Massachusetts Institute of Technology *Spring Term 2005*

8.02X Electricity and Magnetism

Quiz #4 SOLUTIONS

Wednesday, May 4 10:05-10:55am

The quiz has four questions. It is a closed book quiz. No calculators are allowed. A letter-size formula sheet can be used, but has to be signed and submitted together with the quiz.

LAST NAME

FIRST NAME

MIT ID#

Problem #1

a) $$L = \frac{N\Phi}{I} = \frac{NBA}{I} = \frac{n l B \pi R^2}{I}$$

$$B = \mu_0 n I$$

$$\therefore \boxed{L = \mu_0 n^2 l \pi R^2}$$

l = length of solenoid

b) $$M = \frac{N_2 \Phi_{21}}{I_1}$$

$N_2 = 1$

$$\Phi_{21} = B \cdot \frac{a^2}{\sqrt{2}}$$

$$= \mu_0 n I_1 \frac{a^2}{\sqrt{2}}$$

$$\therefore \boxed{M = \frac{\mu_0 n a^2}{\sqrt{2}}}$$

Problem 2 (25 points)

Shown below are plots of voltage V(t) and current I(t) for an RLC series circuit connected to an AC power supply.

(a) How big is the impedance Z of the circuit in Ohm?

$Z = \frac{V_{max}}{I_{max}} = \frac{2}{3}\Omega$ [7 pts]

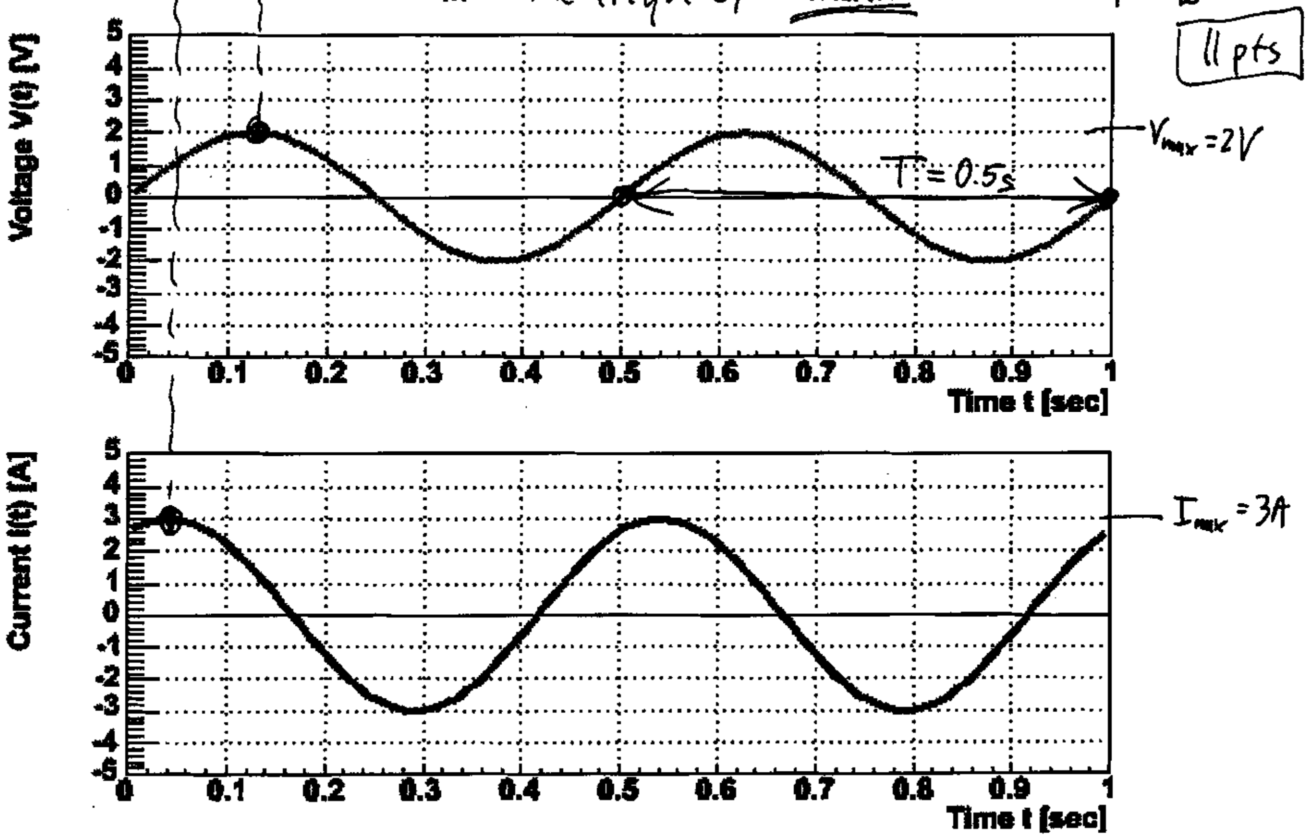
(b) How big is the applied frequency f of the power supply?

