Experiment MF—Magnetic Force

Introduction

The magnetic force on a current-carrying conductor is basic to every electric motor -- turning the hands of electric watches and clocks, transporting tape in Walkmans, starting cars, operating refrigerator compressors, etc.

In this experiment, you will investigate the magnetic force between two current carrying wire coils. One coil has 10 turns and the other has 38 turns. The 10-turn coil will be taped to one end of a pivoted balance beam. The beam pivots on two pins that also make electrical contact, allowing current to flow onto the beam and through the 10-turn coil (figure 1).
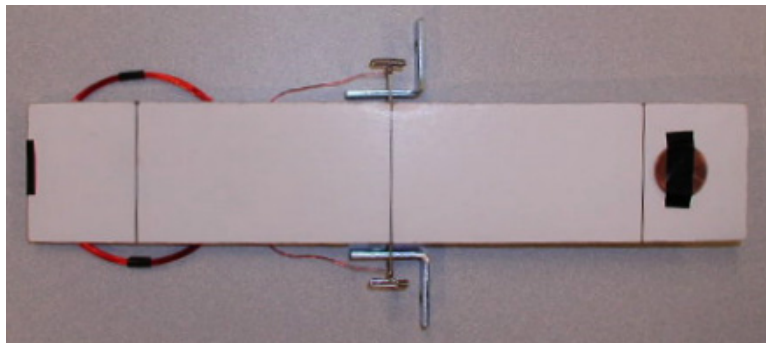

Figure 1: Top view of balance with 10-turn coil taped to underside of foam core

The 38-turn coil will be positioned on the table directly below the 10-turn coil. A current traveling through both coils will produce a magnetic force between the coils. You will measure the magnitude of this force, by noting when the magnetic force between the coils is balanced by the weight of squares of aluminum foil that are placed on the other end of the balance beam.
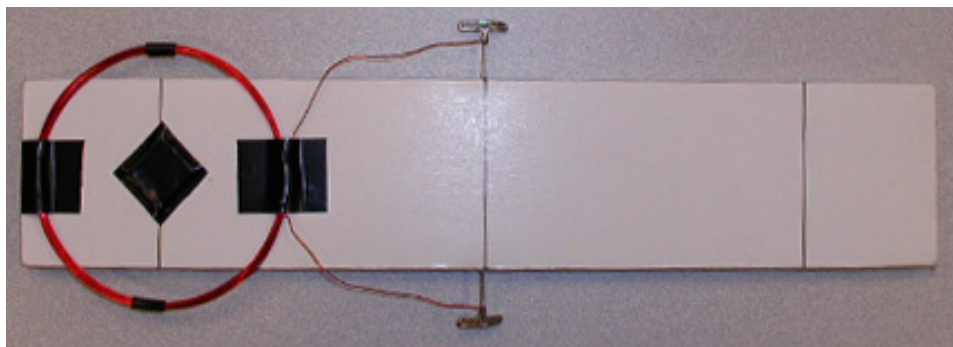

Figure 2: Underneath side of balance beam with 10-turn coil

Derivation of the force equation:

Consider the case when the current in the two coils are flowing in the same direction. Calculate the force by first calculating the magnetic field, $\vec{B}_1$, of the lower coil, (coil #1) at the upper coil. Then we use the magnetic force law,

$$d\vec{F}_2 = I d\vec{r}_2 \times \vec{B}_1,$$

to calculate the force, $d\vec{F}_2$, on a small current element, $I\, d\vec{r}_2$, in the upper coil #2 (figure 3). We then sum up the total force on all n_2 turns in the upper coil,

