Voltage, Current and Resistance

Electric Charge

There are two types of observed ***electric charge***, positive and negative. Positive and negative are arbitrary names derived from Ben Franklin's experiments. He rubbed a glass rod with silk and called the charges on the glass rod positive. He rubbed sealing wax with fur and called the charge on the sealing wax negative. Like charges repel and opposite charges attract each other. The unit of charge, Q, is called the ***coulomb*** $[C]$. Charge of ordinary objects is quantized in integral multiples $Q = +Ne$ or $Q = -Ne$ where $e = 1.6 \times 10^{-19} C$, and N is some positive integer. The electron carries one unit of negative charge, $q_{electron} = -e$, and the proton carries one unit of positive charge, $q_{proton} = +e$.

Voltage Sources

Batteries, generators, power supplies are devices that convert some other form of energy into electrical energy. When the terminals of a battery are connected to a wire, forces act on charges, and produce a flow of charge in the wire, an electric current. Here the electrical energy comes from chemical reactions inside the battery. There are many sources of electromotive force: solar cells, generators, and alternators are a few examples.

Voltage Difference

The ***voltage difference*** $\Delta V \equiv V_B - V_A$ between points A and B is defined to be the negative of the work done ΔW per charge, q, in moving the charge, q, from any point A to any point B

$$\Delta V = -\frac{\Delta W}{q}.$$

Voltage difference is also called ***electric potential difference.*** The unit of voltage difference is the volt $[V]$.

$$[V] = [volt] = [joule/coulomb] = [J/C].$$

The work done ΔW in the definition of the voltage difference is the work done by the electric force

$$\Delta W = \int_A^B \vec{F} \cdot d\vec{r}$$

Work-Energy

A positive charge free to move will go from a higher potential to a lower potential. Notice that ΔV is negative and $q > 0$, hence the work done by the electromotive force is positive, ($\Delta W = -q\Delta V > 0$). This positive work can be converted to mechanical energy in the form of increased kinetic energy, ($\Delta W = \Delta K$), or converted to heat, ($\Delta W = \Delta Q_{heat}$). A negative charge free to move will go from a lower potential to a higher potential.

Current

Electric currents are flows of electric charge. The ***electric current*** through a wire is defined to be the total net charge flowing across any cross-sectional area of the wire per second. The unit of current is the amp $[A]$ with 1 amp = 1 coulomb/sec. Common currents range from mega-amperes in lightning to nanoamperes in your nerves.

There are two different systems of units, the SI or Système International d'Unités, and the CGS (centimeter, grams, sec). In CGS units, charge is a fundamental quantity. The unit for charge is the electrostatic unit $[esu]$. In the SI system, current is the fundamental quantity, and electric charge is a derived unit. This means that one coulomb is defined as follows. If one amp of current is flowing through a wire, then the total charge that moves across any cross-section of the wire in one second is defined to be one coulomb of charge.

The idea that current, I, is the rate of change of charge, Q, in time can be described mathematically by the relation

$$I = dQ/dt \,.$$

Since flow has a direction, we have implicitly introduced a convention that the direction of current corresponds to the direction positive charges are flowing. Inside wires the flowing charges are negatively charged electrons. So the electrons are flowing opposite to the direction of positive current.

There are many kinds of electric current: direct or alternating, high or low frequency, steady or transient, constant, slowly varying, pulsating or fluctuating. Electric currents flow in conductors: solids (metals, semiconductors), liquids (electrolytes) and ionized gases. Electric currents don't flow (much) in non-conductors or insulators.

Power Supplies

The rate of doing work is called ***power***. A voltage source ΔV that produces a current I has a power output

$$P = \Delta V I \,.$$

Voltage sources are commonly referred to as power supplies. The unit of power is the watt, $[W]$;

$$[W] = [watt] = [volt][amp] = [V][A] \,.$$

Since power is the rate of change of energy with time, the units of watts are also

$$[W] = [watt] = [joule/\sec] = [J/s].$$

Electric Circuits

Electrical circuits connect power supplies to `loads' such as resistors, motors, heaters, or lamps. The connection between the supply and the load is made with insulating wires that are often called `leads' and soldering, or with many kinds of connectors and terminals. Energy is delivered from the source to the user on demand at the flick of a switch. Sometimes many circuit elements are connected to the same lead, which is the called a `common lead' for those elements.

Various parts of circuits, called *circuit elements*, can be in *series* or in parallel, or *series-parallel*. Elements are ***in parallel*** *when* they are connected `across' the same voltage difference (see Figure 1).

