}} &= E_0\left[e^{i(\omega t-kR)} + e^{i(\omega t-kR-\delta)} + e^{i(\omega t-kR-2\delta)} + \cdots + e^{i(\omega t-kR-(N-1)\delta)}\right] \\
&= E_0 e^{i(\omega t-kR)} \underbrace{\left[1 + e^{-i\delta} + e^{-2i\delta} + \cdots + e^{-(N-1)i\delta}\right]}_{=\sum_{m=0}^{N-1}(e^{-i\delta})^m} \\
&= E_0 e^{i(\omega t-kR)}\left(\frac{1-e^{-i\delta N}}{1-e^{-i\delta}}\right) \\
&= E_0 e^{i(\omega t-kR)}\left(\frac{e^{-i\delta N/2}(e^{+i\delta N/2}-e^{-i\delta N/2})}{e^{-i\delta/2}(e^{+i\delta/2}-e^{-i\delta/2})}\right) \\
&= E_0 e^{i(\omega t-kR)} e^{-i(\delta(N-1)/2)}\frac{\sin(N\delta/2)}{\sin(\delta/2)}
\end{aligned}$$

$$\langle I\rangle \propto |\vec{E}|^2 = \vec{E}\cdot\vec{E}^* \;\Rightarrow\; \langle I\rangle = I_0\left[\frac{\sin(N\delta/2)}{\sin(\delta/2)}\right]^2$$



1. At $\delta = 0$ 
2. As we increase δ 
3. $\delta = \dfrac{2\pi}{N}$ 

N-radiators $\Rightarrow N-2$ secondary maximum. Width of principle maximum: $\dfrac{4\pi}{N} \propto \dfrac{1}{N}$
Corresponding resolution:

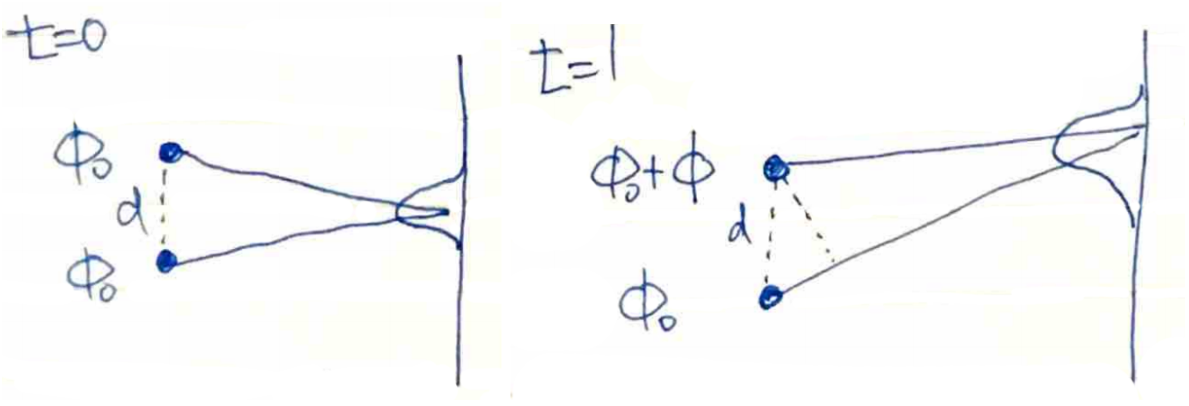


$$\frac{d\sin\theta}{\lambda} = \frac{2\pi}{N} \qquad \sin\theta = \frac{2\pi\lambda}{Nd}$$

We learn that: to get high resolution (i.e. small θ)

1. Use small λ
2. Large d
3. Large N

Sweep?
If we want a sweep frequency ϕ we add additional phase difference between the sources $\Delta\phi = \phi \cdot t$