$T = 0.5s \Rightarrow f = \frac{1}{T} = 2Hz$ or $\omega = 2\pi f = \frac{2\pi}{T} = 4\pi/s$ [7 pts]

(c) Is the applied frequency smaller than, equal to or bigger than the resonance frequency of the circuit? Explain your answer very briefly.

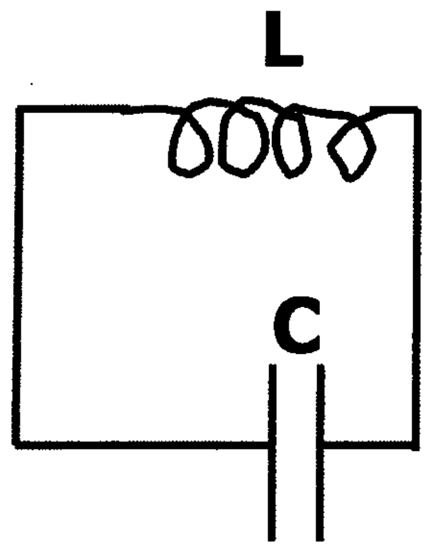
current comes first ... the circuit resembles a capacitor ... $\frac{1}{\omega C} > \omega L$ ... $\omega < \frac{1}{\sqrt{LC}}$ ...

... the frequency is smaller than resonant ω_{LC} [11 pts]



Problem 3 (25 points)

Shown below is an ideal LC circuit, consisting of an inductor L and a capacitor with C=4F. At time t=0, the voltage across the capacitor is 2V. The total energy stored in the circuit is 16J. The natural frequency of oscillation f = ω/(2π) is 2Hz.



(a) Qualitatively, give an example of a mechanical system that is analogous to this circuit. Explain briefly the correspondence between the mechanical system and the LC circuit.

(b) On the graph below, show quantitatively how the energy stored in the inductor varies as a function of time, starting at t=0.