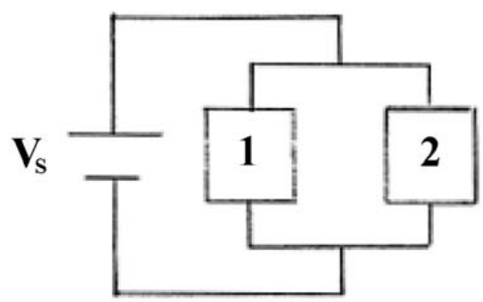


Figure 1: parallel elements

Generally, loads are connected in parallel across the power supply. When the elements are connected one after another, so that the current passes through each element without any branches, the elements are ***in series*** (see Figure 2).

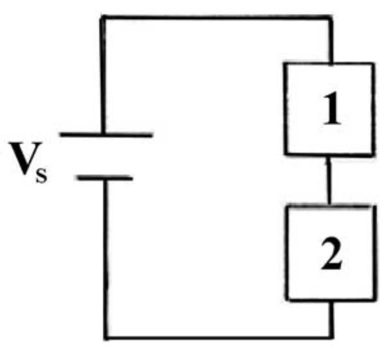


Figure 2: series elements

There are pictorial diagrams that show wires and components roughly as they appear, and schematic diagrams that use conventional symbols, somewhat analogous to road maps.

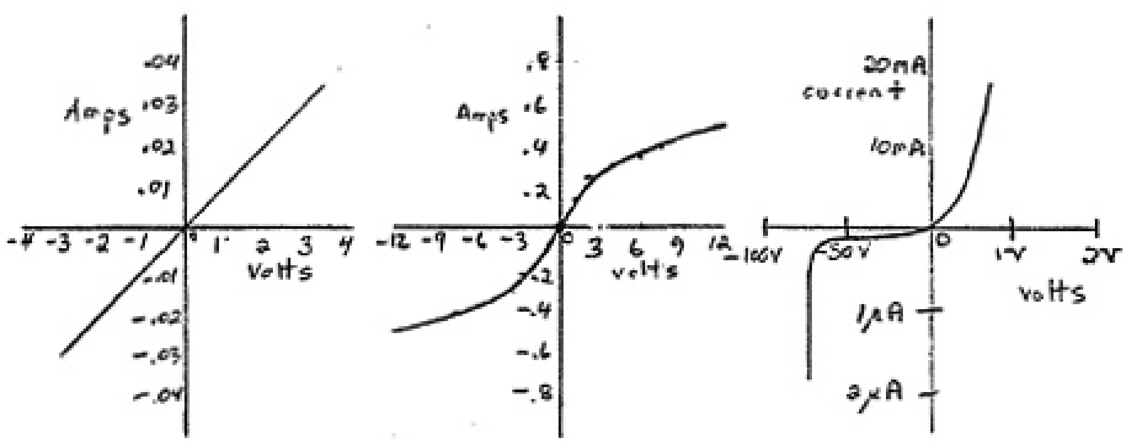
Often there is a switch in series; when the switch is open the load is disconnected; when the switch is closed, the load is connected.

One can have closed circuits, through which current flows, or open circuits in which there are no currents. Sometimes, usually by accident, wires may touch, causing a short circuit. Most of the current flows through the short, while very little will flow through the load. This may burn out a piece of electrical equipment like a transformer. To prevent damage, a fuse or circuit breaker is put in series. When there is a short the fuse blows, or the breaker opens.

In electrical circuits, a point (or some common lead) is chosen as the '*ground'***.** This point is assigned an arbitrary voltage, usually zero, and the voltage *V at any point in the circuit* is defined as the voltage difference between that point and ground.

Resistance and Ohm's Law

When a voltage difference, ΔV, is applied to a circuit element, a current flows through it. The amount of the current is a function of the voltage. The current-versus-voltage relationship ($I - \Delta V$ curve) is an empirical property of the element. Three examples are shown in Figure 3. Figure 3a shows a linear relation when the element is carbon composition resistor, Figure 3b shows a more complicated non-linear relation for the $8W$ filament of the 1157 lamp, and Figure 3c shows the unsymmetrical non-linear relation for a diode.