$$\vec{F}_2 = n_2 \int_{coil\#2} d\vec{F}_2.$$

In our experiment set-up, the two coils are a separated by a distance d that is much smaller than the radius r of either coil. As a first approximation, we can treat the two coils as if they were parallel wires separated by a distance d. Then we can use Ampère's Law to calculate the magnetic field, $\vec{B}_1$, from the lower coil and compute the force on the upper coil.
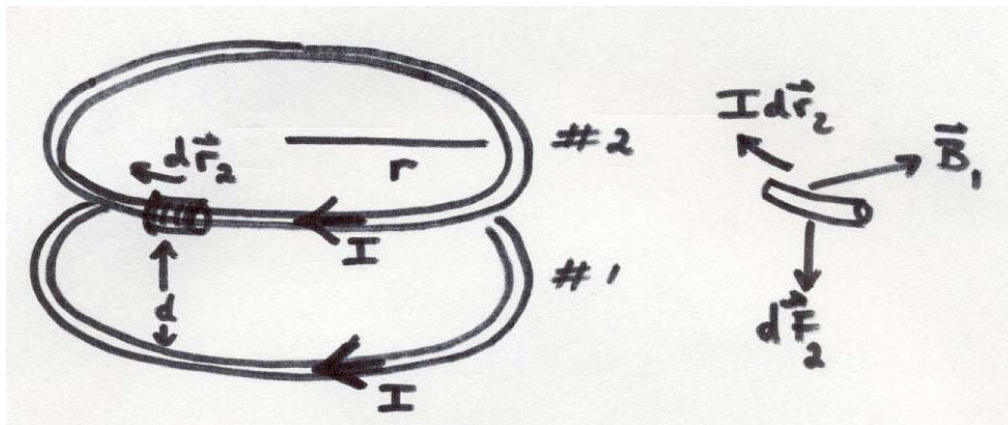

Figure 3: Force diagram on coil # 2 for attractive force

Note that in our approximation when we calculate the force, $d\vec{F}_2$, on a small current element, $I\, d\vec{r}_2$, in the upper coil, we are neglecting the contribution to the magnetic field, $\vec{B}_1$, from parts of the lower coil that are not directly below the small current element in the upper coil. (This will over-estimate the force somewhat (can you see qualitatively why?) but the error is not more than about 10% with your arrangement).

Each turn in coil #1 has a current I which produces a tangential magnetic field $\vec{B}_{turn}$ at $d\vec{r}_2$ (a distance d away), whose magnitude is then approximately the field of a single long straight wire,

$$B_{turn} = \frac{\mu_0 I}{2\pi d}.$$

The constant $\mu_0 = 4\pi \times 10^{-7} T - m/A$ and is called the *magnetic permeability of space.* The magnitude of the total field $\vec{B}_1$ at $d\vec{r}_2$ is equal to n_1 times the field due to a single turn

$$B_1 = \frac{\mu_0 n_1 I}{2\pi d}.$$

In figure 3, the direction of the magnetic field at $d\vec{r}_2$ points towards the center of the circle and is perpendicular to the current element $I\, d\vec{r}_2$, in the upper coil.

The magnetic force $d\vec{F}_2$ exerted on $d\vec{r}_2$ is given by:

$$d\vec{F}_2 = I d\vec{r}_2 \times \vec{B}_1.$$

Since the currents are flowing in the same direction in the two coils, the direction of this force at $d\vec{r}_2$ points downwards towards coil #1. Thus the force is attractive between the coils when the currents flow in the same direction. The magnitude of this force is given by

$$dF_2 = \frac{I dr_2 \mu_0 n_1 I}{2\pi d}.$$

Every element $d\vec{r}_2$ of coil #2 experiences this force, in the same direction -- vertically down, (if the current directions are such as to produce attraction). The total length of wire in coil #2 is $n_2 2\pi r$, where r is the radius of both coil #1 and coil # 2. Thus the magnitude of the total force of attraction on coil #2 is given by:

$$F_2 = n_2 \int dF_2 = n_2 \int \frac{I dr_2 \mu_0 n_1 I}{2\pi d} = n_2 \frac{2\pi r \mu_0 n_1 I^2}{2\pi d} = \mu_0 n_1 n_2 I^2 \frac{r}{d}$$

where r is the coil radius, d the separation between the center of the two coils (see the figure above), and n_1 and n_2 the number of turns in each of the coils. By Newton's Third Law of Motion, the total force on coil #1 is equal and opposite to the force on coil #2.

