*Massachusetts Institute of Technology* *Spring Term 2005*

# 8.02X Electricity and Magnetism

## Quiz #3 Solutions

**Monday, April 11** **10:05-10:55am**

*The quiz has four questions. It is a closed book quiz. No calculators are allowed.*
*A letter-size formula sheet can be used, but has to be signed and submitted together with the quiz.*

**LAST NAME**

**FIRST NAME**

**MIT ID#**

## Problem 1 (~~20~~ 25 points) Experiment EB

Suppose that experiment EB is performed with a gas that has an ionization potential of $V_{ion} = 10V$. For a gap of d=0.1mm you observe electric breakdown at a voltage difference across the spark gap of $V_{gap} = 1000V$.

(a) What is the mean free path of the electrons in the gas?

$$\frac{V_{ION}}{\lambda_{mfp}} = \frac{V_{gap}}{d} \quad b/c \quad q\,\lambda_{mfp} \cdot \frac{V_{gap}}{d} \geq q \cdot V_{ion}$$

$$\Rightarrow \frac{10V}{\lambda_{mfp}} = \frac{1000V}{10^{-4}m} \Rightarrow \lambda_{mfp} = 10^{-6}m = 1\mu m$$

Assume the experiment was repeated using the same gas and the same gap d = 0.1mm, but in an enclosure with only half the pressure and therefore only half the density of molecules? At which voltage would breakdown occur under these conditions? Explain your answer in a few sentences.

Same gas → same $V_{ION}$

Lower density → $\lambda_{mfp}$ increases

$$\frac{V_{ION}}{\underset{\substack{\downarrow \\ \text{doubles}}}{\lambda_{mfp}}} = \frac{V_{gap}}{d} \Rightarrow V_{gap} = 500V$$

The lower density allows electrons to travel farther before hitting a molecule. Therefore the field necessary for them to pick up enough energy to ionize molecules can be lower. For the same d, that means smaller $V_{gap}$.

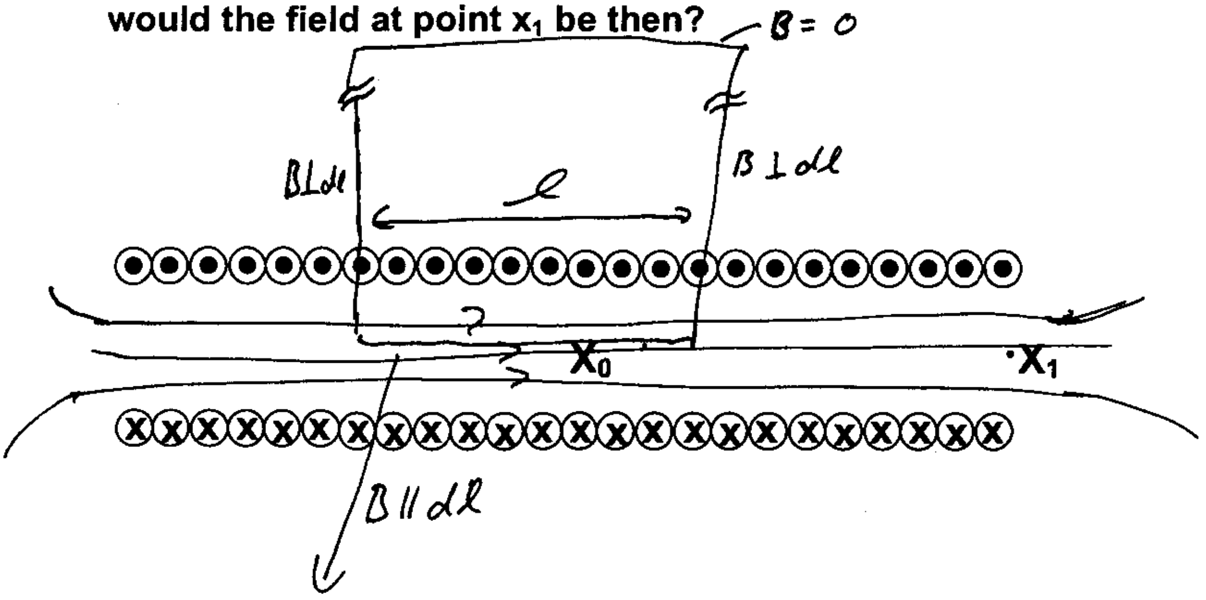
**Problem 2 (25 points)**

**Shown below is the cross-section of a long solenoid with length L and number of windings N. The solenoid carries a current I.**

5 **(a) Using fieldlines, sketch the magnetic field created by the solenoid.**

15 **(b) Using Ampere's Law and symmetry arguments, derive an expression for the magnitude of the magnetic field at the center ($X_0$) of the solenoid. Show work!**

5 **(c) Assume an identical solenoid was placed in close proximity to the first one, to the right of the first solenoid, carrying the same current I in the same direction. How big would the field at point $x_1$ be then?**



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow B \cdot \ell = \mu N_\ell I \Rightarrow \underline{\underline{B = \mu_0 \frac{N}{L} \cdot I}}$$

c) $B(x_1) = B(x_0)$ b/c then it would look like one long solenoid (and at the edge the field of the two solenoids would add up)

