Class 30: Outline

Hour 1:

Traveling & Standing Waves

Hour 2:

Electromagnetic (EM) Waves

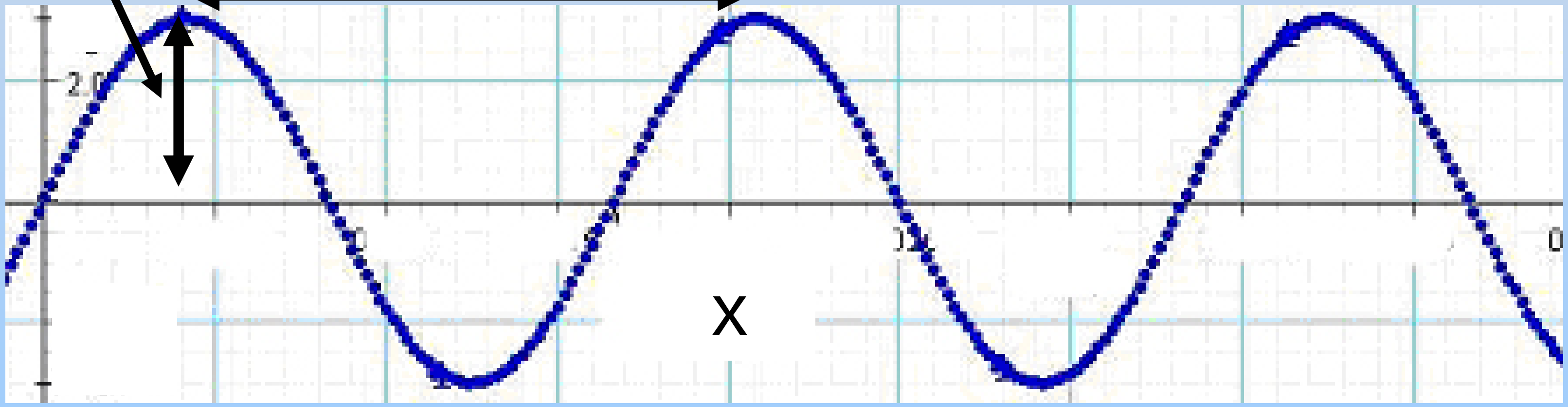
Last Time:
Traveling Waves

Traveling Sine Wave

Now consider $f(x) = y = y_0 \sin(kx)$:

$$\text{Wavelength}\,(\lambda) = \frac{2\pi}{\text{wavenumber}\,(k)}$$

Amplitude (y_0)