3a: Carbon composition resistor 3b: Lamp 3c: Diode

Figure 3: $I - \Delta V$ curves for various elements

When the $I - \Delta V$ curve is linear, the ***resistance*** R is defined to be the slope of the curve.

$$R = \frac{\Delta V}{I}.$$

This is known as ***Ohm's Law***, commonly stated as follows: the voltage drop, $\Delta V > 0$, across a resistor is $\Delta V = I\,R$. The unit of resistance is the ohm $[\Omega]$, with $[\Omega] = [\Omega]/[V]$ since (1 ohm) =

(1 volt)/(1 amp). The resistance of a resistor may not be constant but may depend on a number of variables such as temperature or applied voltage.

Power Dissipated by a Resistor

The power dissipated by a resistor as heat, called Joule heating, is given by

$$P = \Delta V I = I^2 R = \frac{\Delta V^2}{R}.$$

Resistors are rated by the power they can safely dissipate.

Current Conservation

A node is a point in a circuit where three or more elements are soldered together. At any point where there is a junction between various current carrying branches, the sum of the currents into the node must equal the sum of the currents out of the node.

$$I_{in} = I_{out}$$

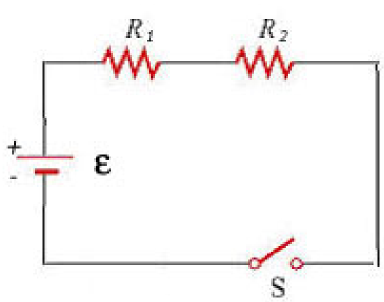
Loop Rule

The sum of the voltage drops, ΔV_i, across any circuit elements that form a closed circuit is zero. This is just the statement that the electric field does zero work per charge in moving a charge around a closed path.

$$\sum_{i=1}^{i=N} \Delta V_i = 0.$$

Example 1

Consider the following closed circuit consisting of one branch that has an electromotive (voltage) source $\mathcal{E}$, a switch S, and two resistors, R_1 and R_2, with $R_1 = 4R_2$. When the switch, S is closed, current will flow in the circuit. In this circuit there is only one branch, so there is only one current that flows in this circuit. This current, flowing through the wire, also flows inside the voltage source.



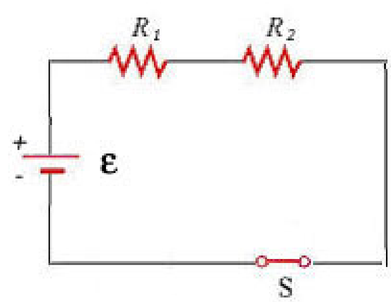


Figure 4a: One loop circuit, open switch **Figure 4b): One loop circuit, closed switch**

A graph of the voltage vs. position along the loop (see Figure 5) shows that the highest voltage is immediately after the battery. The voltage drops as each resistor is crossed. Note that the voltage is essentially constant along the wires. This is because the wires have a negligibly small resistance compared to the resistors.