The magnetic force is balanced by aluminum foil weights that are placed on the other side of the pivot, at an equal distance from the pivot as the center of the upper coil. If the weights are all the same--2 cm x 2 cm of foil, their weight will be:

$$F_{grav} = nmg = n\rho A t g.$$

where $g = 9.8 m/s^2$, $A = 4.0 cm^2$, $\rho = 2.7 \times 10^3 kg/m^3$, $t = 1.8 \times 10^{-5} m$, and n is the number of foils. The balance just begins to tip when the torque from the magnetic force equals the torque from the aluminum weight. Since the moment arms are equal, the forces must also be equal,

$$F_{grav} = F_2.$$

Thus we have

$$n\rho A t g = \mu_0 n_1 n_2 I^2 \frac{r}{d}.$$

This equation shows us that the current squared depends linearly on the number of foils present, i.e.

$$I^2 = \frac{\rho A t g d}{\mu_0 n_1 n_2 r} n.$$

This is why you were asked to plot I^2 vs. n. The slope of the I^2 vs. n plot is given by

$$slope = \frac{\rho A t g d}{\mu_0 n_1 n_2 r}.$$

Then from the best straight line fit of your data, you can calculate the magnetic permeability of space using

$$\mu_0 = \frac{1}{slope} \left(\frac{\rho A t g d}{n_1 n_2 r} \right).$$

In this experiment you depended on the multimeter calibration in amperes when you measured current. It is this very sort of experimental apparatus that was long used to define the current unit, the ampere, in terms of geometry and mechanical forces, setting the constant μ_0 in Ampere's law at exactly $\mu_0 = 4\pi \times 10^{-7} \, N/A^2$.

You can read your result backwards, so to speak. Find out what value your own measurements, using the ampere as the unit of current, yield for the constant μ_0.

Apparatus

The wire coils are made by winding #26 enameled wire around a soda can. Before winding, prepare the soda can by taping a sheet of paper around it -- this paper will help the wire coil slide off the can. Then wind the wire around the can leaving about 8" of wire at each end of the coil. After making the coil, carefully slide it off the can. Then use a couple of pieces of tape to hold them in their round shape.

Figures 1 and 2 above show the completed apparatus. Figures 4 through 6 provide a general idea of how the balance is constructed. Begin by drawing 3 lines across the beam, one at the center of the beam and the others at positions 1-1/2" from each end of the beam. Stick the T-pins into the foam-core to make a pivot. Place and align each pin on the center line.

Using a line you marked on the beam as a guide, tape the lighter, ten-turn coil in place. Cut the leads to the 10-turn coil to convenient lengths that will allow their ends to be wrapped around and soldered to the pins. Remove about 1/2" of the enamel insulation at the ends of the leads to the two coils. The enamel insulation on those wires is tenacious; folding a piece of

emery cloth in half and drawing the wire through it will remove the insulation. Alternatively strip off the enamel with your wire stripper. You will see a bright copper color when the insulation has gone, but check with your ohmmeter to be sure. Wind the bare ends of the wire a couple of turns around the pins. Solder the wires to the pins.
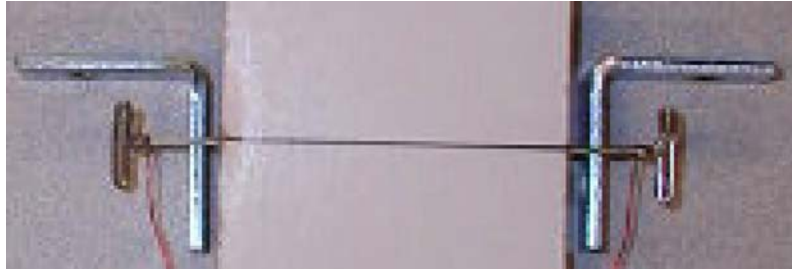