What is $g(x,t) = f(x+vt)$? Travels to left at velocity v

$$y = y_0 \sin(k(x+vt)) = y_0 \sin(kx+kvt)$$

Traveling Sine Wave

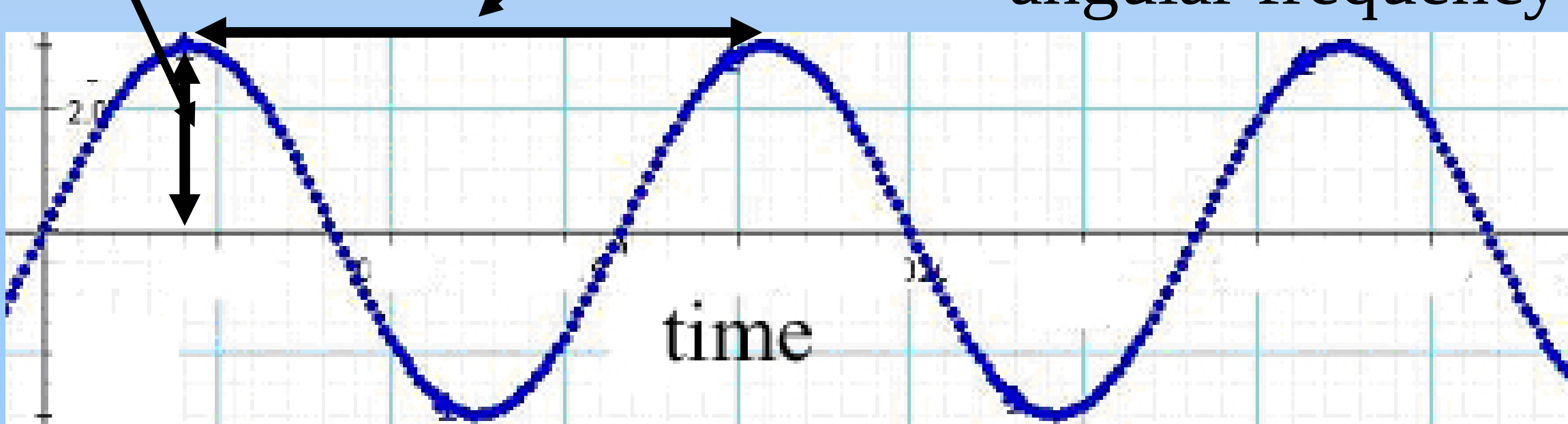
$$y = y_0 \sin\left(kx + kvt\right)$$

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$$\text{Period } (T) = \frac{1}{\text{frequency } (f)}$$

$$= \frac{2\pi}{\text{angular frequency } (\omega)}$$

Amplitude (y_0)



Traveling Sine Wave

$$y = y_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

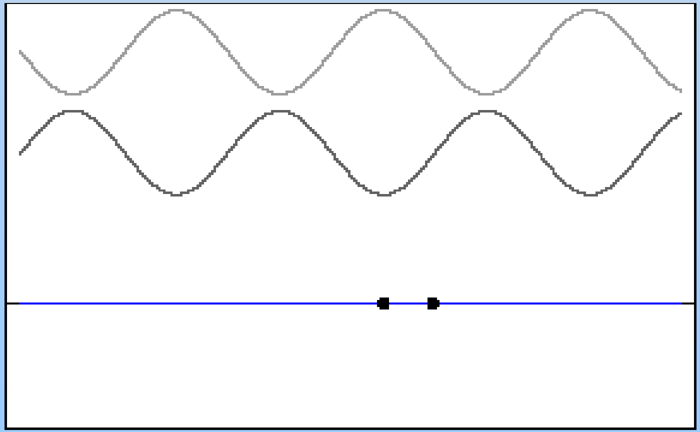
This Time:
Standing Waves

Standing Waves

What happens if two waves headed in opposite directions are allowed to interfere?