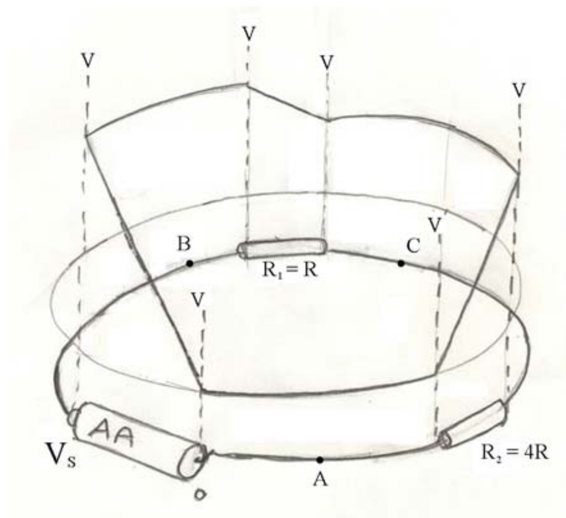


Figure 5a) Voltage changes around a closed loop

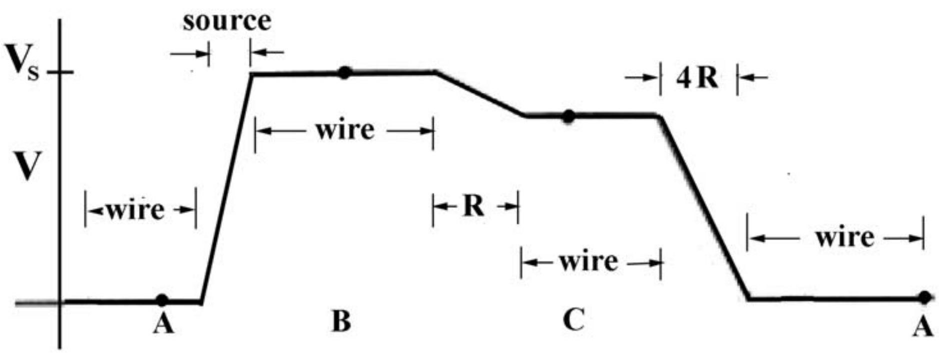
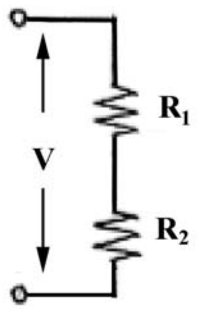
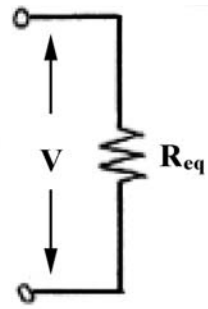


Figure 5b: Voltage Difference between points on a closed circuit and common

Resistors in Series

The two resistors, R_1 and R_2, in Figure 6 are connected in series to a voltage source, V_S. By current conservation, the same current, I, is flowing through each resistor.





6a: Resistors in series **6b: Equivalent resistance**

The total voltage drop across both elements is the sum of the voltage drops across the individual resistors

$$\Delta V = IR_1 + IR_2 .$$

Two resistors in series can be replaced by one equivalent resistor, R_{eq} (Figure 6b). The voltage drop across the equivalent resistor is given by

$$\Delta V = IR_{eq} .$$

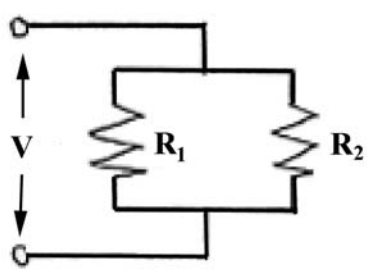
Therefore when any number of resistors are placed in series, the equivalent resistance is just the sum of the original resistances.

$$R_{eq} = R_1 + R_2 + ...$$

Notice that if one resistor, R_1, is much bigger than the other resistor, R_2, then the equivalent resistor, R_{eq} is approximately equal to the larger resistor, R_1.

Resistors in parallel

Consider two resistors, R_1 and R_2, that are connected in parallel across a voltage source ΔV (Figure 7a).



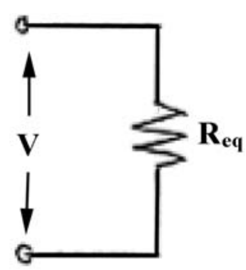


Figure 7a: resistors in parallel **7b: equivalent resistance**

By current conservation, the current, I, that flows through the voltage source must divide into a current, I_1, that flows through resistor, R_1, and a current, I_2, that flows through resistor, R_2. Each resistor individually satisfies Ohm's law, $\Delta V = I_1 R_1$ and $\Delta V = I_2 R_2$. Therefore current conservation becomes

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V(\frac{1}{R_1} + \frac{1}{R_2}).$$

The two resistors in parallel can be replaced by one equivalent resistor, R_{eq}, with the identical voltage drop, ΔV (Figure 7b), and the current, I, satisfies

$$I = \frac{\Delta V}{R_{eq}}.$$

Comparing these results, the equivalent resistance for two resistors that are connected in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

This result easily generalizes to any number of resistors that are connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

When one resistor, R_1, is much bigger than the other resistor, R_2, then the equivalent resistor, R_{eq}, is approximately equal to the smaller resistor, R_2, because

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \simeq R_2.$$

This means that almost all of the current that enters the node point will pass through the branch containing the smaller resistance. So when a short develops across a circuit, all of the current passes through this path of nearly zero resistance.

