Massachusetts Institute of Technology

Department of Physics

Physics 8.033 21 November 2006

Quiz 2

Name: (Last, First) ______________________________ (please print).

Recitation number (circle one): 1 2 3

- Record all answers and show all work in this exam booklet. If you need extra space, use the back of the page.
- All scratch paper must be handed in with the exam, but will not be graded.
- This exam is closed book. You may use your handwritten notes if they are clearly labeled with your name and you hand them in with your exam.
- Whenever possible, try to solve problems using general analytic expressions. Plug in numbers only as a last step.
- Please make sure to answer all sub-questions.
- Good Luck!

Problem	Max	Grade	Grader
1	25		
2	25		
3	25		
4	25		
Total	100		

Question 1: Quick answers [25 Points]

(a). **(5 pts)** Consider the following reactions:

- (A) ______________ A 100 MeV photon decays into an electron-positron pair. *answer:*E
- (B) ______________ A neutron decays into an electron-positron pair and a photon. *answer:*B
- (C) ______________ A neutron decays into a proton, an electron and a neutrino.*answer:*L
- (D) ______________ A proton decays into a neutron, a positron and a neutrino. *answer:*E
- (E) ______________ A neutron decays into a proton and a photon.*answer:*Q

For each one, write **one** of the letters from the option list below.

- L violates lepton number conservation
- B violates baryon number conservation
- P violates parity conservation
- E violates energy-momentum conservation
- Q violates charge conservation
- N violates none of the above conservation laws

(b). **(9 pts)** Give each of the following quantities to the nearest power of 10 (don't show calculations, being off by one power of 10 is OK):

- (A) ______________ Age of our universe when most He nuclei were formed *answer:*1 min
- (B) ______________ Age of our universe when hydrogen atoms formed.*answer:*400000 yrs
- (C) ______________ Age of our universe today. *answer:*10 Gyr
- (D) ______________ Number of stars in our Galaxy. *answer:*1e11
- (E) ______________ Light travel time to closest star (Sun!:) in minutes. *answer:*8
- (F) ______________ Hydrogen binding energy in eV/c^2 .*answer:*13.6
- (G) ______________ Electron mass in eV/c^2 .*answer:*500000
- (H) ______________ Neutron mass in eV/c^2 .*answer:*10^9
- (I) ______________ Light travel time to 2nd closest star in years.*answer:*3

(c). **(9 pts)** Indicate whether each of the following statements are true of false.

- (A) TRUE / FALSE If our Universe is only X billion years old, then we can only see objects that are now less than X billion light years away *answer:*F
- (B) TRUE / FALSE Space must be infinite, because it cannot end with a boundary without more space on the other side. *answer:*F
- (C) TRUE / FALSE Leptons do not feel the strong interaction. *answer:*T
- (D) TRUE / FALSE No experiment inside an isolated sealed lab in space can distinguish between whether it is uniformly accelerating or in a uniform gravitational field. *answer:*T
- (E) TRUE / FALSE A clock by the ceiling runs faster than one by the floor. *answer:*T
- (F) TRUE / FALSE Hubble's law implies that the Big Bang was an explosion localized near the comoving position of our Galaxy.
- (G) TRUE / FALSE The expansion of our galaxy is governed by the Friedmann equation. *answer:*F
- (H) TRUE / FALSE Two galaxies can recede from each other faster than the speed of light. *answer:*T
- (I) TRUE / FALSE We know that our entire observable universe was once at infinite density *answer:*F

(d). **(2 pts)** A tritium (H^3) nucleus contains ______ up quarks and ______ down quarks. *answer:*p+2n = 4 + 5

Question 2: The Proton-Proton Cycle `[25 Points]`

In the Sun, one of the processes in the He fusion chain is $p+p+e^- \to d+\nu$, where d is a deuteron. Make the approximations that the deuteron rest mass is $2m_p$, and that $m_e \approx 0$ and $m_\nu \approx 0$, since both the electron and the neutrino have negligible rest mass compared with the proton rest mass m_p.

(a). For the arrangement shown in the figure, where (in the lab frame) the two protons have the same energy γm_p and impact angle θ, and the electron is at rest, calculate the energy E_ν of the neutrino in the rest frame of the deuteron in terms of θ, m_p and γ.