Figure 4: wires soldered to pins

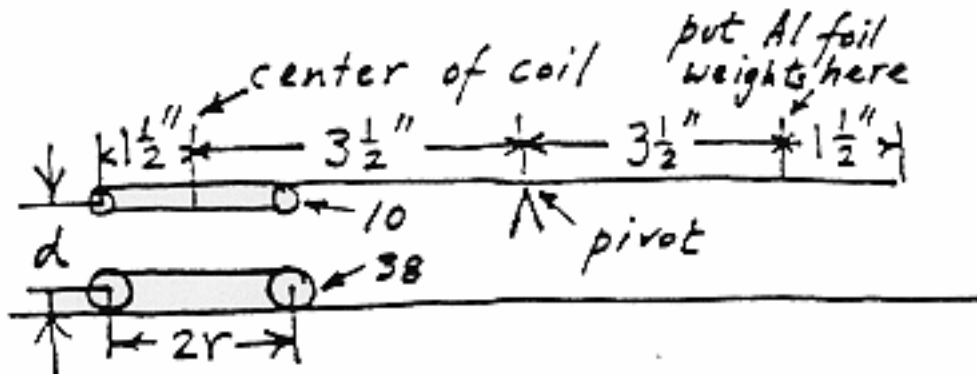**Figure 5: Spacing between the 10-turn and 38-turn coils**

Figure 6: Lay out of coils on foam core and base

Use double-sided sticky tape to hold the corner braces and the 38-turn coil to your desk. Arrange things so the coils are parallel with the beam in its horizontal balanced position. The coil end of the beam is much heavier than the other end. You can balance it by taping one or two pennies, appropriately located, on the light end. As you tape the pennies into position, be careful not to bear down on the beam while it's resting on the bearings: you'll distort the pin positions. Remember that the beam will start to swing slowly as it nears balance, and that sensitivity to a few square centimeters of aluminum foil is quite good enough. (A square centimeter of the aluminum foil provided has a mass of about $4.3 mg$.) You can use a piece of aluminum foil or a sliver of tape for the final adjustment of balance. Later, it may help to make a stop out of wadded foil placed on the desk at the center of the bottom coil. Press down gently until the

beam is where you want it. In the experiment it will lift off when the unbalanced force of gravity (weight) just exceeds the magnetic force.

If the electrical path through the touching surfaces of pin and bearing appears erratic or intermittent, use emery cloth to clean the surfaces.

Experiment

Depending upon the winding direction of the coils, the current through them will generate magnetic forces such that the coils will either attract or repel each other causing them to move. Exchanging the ends of *one* coil in the circuit will reverse the direction of the force (although reversing the direction of the current in *both* coils will not).

Connect the following components in a series circuit as shown in the figure 7:

•   The coil on the balance beam. (Connect it into your circuit by clipping alligator clips to the corner braces.

•   The other coil of the balance.

•   The $8W$ filament of the 1157 bulb.

•   Your LVPS

•   The piece of #16 alloy 800 resistance wire from your red box, a stiff shiny wire about $1.3mm$ in diameter with a resistance of $1.02\ \Omega/m$.