Voltage Divider

Consider a voltage source, ΔV_{in}, that is connected in series to two resistors, R_1 and R_2

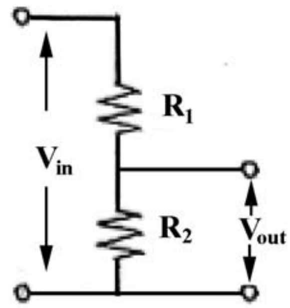


Figure 8: voltage divider

The voltage difference, ΔV_{out}, across resistor, R_2, will be less than ΔV_{in}. This circuit is called a *voltage divider*. From the loop rule,

$$\Delta V_{in} - IR_1 - IR_2 = 0.$$

So the current in the circuit is given by

$$I = \frac{\Delta V_{in}}{R_1 + R_2}.$$

Thus the voltage difference, ΔV_{out}, across resistor, R_2, is given by

$$\Delta V_{out} = IR_2 = \frac{\Delta V_{in}}{R_1 + R_2} R_2.$$

Note that the ratio of the voltages characterizes the voltage divider and is determined by the resistors

$$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{R_2}{R_1 + R_2}.$$

Internal Resistance of a Voltage Source

Voltage sources have an intrinsic internal resistance (that may vary with current, temperature, past history, etc.). This means that when a load is place across the power supply, the voltage across the terminals, ΔV_{load}, will drop. When an external load is connected across the power supply, the circuit diagram looks like (see Figure 9):

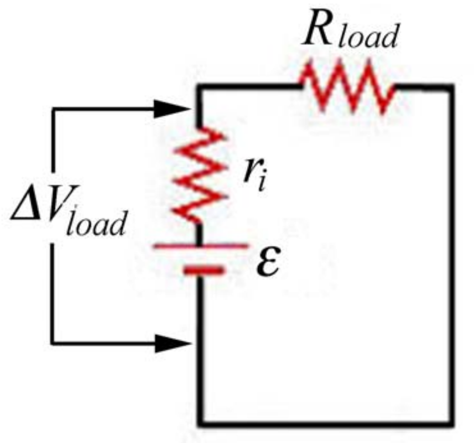


Figure 9: Internal resistance of a voltage source

The terminal to terminal voltage, ΔV_{load}, across the power supply when the load is connected is given by

$$\Delta V_{load} = \Delta V_{no-load} - IR_{\text{int}}$$

The external circuit has a voltage drop given by Ohm's law:

$$\Delta V_{load} = IR_{load}$$

Therefore the current in the circuit is

$$I = \frac{\Delta V_{load}}{R_{load}}.$$

The internal resistance can now be calculated

$$R_{\text{int}} = \frac{\Delta V_{no-load} - \Delta V_{load}}{I}.$$

The loop rule for the circuit law yields

$$\Delta V_{no-load} - IR_{\text{int}} - IR_{load} = 0,$$

so the current in the circuit can also be expressed in terms of the no-load voltage and the load resistance

$$I = \frac{\Delta V_{no-load}}{R_{load} + R_{\text{int}}}.$$

The power lost to the internal resistance is given by

$$P_{joule} = I^2 R_{\text{int}} = (\frac{\Delta V_{no-load}}{R_{load} + R_{\text{int}}})^2 R_{\text{int}}$$

When the terminals of a power supply are shorted by a wire with negligible resistance, there is an upper limit to the short-circuit current

$$I_{sc} = \frac{\Delta V_{no-load}}{R_{\text{int}}}.$$

Voltage-Current Measurements

Any instrument that measure voltage or current will disturb the circuit under observation. Some devices, ammeters, will indicate the flow of current by a meter movement. There will be some voltage drop due to the resistance of the flow of current through the ammeter. An ideal ammeter has zero resistance, but in the case of your MMM, the resistance is 1Ω on the 250mDCA range. The drop of $0.25V$ may or may not be negligible, but you can correct for it. Again, knowing the meter resistance allows one to correct for its effect on the circuit.

An ammeter can be converted to a voltmeter by putting a resistor, R, in series with the coil movement. The voltage across some circuit element can be determined by connecting the coil movement and resistor in parallel with the circuit element. This causes a small amount of current to flow through the coil movement. The voltage across the element can now be determined by measuring I and computing the voltage from $\Delta V = IR$ which is read on a calibrated scale. The larger the resistance, R, the smaller the amount of current is diverted through the coil. Thus an ideal voltmeter would have an infinite resistance.

Resistor Value Chart

0	Black
1	Brown
2	Red
3	Orange
4	Yellow
5	Green
6	Blue
7	Violet
8	Gray
9	White
–1	Gold
–2	Silver

The colored bands on a composition resistor specify numbers according to the chart above (2-7 follow the rainbow spectrum). Starting from the end to which the bands are closest, the first two numbers specify the significant figures of the value of the resistor and the third number represents a power of ten by which the first two numbers are to be multiplied (gold is 10^{-1}). The fourth specifies the 'tolerance' or accuracy, gold being 5% and silver 10%.

Example: 43Ω 5% tolerance is represented by yellow, orange, black, gold.