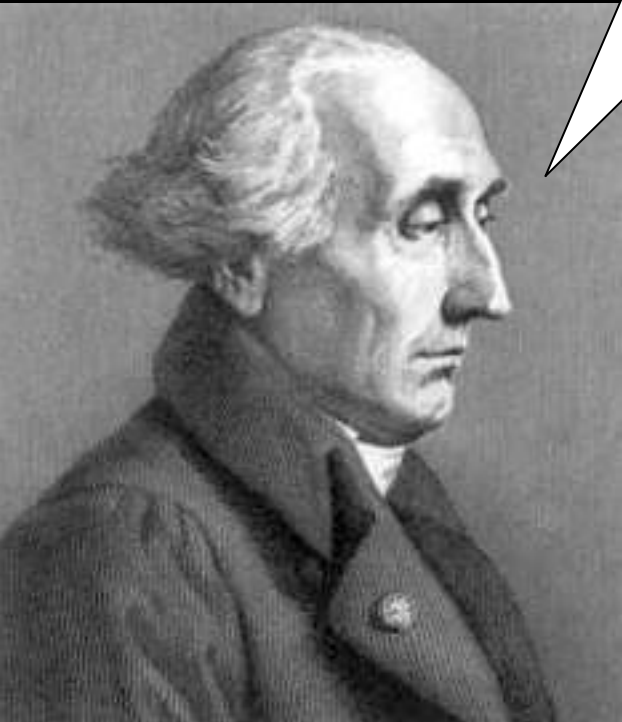


Welcome
back
to 8.033!



Joseph Louis Lagrange, French
(1736-1813)

Summary of last two lectures: Variational calculus

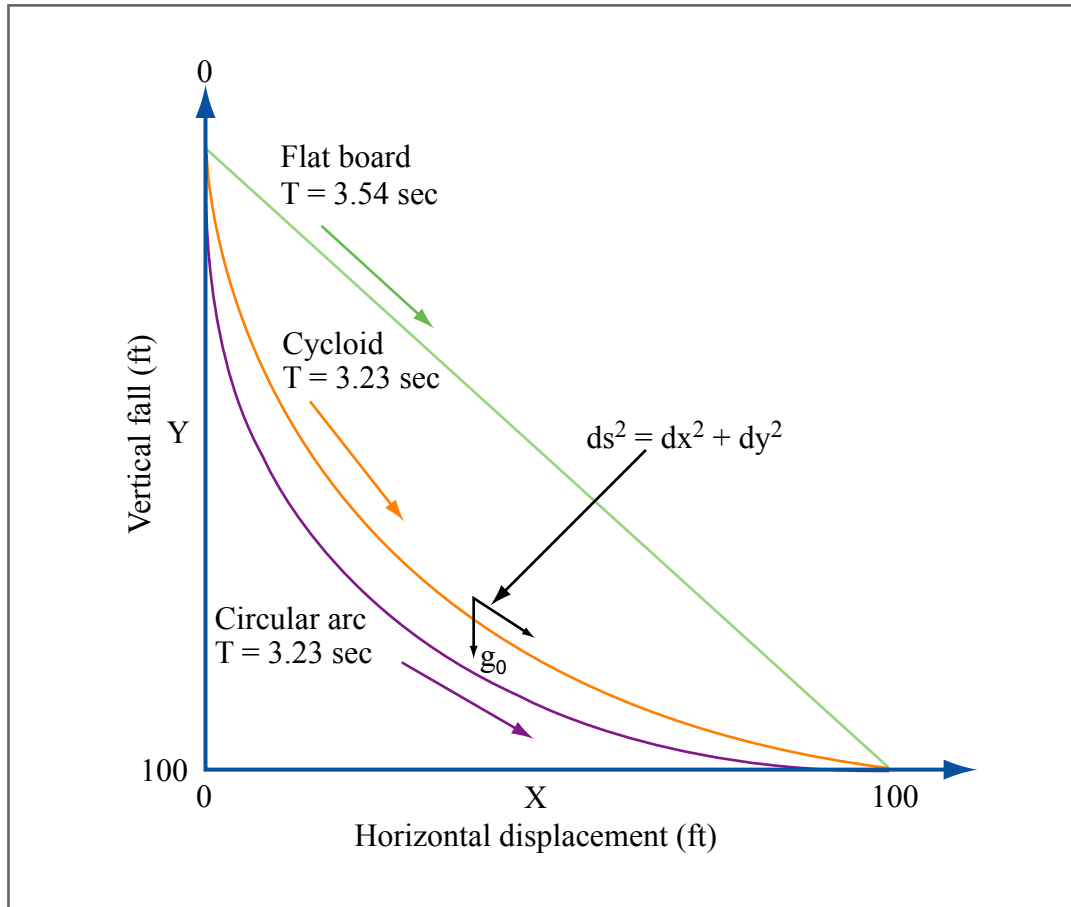


Figure by MIT OCW.

MIT Course 8.033, Fall 2006, Lecture 9

Max Tegmark

Today:

- Using the Euler-Lagrange equation
 - brachistochrone problem
 - catenary
- Midterm review

Photograph of a brachistochrone experiment. Image removed due to copyright restrictions.

The Standard Model Lagrangian

$$\begin{aligned}
 & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^b g_\mu^c - \frac{1}{4}g_s^2 f^{abcd} f^{abcd} g_\mu^a g_\nu^b g_\mu^c g_\nu^d + \\
 & \frac{1}{2}ig_s^2 (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \right. \\
 & \left. \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h - igc_w [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\mu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - \\
 & W_\nu^- \partial_\nu W_\mu^+)] - ig s_w [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - \\
 & W_\nu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \\
 & \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - Z_\mu^0 Z_\nu^0 W_\nu^+ W_\mu^-) + \\
 & g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha [H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
 & \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
 & g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig [W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
 & W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \\
 & \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
 & ig s_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
 & ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
 & \frac{1}{4}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma \partial \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + \\
 & m_d^\lambda) d_j^\lambda + ig s_w A_\mu [-\bar{e}^\lambda \gamma e^\lambda + \frac{2}{3}(\bar{u}_j^\lambda \gamma u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma d_j^\lambda)] + \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \\
 & \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) u_j^\lambda) + \\
 & (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1 + \\
 & \gamma^5) C_{\lambda e} d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\lambda C_{\lambda e}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)] + \\
 & \frac{ig}{2\sqrt{2}} \frac{m_\lambda^2}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \frac{g}{2} \frac{m_\lambda^2}{M} [H (\bar{e}^\lambda e^\lambda) + \\
 & i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\lambda (\bar{u}_j^\lambda C_{\lambda e} (1 - \gamma^5) d_j^\lambda) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda e} (1 + \\
 & \gamma^5) d_j^\lambda) + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}_j^\lambda C_{\lambda e}^\dagger (1 + \gamma^5) u_j^\lambda) - m_u^\lambda (\bar{d}_j^\lambda C_{\lambda e}^\dagger (1 - \gamma^5) u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + \\
 & igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + \\
 & igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + \\
 & igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \partial_\mu \bar{X}^- X^+) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^- - \partial_\mu \bar{X}^- X^+) - \\
 & \frac{1}{2}g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} ig M [\bar{X}^+ X^0 \phi^+ - \\
 & \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} ig M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + ig M s_w [\bar{X}^0 X^- \phi^+ - \\
 & \bar{X}^0 X^+ \phi^-] + \frac{1}{2}ig M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$



(From T.D. Gutierrez)