*answer:*Use the fact that the quantity $E^2 - p^2c^2$ is invariant. In the deutron's rest frame, after the collison:

$$E^2 - p^2c^2 = (2m_pc^2 + E_\nu)^2 - E_\nu^2 \tag{2.1}$$

$$= 4m_p^2c^4 + 4m_pc^2E_\nu = 4m_pc^2(m_pc^2 + E_\nu) \tag{2.2}$$

In the lab frame, before collison:

$$E^2 - p^2c^2 = (2E_p)^2 - (2p_p \cos\theta c)^2 \tag{2.3}$$

$$= (2\gamma m_pc^2)^2 - (2\gamma\beta m_p \cos\theta c^2)^2 \tag{2.4}$$

Use $\gamma^2\beta^2 = (\gamma^2 - 1)$ in the second term and simplify the algebra to find

$$E^2 - p^2c^2 = 4m_p^2c^4(\gamma^2 - (\gamma^2 - 1)\cos^2\theta) \tag{2.5}$$

Equating the invariants in the two frames, we have

$$4m_pc^2(m_pc^2 + E_\nu) = 4m_p^2c^4(\gamma^2 - (\gamma^2 - 1)\cos^2\theta) \tag{2.6}$$

$$\Rightarrow E_\nu = m_pc^2(\gamma^2 - 1)\sin^2\theta \tag{2.7}$$

(b). For the special case where the deuteron remains at rest in the lab frame and $\theta = 30°$, solve for γ and calculate the energy of all particles (the deuteron, the neutrino, one of the protons) in terms of the proton rest mass m_p.

*answer:*The deutron's rest frame *is* the lab frame. Also, $\theta = 30°$. Use conservation of energy, along with the result from the previous part to find:

$$2\nu m_pc^2 = 2m_pc^2 + E_\nu \tag{2.8}$$

$$= 2m_pc^2 + m_pc^2(\gamma^2 - 1)/4 \tag{2.9}$$

$$\Rightarrow 2\gamma = 2 + \gamma^2/4 - 1/4 \tag{2.10}$$

$$\Rightarrow \gamma = 7, 1 \tag{2.11}$$

$\gamma = 1$ is obviously not the solution. Thus, $\gamma = 7$ and the energies are: $E_p = 7m_p, E_\nu = 12m_p, E_d = 2m_p$

Question 3: Coulomb's Law generalized [25 Points]

In an inertial fram S, the position $\mathbf{r}_q$ of a point charge q moves according to $\mathbf{r}_q(t) = v\hat{\mathbf{z}}t$, i.e. with velocity v in the $\hat{\mathbf{z}}$-direction, passing the origin at $t = 0$. In the moving frame S' where the charge is at rest at the origin, Coulomb's law states that the electric field is

$$E' = A\frac{\mathbf{r'}}{r'^3},$$

where $A = q/4\pi\varepsilon_0$. Show that in the frame S, the electric field at $t = 0$ is s

$$\mathbf{E} = A\frac{(1-\beta^2)}{(1-\beta^2\sin^2\theta)^{3/2}}\frac{\mathbf{r}}{r^3},$$

where θ is the usual polar angle ($z = r\cos\theta$, $x^2 + y^2 = r^2\sin^2\theta$).

*answer:*Let us convert all quantities to the cartesian coordinates. In the frame S', the components of the electric field are:

$$\vec{E'} = E'_x\hat{x'} + E'_y\hat{y'} + E'_z\hat{z'} \tag{3.1}$$

$$E'_x = \frac{Ax'}{(x'^2 + y'^2 + z'^2)^{3/2}} \tag{3.2}$$

$$E'_y = \frac{Ay'}{(x'^2 + y'^2 + z'^2)^{3/2}} \tag{3.3}$$

$$E'_z = \frac{Az'}{(x'^2 + y'^2 + z'^2)^{3/2}} \tag{3.4}$$

We can now Lorentz transform the fields and coordinates from S' to S. First the coordinates,

$$x = x' \tag{3.5}$$

$$y = y' \tag{3.6}$$

$$\gamma z = z' \tag{3.7}$$

and then the fields,

$$E_x = \gamma E'_x = \frac{A\gamma x}{(x^2 + y^2 + \gamma^2 z^2)^{3/2}} \tag{3.8}$$