a) $L\frac{di}{dt} + \frac{q}{C} = 0$

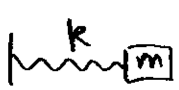
$i = \frac{dq}{dt}$

$\Rightarrow L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$

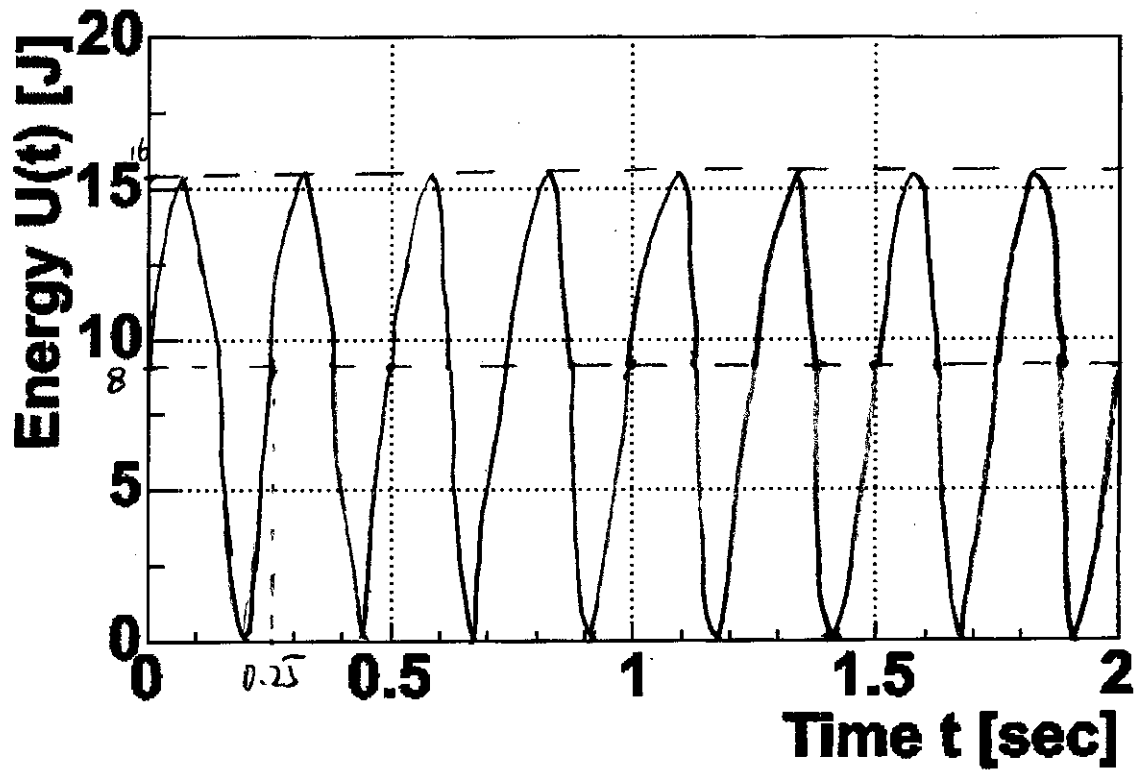
$L \leftrightarrow m$

$\frac{1}{C} \leftrightarrow k$

spring - mass system.



Newton's law: $m\frac{d^2x}{dt^2} + kx = 0$



$U_{tot} = U_L + U_C = \frac{1}{2}Li^2 + \frac{1}{2}CV^2 = 16\ J$ is conserved

At $t=0$, $V = 2V$, i.e. $U_C = \frac{1}{2} \times 4 \times 2^2 = 8J$

$\Rightarrow U_L = U_{tot} - U_C = 16J - 8J = 8J$

The frequency $f = \frac{\omega}{2\pi} = 2\ Hz$

hence the period $T = \frac{2\pi}{\omega} = \frac{1}{2}S = 0.5S$

But T is the period of charge / current oscillation, the period of energy oscillation should be $\frac{T}{2} = 0.25S$.

This is because:

$$U_L \propto i^2 \propto \cos^2(\omega t + \varphi) = \frac{1}{2}(1 + \cos 2(\omega t + \varphi))$$

i.e. ω' (energy oscillation) $= 2\omega$

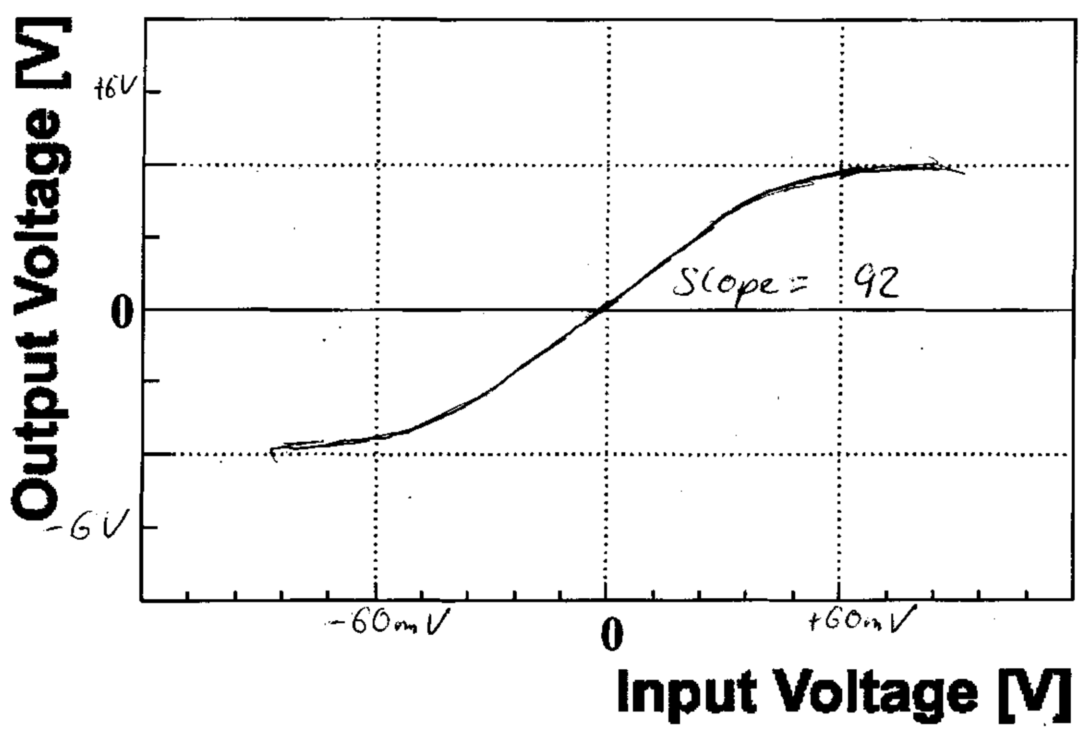
$\Rightarrow T'$ (energy oscillation) $= \frac{T}{2} = 0.25S$

