Massachusetts Institute of Technology

Physics 8.03SC Fall 2016

Homework 3

Problems

Problem 3.1 (20 pts)

Here we consider a double pendulum, each with one degree of freedom, as shown in the figure above. Mass M_1 is connected by a massless rigid rod of length L to a fixed origin. Its x-coordinate is X_1 and it makes an angle θ_1 with respect to the vertical (y-axis). Mass M_2 is connected by a massless rigid rod of length L to mass M_1. Its x-coordinate is X_2 and it makes an angle θ_2 with respect to the vertical direction.

You may assume that the 'hinges' at the origin and on mass M_1 are frictionless. Gravity points downward in the y-direction. You may assume that all oscillations are small, i.e., that θ_1 and θ_2 are small and terms of order $O(\theta^2)$ are negligible.

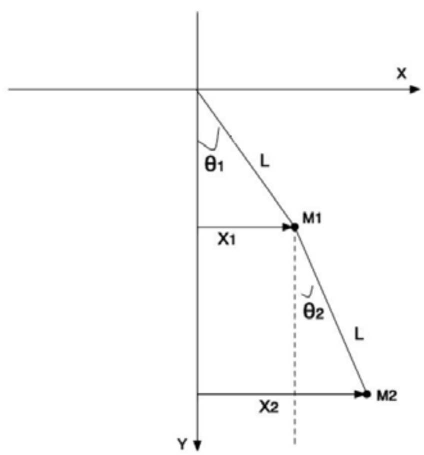

Figure 1: Two circuits

a. Write down the equations of motion for the two oscillating masses.

b. Determine the two normal mode frequencies of oscillation in the small amplitude limit as a function of ω_0, and α, where $\omega_0^2 = \frac{g}{L}$ and $\alpha = \frac{M_2}{M_1}$. (You don't have to solve the corresponding amplitude ratios.)

c. What are the two normal mode frequencies if $\alpha \to \infty$? Do your results make sense?

d. What are the two normal mode frequencies if $\alpha \to 0$? Do your results make sense?

Problem 3.2 (20 pts)

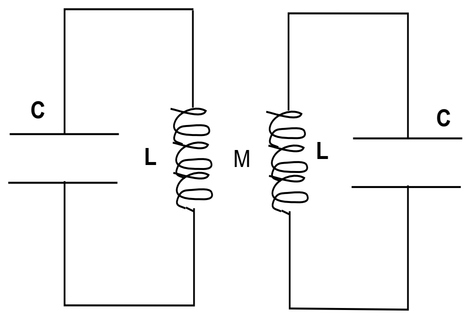

Figure 2: Two circuits

Consider two identical LC circuits as shown in Figure 2. The two inductors are brought close together such that their mutual inductance M results in a coupling between the currents flowing in the two circuits.

a. Find the frequencies of normal modes as a function of given parameters.

b. What current patterns correspond to these normal modes? Could you use symmetry arguments to discover these modes?

Problem 3.3 (20 pts)

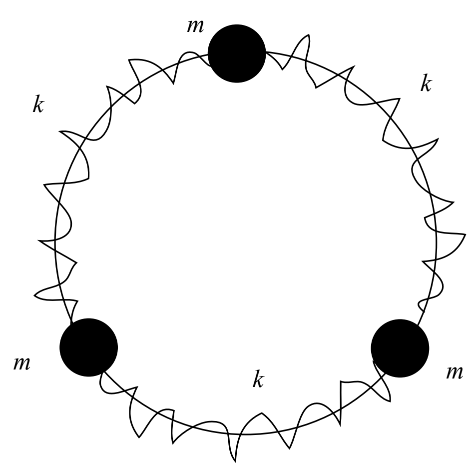

Figure 3: Three Masses on Circle

Consider three identical masses constrained to move on a frictionless circle. The masses are connected with identical springs each with spring constant k (see Figure 3). The circle is large such that you can ignore any effects related to the curvature. The circle is horizontal such that gravity can be ignored.

a. Find equations of motion for the three masses in terms of the small displacements from the equilibrium position of each mass.

b. Determine the frequencies and the relative amplitudes for each of the normal modes. Make a simple sketch of the motion of the masses for each of the normal modes. How many different frequencies are there in this system?

c. Because the masses are connected in a circle some of the results of normal mode calculations do not correspond to oscillatory motion. Explain why.

Problem 3.4 (20 pts)

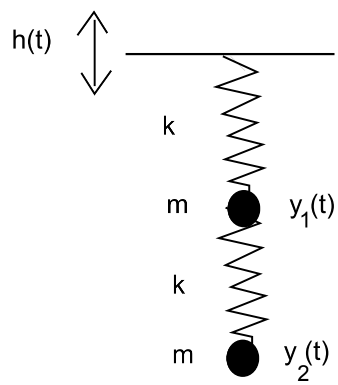

Figure 4: Two Masses Hanging from an Oscillating Support

Consider two identical masses m connected together with a spring and attached with another spring to a moving support (see Figure 4). The support is oscillating vertically and its position is given by $h(t) = A\cos(\omega t)$. The Hooke constant of the two identical springs is k. Ignore effects of damping.

a. Find coupled differential equations that govern displacements from equilibrium of the masses $y_1(t)$ and $y_2(t)$. Express your results in terms of $\omega_0^2 = \frac{k}{m}$. Note that the effect of gravity results in a shift of equilibrium position but it does not affect the harmonic motion.

b. Find the steady state response of the positions of two masses $y_1(t)$ and $y_2(t)$. Make a careful sketch of the amplitude as a function of the driving frequency ω for each of the masses.

c. By inspecting the results of b), give the frequencies and amplitude ratios for the normal modes of the undriven system.

Problem 3.5 (20 pts)

Two identical beads, each of mass m, are equally spaced along a massless string of length $3a$ (see Figure 5). Consider the system to be on a frictionless horizontal surface. Initially both ends of the string are fixed

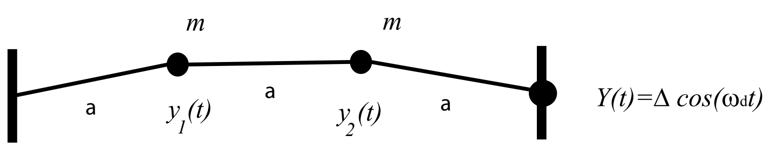

Figure 5: Two Beads

($\Delta = 0$). Assume that the string is under tension T at all times. Beads can execute small amplitude oscillations perpendicular to the string (displacements are exaggerated in the Figure!).

a. Find equations of motion for the two beads in terms of displacement from the equilibrium $y_1(t)$, $y_2(t)$.

b. Find and sketch the motion of the normal modes and calculate the normal mode frequencies for the system.

Assume now that the rightmost attachment point undergoes harmonic oscillation $y(t) = \Delta \cos(\omega_d t)$.

c. Find the steady state amplitude of the motion of the two masses as a function of the driving frequency ω_d and the amplitude Δ.