•   Connect your MMM with the clips $0.2\,m$ apart along the #16 resistance wire. You will measure the current flowing through the circuit by measuring the voltage across the resistance wire.
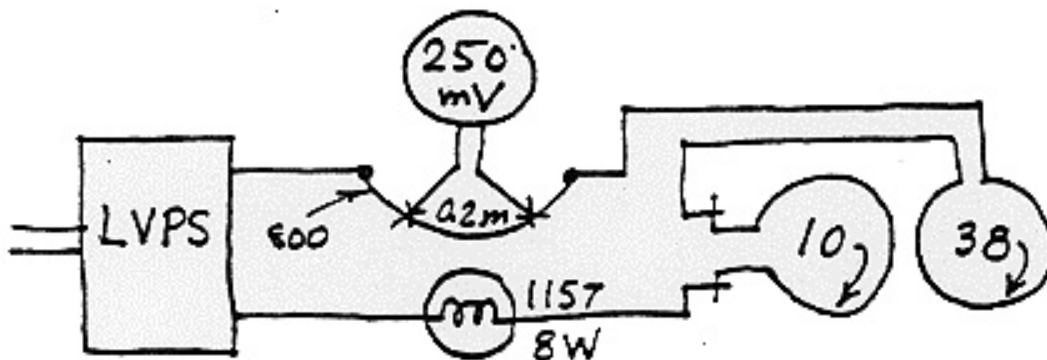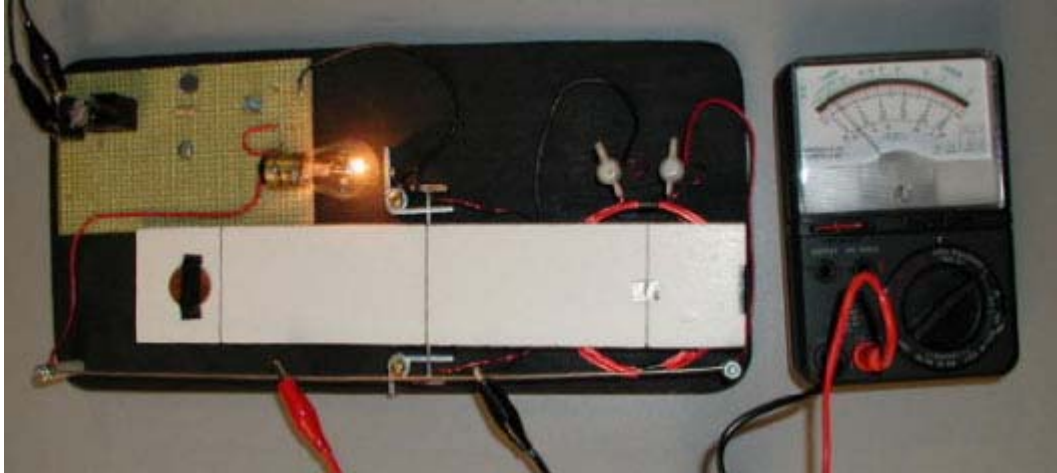

Figure 7: Wiring diagram

Figure 8: Experimental set-up

Plug in the wall transformer of the LVPS; the lamp will tell you if current is flowing. Adjust the pot of the LVPS; the MMM should read on the 50μ(250mV) scale between $20\,mV$ and $100\,mV$ corresponding to currents from $0.1\,A$ to $0.5\,A$.

When you have things working, convince yourself that the balance can show the magnetic effect, exchanging leads to see both attractive and repulsive forces.

The idea is to balance the magnetic force with the force of known masses of foil acting on the balance. Several arrangements of the experiment's parameters are possible. The balance can be set to attract or to repel, the current can be raised or lowered, and the weights can exert their force at either end of the beam. Think through how these setups vary and what might be the advantages of one over the other.

Having the coils close together is a good idea; the magnetic force, whether attracting or repelling, is strongest in that position. Also the coil spacing is easier to measure, and more reproducible, under those conditions. But should the coils be wired to repel or attract? Both ways work. But the 'attraction' choice is preferable because once the coils start to separate the attractive forces are reduced in two ways simultaneously: by the lowered current and by the increased separation of the coils.

Taking Data

So turn the current up high. Set the coils to attract, and put weights, squares of aluminum foil, on the line at the far end of the balance, but not enough to overcome the magnetic attraction. Then, bringing the current down slowly and as smoothly as you can, find the current where the reduced attraction lets the balance tip. Suitable weights are $2\,cm \times 2\,cm$ squares of foil. Cut out three weights of that size. Gentle tapping of the table may help overcome any tendency to stick.

Make several measurements of the current at which the balance tips with all three weights. Then, remove one weight at a time and again measure the critical current. Repeat the measurements to make sure they are ok and tabulate the data, number of weights versus current.