Problem 4 (25 points) AMP experiment

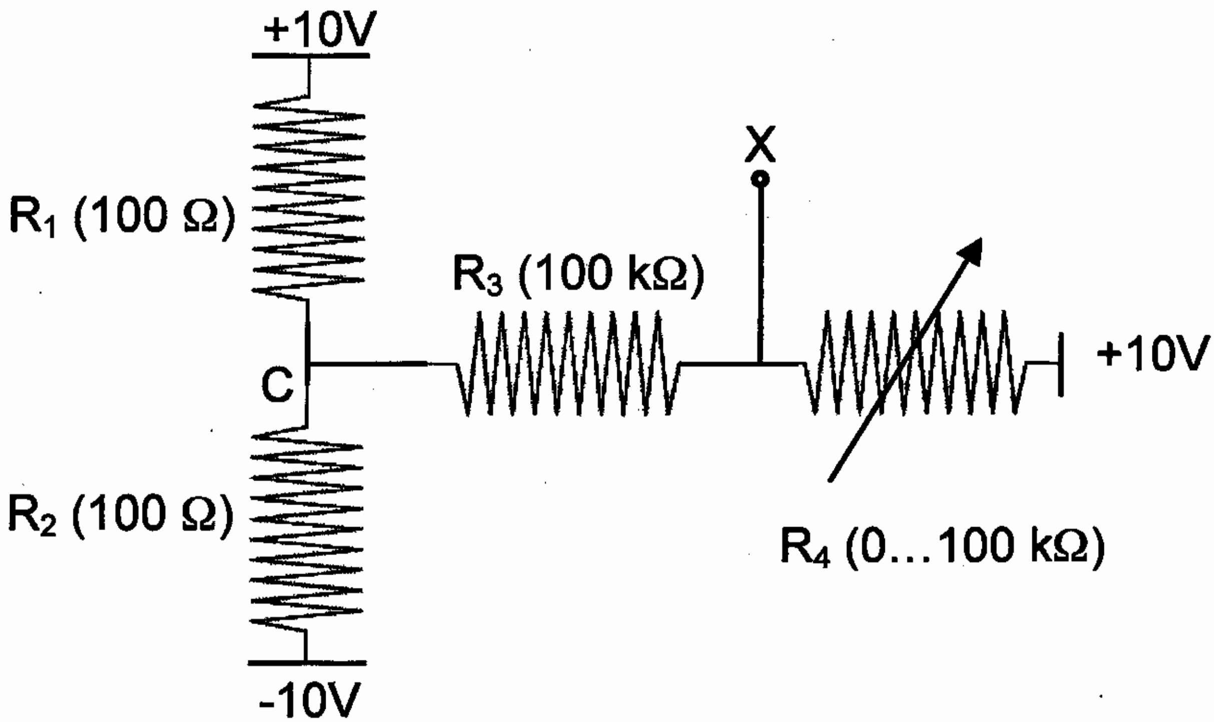
For the AMP experiment, you built an amplifier using an op-amp with an amplifaction of 10^5.

(a) How big was the nominal gain of the amplifier? 92

(b) On the graph below, sketch the relationship between input and output voltage for the amplifier. Make sure to indicate the magnitude of input and output voltages.



(c) Shown below is a voltage divider that provides a variable voltage at point X. The resistance of the potentiometer R_4 can be varied from 0 to 100 kΩ. What is the minimal and maximal electric potential at X relative to C, when the potentiometer is turned from one endpoint to the other? ***(note that the fact that $R_3 >> R_1, R_2$ allows you to avoid complicated algebra!).***



As $R_3 \gg R_1$ and R_2 $\Rightarrow$ CURRENT through R_3 << CURRENT THROUGH R_1, R_3 $\Rightarrow$ $I(R_1) \approx I(R_2)$

$\Rightarrow$ $V@C = 0$

IF $R_4 = 0 \Rightarrow V_X = +10V$

IF $R_4 = 100\,k\Omega = \frac{1}{2}(10V - V@C) = +5V$