**Problem 3 (25 points)**

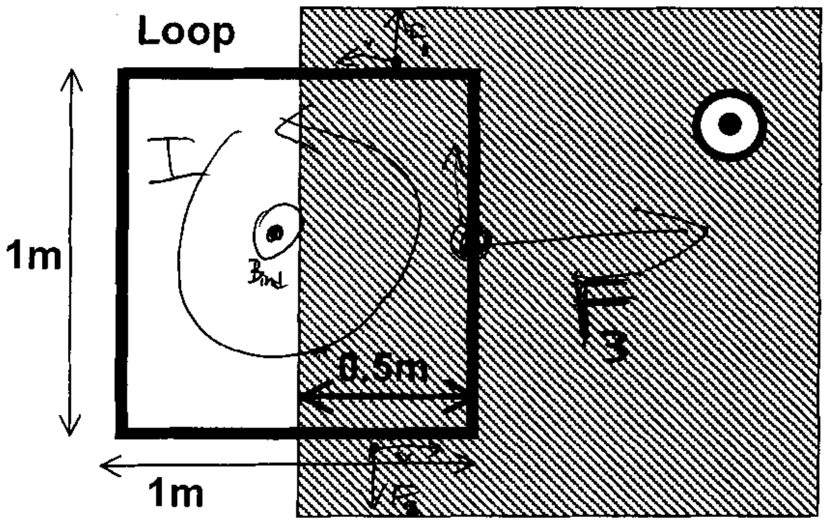
**Shown below is a square conducting loop. The loop is not movable. The sides of the loop have length 1m. The right half of the loop is inside a uniform external magnetic field, which points out of the paper plane. The resistance of the loop is 1 Ohm.**

**(a) At time t=0, the magnitude of the field is B=2T. What is the magnitude of the magnetic flux through the loop at this time?** $\Phi = B \cdot A = 2T \times 0.5\,m^2 = 1\,Tm^2 = 1\,Wb$ $\boxed{2pts}$ −1 for wrong units

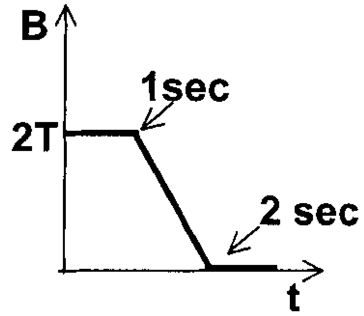
**(b) Starting at time t=1 sec, the field is ramped from B=2T to B=0 over the course of 1 sec with a constant rate. What is the magnitude of the induced EMF at t=1.5 sec during the ramp. Show work!** Because the change is linear, we can take $\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t}$. $\boxed{11\,pts}$
The flux changes from $1\,Tm^2$ to $0\,Tm^2$ in 1s, so $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\Delta\Phi}{\Delta t} = -\frac{(-1\,Tm^2)}{1s} = \underline{\underline{1V}}$

**(c) What is the direction and magnitude of the induced current at t=1.5 sec?** The flux $\Phi$ decreases, so the induced field wants to <u>increase it</u>. Therefore $B_{ind}$ is ⊙ and the current is <u>counterclockwise</u>. $I = \frac{\mathcal{E}}{R} = \frac{1V}{1\Omega} = \underline{\underline{1A}}$ $\boxed{7pts}$

**(d) What is the direction and magnitude of the net magnetic force on the loop at t=1.5 sec?**
The forces on the top & bottom cancel out, the only force left is $F_3 = B \cdot I \cdot L =$
$= (1T) \cdot (1A) \cdot (1m) = \underline{\underline{1N}}$



**Magnetic Field, pointing out of paper plane** $\boxed{5pts}$



**Problem 4 (25 points)**

**Shown below is the cross-section of a parallel plate capacitor carrying a charge +Q (top) and -Q (bottom). The potential difference between the plates is $\Delta V$, the plates are separated by a distance *d*.**

**An electron with charge e = $-1.6\ 10^{-19}$C and velocity v is entering the capacitor from the left.**

**(a) On the figure, show the direction of the electric field in the capacitor.**

**(b) What direction should an external magnetic field have, such that the electron is not deflected inside the capacitor?** Since $e < 0$, the electric force $\vec{F}_E = e\vec{E} = |eE|\,\hat{y}$
therefore the magnetic force must be in $-\hat{y}$ direction:
$\vec{F}_B = e\vec{v} \times \vec{B} = -|e||v|$ [illegible] $\hat{x} \times \hat{B} \propto -\hat{y} \Rightarrow \hat{x} \times \vec{B} \propto \hat{y}$

$\Rightarrow \vec{B} = |B|(-\hat{z})$
as shown in figure

**(c) What should the magnitude of the field be in terms of the quantities given, such that the electron is not deflected inside the capacitor? Show work!**

Since $\vec{v}, \vec{F_E}, \vec{F_B}$ are perpendicular to each other:

$\vec{F}_{total} = e[\vec{E} + \vec{v} \times \vec{B}] = 0 \Rightarrow \vec{E} = -\vec{v} \times \vec{B}$

$\Rightarrow |E| = |vB| \Rightarrow |B| = \left|\frac{E}{v}\right|$ Note $|E| = \frac{\Delta V}{d}$

$\Rightarrow \boxed{B = \frac{\Delta V}{v d}}$