$$E_1 = E_0 \sin(kx - \omega t) \qquad E_2 = E_0 \sin(kx + \omega t)$$

Superposition: $E = E_1 + E_2 = 2E_0 \sin(kx)\cos(\omega t)$

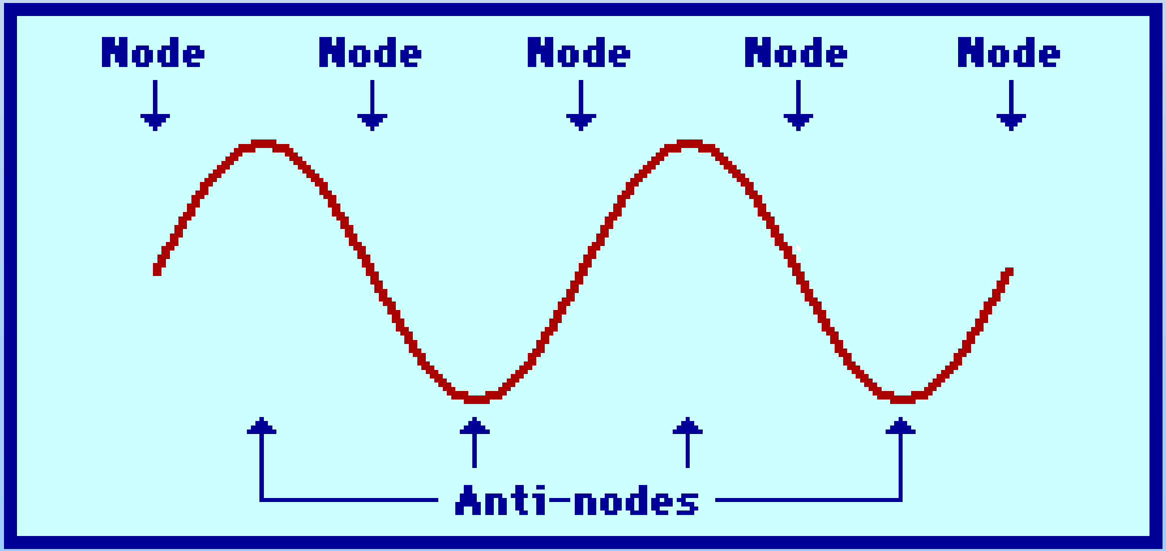


Standing Waves: Who Cares?

Most commonly seen in resonating systems:

Musical Instruments, Microwave Ovens

$$E = 2E_0 \sin(kx)\cos(\omega t)$$



Standing Waves: Bridge

Tacoma Narrows Bridge Oscillation:

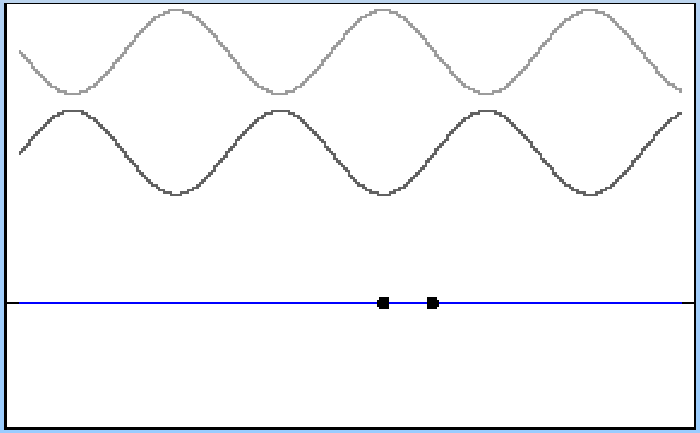
http://www.pbs.org/wgbh/nova/bridge/tacoma3.html