$$E_y = \gamma E'_y = \frac{A\gamma y}{(x^2 + y^2 + \gamma^2 z^2)^{3/2}} \tag{3.9}$$

$$E_z = E'_z = \frac{A\gamma z}{(x^2 + y^2 + \gamma^2 z^2)^{3/2}} \tag{3.10}$$

Note that the primed coordinates have been converted to the unprimed ones using the coordinate transformation. The total magnitude for the electric field in the S frame can be obtained from

$$E^2 = E_x^2 + E_y^2 + E_z^2 \tag{3.11}$$

$$= \frac{A^2\gamma^2 r^2}{(x^2 + y^2 + \gamma^2 z^2)^3} \tag{3.12}$$

$$= \frac{A^2 r^2}{(1-\beta^2)(x^2 + y^2 + z^2/(1-\beta^2))^3} \tag{3.13}$$

$$\Rightarrow E = \frac{A(1-\beta^2)}{r^2(1-\beta^2\sin^2\theta)^{3/2}} \tag{3.14}$$

Since the electric field always has to be radial, $\vec{E} = |E|\hat{r}$.

Question 4: Cosmology [25 Points]

(a). **(10 pts)** Consider a particle coasting in the r-direction (i.e., with constant θ and ϕ) in a flat FRW metric, with no non-gravitational forces acting on it. Use variational calculus to prove that $p \propto 1/a$ (here $p = m_0\gamma u$ is its momentum and $u \equiv a\dot{\mathbf{r}}$ is its velocity relative to nearby comoving observers). *answer:*The particle only has a radial motion $\Rightarrow d\theta = d\phi = 0$. Also, the universe is flat $\Rightarrow k = 0$. Thus, the FRW metric becomes:

$$d\tau^2 = dt^2 - a^2 dr^2 \tag{4.1}$$

$$\Rightarrow \Delta\tau = \int d\tau = \int \sqrt{dt^2 - a^2 dr^2} \tag{4.2}$$

$$= \int dt\sqrt{1 - a^2\dot{r}^2} = \int dt f(t, r, \dot{r}) \tag{4.3}$$

The interval $\Delta\tau$ has to be extremized. The Euler lagrange equations give:

$$\frac{\partial f}{\partial r} - \frac{d}{dt}\left[\frac{\partial f}{\partial \dot{r}}\right] = 0 \tag{4.4}$$

$$\Rightarrow \frac{d}{dt}\left[\frac{a^2\dot{r}}{\sqrt{1 - a^2\dot{r}^2}}\right] = 0 \tag{4.5}$$

$$\Rightarrow \frac{a^2\dot{r}}{\sqrt{1 - a^2\dot{r}^2}} = \text{constant} \tag{4.6}$$

Identifying $a\dot{r} = u$ leave us with $\gamma u \propto 1/a \Rightarrow p = m_0\gamma u \propto 1/a$.

(b). **(2 pts)** Given a), the value of u in the limit $a \to 0$ is ________. *answer:*1(c)

(c). **(2 pts)** Given a), the value of u in the limit $a \to \infty$ is ________. *answer:*0

(d). **(2 pts)** Thus relative to comoving observers, your results show that an object without external forces in an expanding universe
(circle one) REMAINS IN UNIFORM MOTION / SLOWS DOWN / ACCELERATES.
*answer:*slows down

(e). **(3 pts)** Starting with the answer from a), derive how the wavelength λ of a photon depends on a. Your answer should be of the form $\lambda \propto$(function of a).
*answer:*For a photon, $p = hk/2\pi \propto 1/\lambda$. So $\lambda \propto 1/p \propto a$.

(f). **(6 pts)** Solve the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

to obtain a solution of the form $a(t) \propto$ (function of t) for the case where space is flat and the density is dominated by photons, and compute the age of the universe at the time when $H^{-1} = 30$ seconds.
*answer:*For a flat universe domainated by photons,

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \propto a^{-4} \tag{4.7}$$

$$\Rightarrow \dot{a} \propto a^{-1} \tag{4.8}$$

$$\Rightarrow a = At^{1/2} \tag{4.9}$$

$$\Rightarrow H = \dot{a}/a = 1/2t \tag{4.10}$$

Thus, $t = H^{-1}/2 = 15 seconds$.