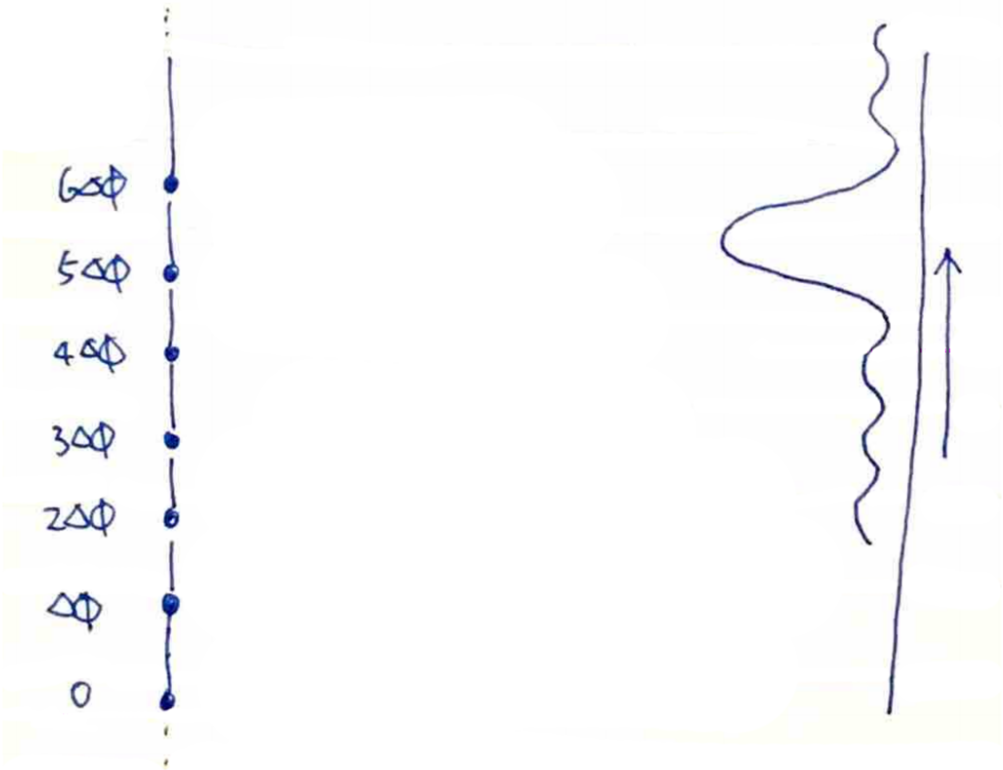


$$\delta = \frac{2\pi}{\lambda} d\sin\theta - \phi \cdot t$$

Where the first term is the phase difference from the optical path length and the second term is the additional phase difference from the source. The Principle Maximum happens at $\delta = 0$ or

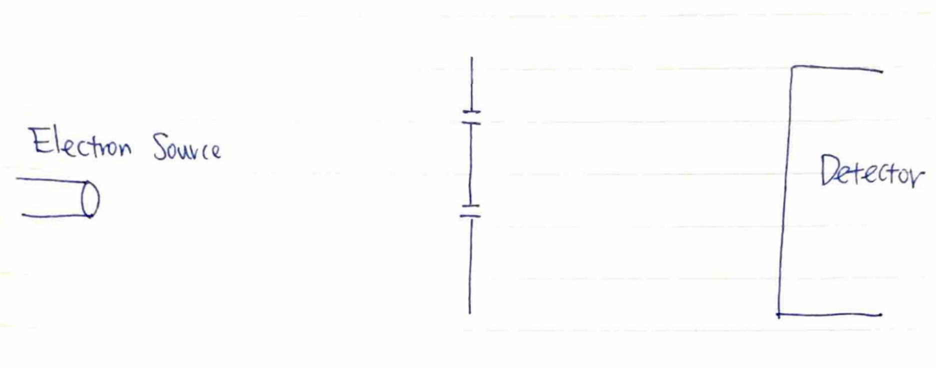
$$\sin\theta = \frac{\phi t \lambda}{2\pi}$$

N source Phased Radar:

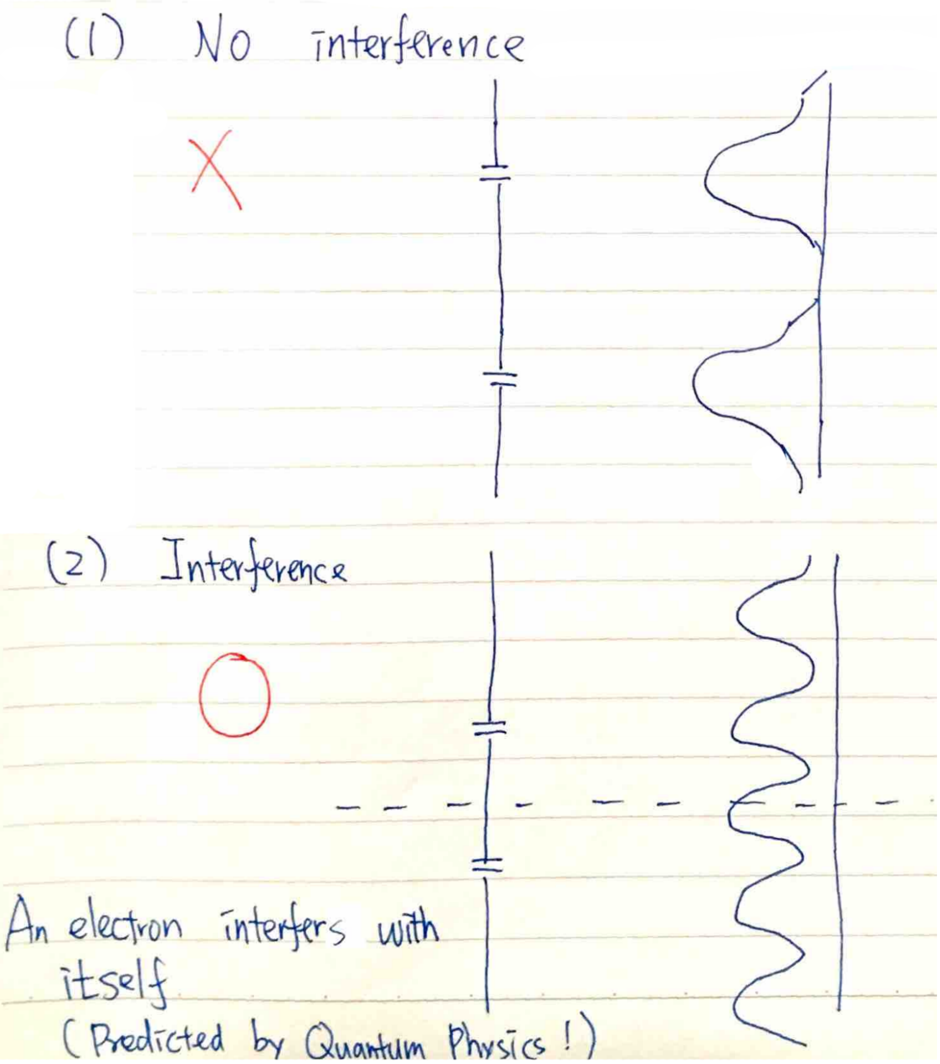


Where we have the additional phase differences on the left which change the direction of the principle maximum. We see interference: light, water, sound, ...

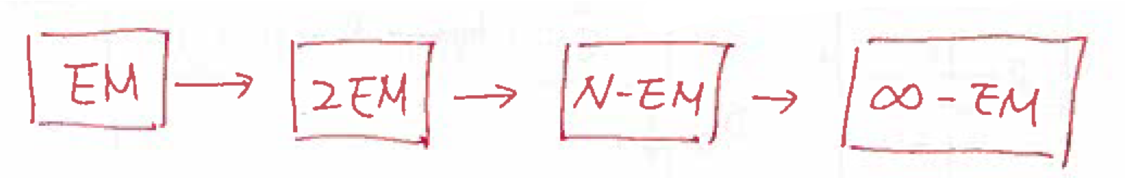
Single Electron Experiment:



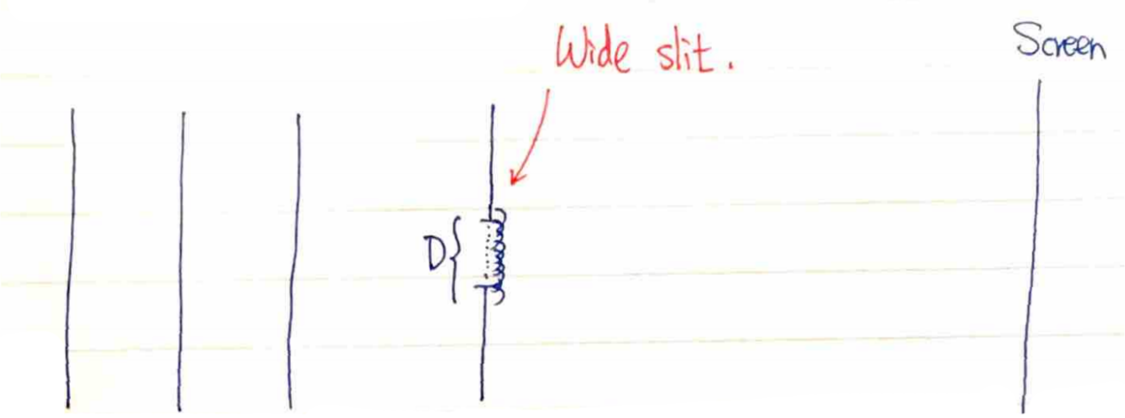
Emit one electron every time.