Plot the data with the number of weights, n, horizontally and current squared, I^2 measured in $[A^2]$, vertically. If no weights were on your balance, and it balanced at zero current, then the origin (0,0) is also a point. Fit a best straight line by eye. Estimate the slope and the error on the slope. This can be done by taking the extremal (maximum and minimum) values of the slope and computing the difference with the best fit slope.

Analyzing the Data

From the best straight line fit of your data, you can calculate the magnetic permeability of space using

$$\mu_0 = \frac{1}{slope}\left(\frac{\rho A t\, g\, d}{n_1\, n_2\, r}\right).$$

Error Analysis

Consider the uncertainties in the quantities that appear in the slope: g, ρ, A, t, d, and r. Which error contributes the most? Which errors can you safely ignore?

Parts List

Red Box
1'   800 wire
1   lamp socket (#1157)
aluminum foil

LVPST Test Kit
1   #1157 tail lamp

Experiment MF Parts Kit
40 *ft*   #26 enameled copper wire
1   2″ by 10″ piece of foam core
2   T-pins
2   1″ corner braces
1   piece of double sided tape
1 *ft*   #22 red insulated stranded wire
1 *ft*   #22 black insulated stranded wire

Experiment MF (Magnetic Force): *Calculating the magnetic permeability*

In your experiment, depending upon the winding direction of the coils, the current through them will generate magnetic forces such that the coils will either attract or repel each other causing them to move.

a) Is your apparatus set up to repel or attract? Briefly explain how you intend to measure the current that flows through the coil that will produce a magnetic force that will just balance the weight of the foil.

b) What is the radius r of your coils? What is the distance d between your coils?

c) You will measure the current that will produce a magnetic force that will just balance one, two, and three squares of the foil. Suitable weights are 2 cm by 2 cm squares of foil. For each number of weights, n, make several measurements of the current that balances the weight. Average your values.

d) Make a table of your data with columns for weight (in terms of the number of weights, n), the current, I, in $[A]$, and the current squared, I^2, in $[A^2]$.

e) Make a plot of the current squared I^2, in $[A^2]$, vertically vs. the weight (in terms of the number of weights, n) horizontally. If no weights were on your balance, and it balanced at zero current, then the origin is also a point.

f) Use a linear regression to find the slope and intercept of a straight-line fit of your plot. (You may also find the best fit straight-line by eye.)

g) Estimate the error on the slope. This can be done by taking the extremal (maximum and minimum) values of the slope and computing the difference with the best fit slope.

h) Then from the best straight-line fit of your data, calculate the magnetic permeability of space using

$$\mu_0 = \frac{1}{slope}(\frac{\rho Atgd}{n_1 n_2 r}),$$

where $g = 9.8 m/s^2$, A is the area of your foil, $\rho = 2.7 \times 10^3 kg/m^3$ is the density of aluminum, $t = 1.8 \times 10^{-5} m$ is the thickness of the aluminum, $n_1 = 38$, $n_2 = 10$, r is the radius of your coils, and d is the separation between your coils.

Consider the uncertainties in the quantities that appear in the slope: g, ρ, A, t, d, and r. Which error contributes the most? Which errors can you safely ignore?

Problem 1: *Experiment Magnetic Force*

In the magnetic force experiment, a current $I = 0.5A$ is passed in series through a 38 turn coil taped to a table and a 10 turn coil which is taped to a balance directly above the 38 turn coil. The distance between the 10 turn coil and the 38 turn coil is $d = 0.5cm$. Each coil has a radius of $r = 6.0cm$.

a) Calculate the magnitude of the magnetic field originating from the 38 turn coil at any point on the 10 turn coil. Clearly indicate any approximations you make.

b) Calculate the magnitude of the magnetic field originating from both coils along the central axis at a height $z = 0.5cm$ above the plane of the 38 turn coil. You may take this point as the center of the 10 turn coil.