We learned the interference of two EM waves to N EM waves.



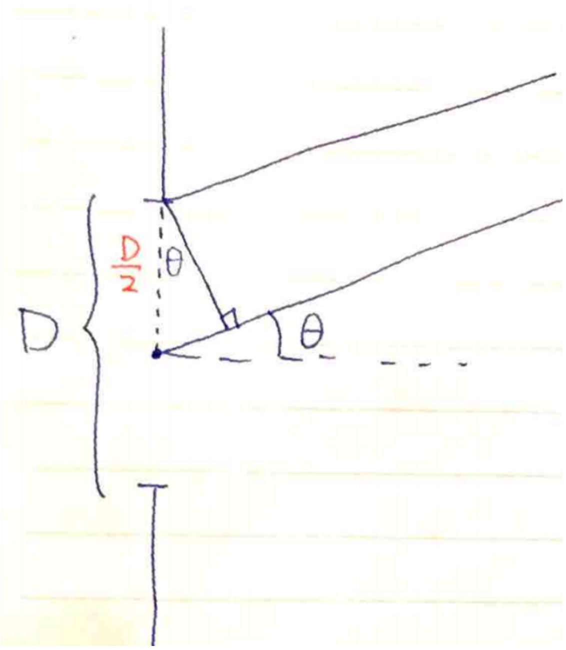
We call the interference of infinite number of EM waves “diffraction”.



We have ∞ point like spherical EM wave sources. This situation: we will see the “interference” between all the spherical wave sources.

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage.

Usually we use “interference” when we are talking about a few sources and “diffraction” when we are talking about many sources.

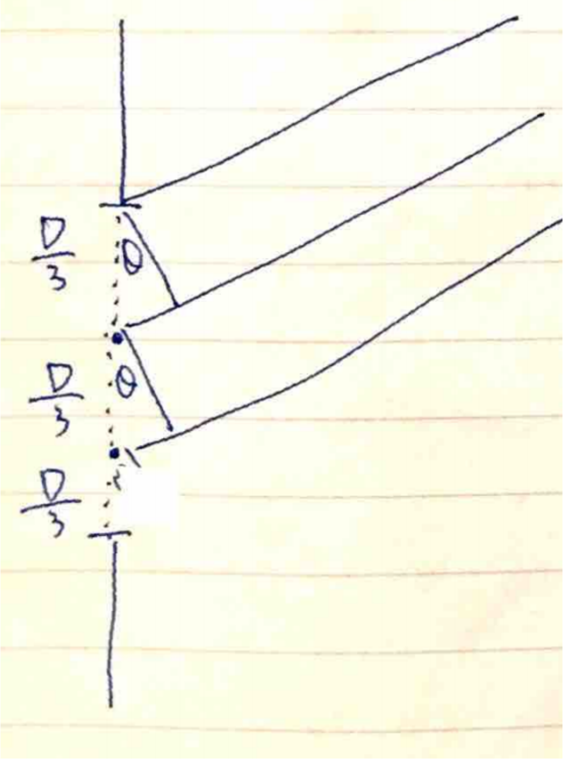


$$\delta = \frac{D}{2}\sin\theta\frac{2\pi}{\lambda}$$

Destructive interference: $\delta = \pi$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{D} \cdots \text{minimum!}$$

We can also divide the slit into 3 pieces.



Destructive

$$\delta = \frac{D}{3}\sin\theta\frac{2\pi}{\lambda} = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\Rightarrow \sin\theta = \frac{\lambda}{D}, \frac{2\lambda}{D}$$

$$\vdots$$

Divide into N pieces

$$\Rightarrow \sin\theta = \frac{\lambda}{D}, \frac{2\lambda}{D}, \cdots, \frac{(N-1)\lambda}{D}$$

8.03SC Physics III: Vibrations and Waves
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