Group Work: Standing Waves

Do Problem 2

$$E_1 = E_0 \sin(kx - \omega t) \qquad E_2 = E_0 \sin(kx + \omega t)$$

Superposition: $E = E_1 + E_2 = 2E_0 \sin(kx)\cos(\omega t)$



Last Time:
Maxwell's Equations

Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

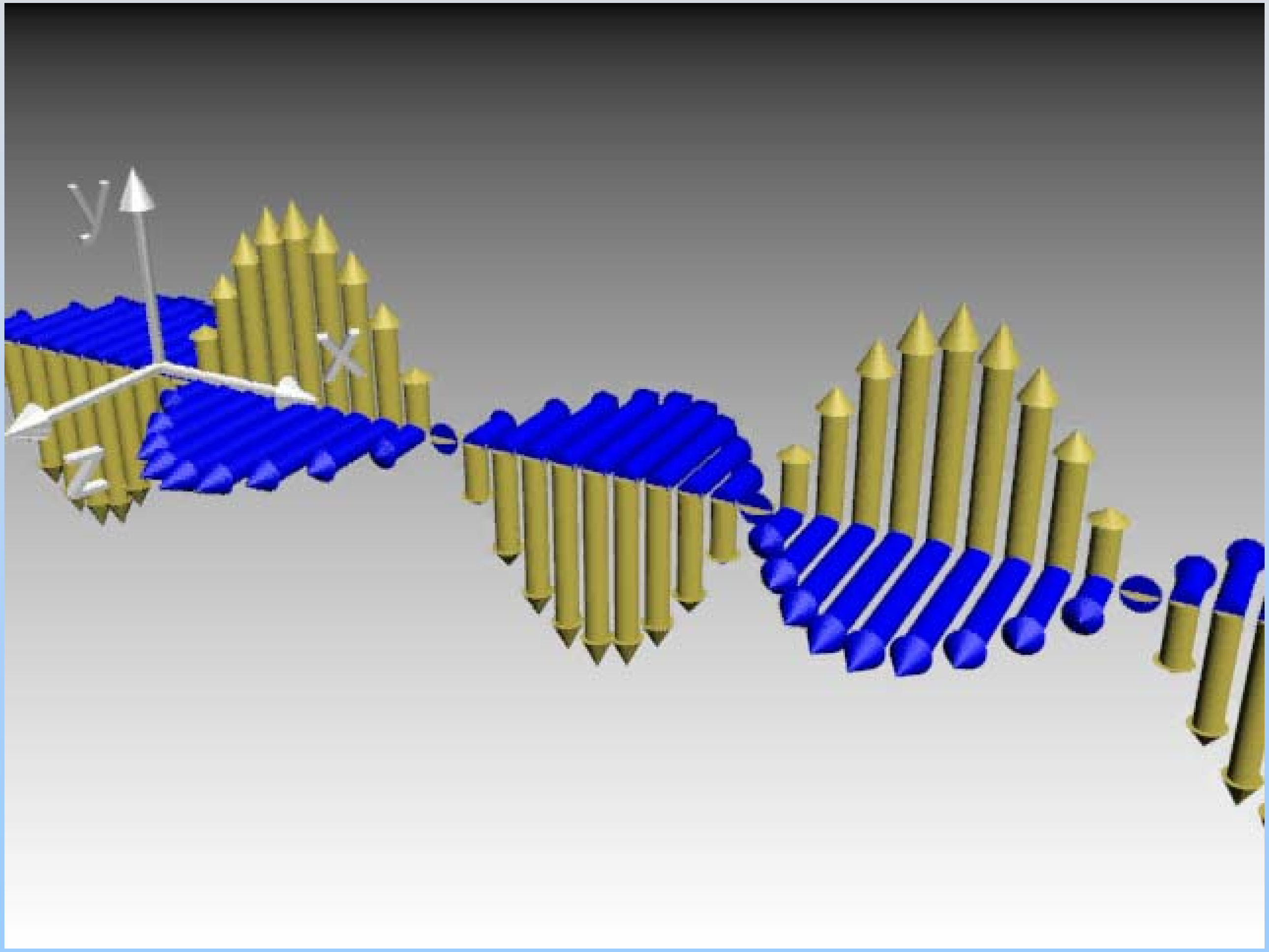
$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

Which Leads To…
EM Waves

Electromagnetic Radiation: Plane Waves



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html

Traveling E & B Waves

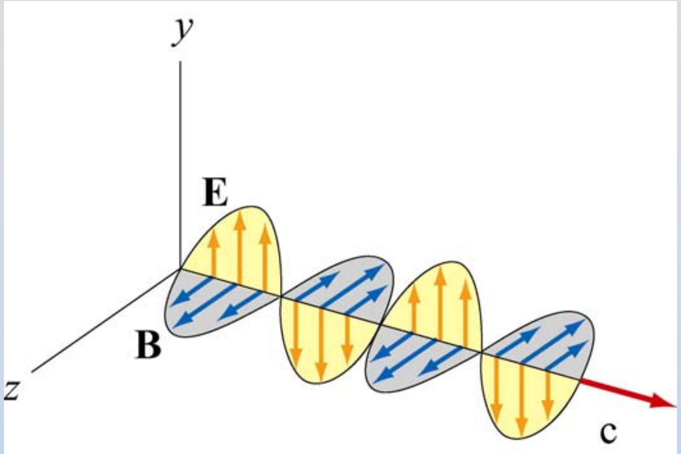
$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

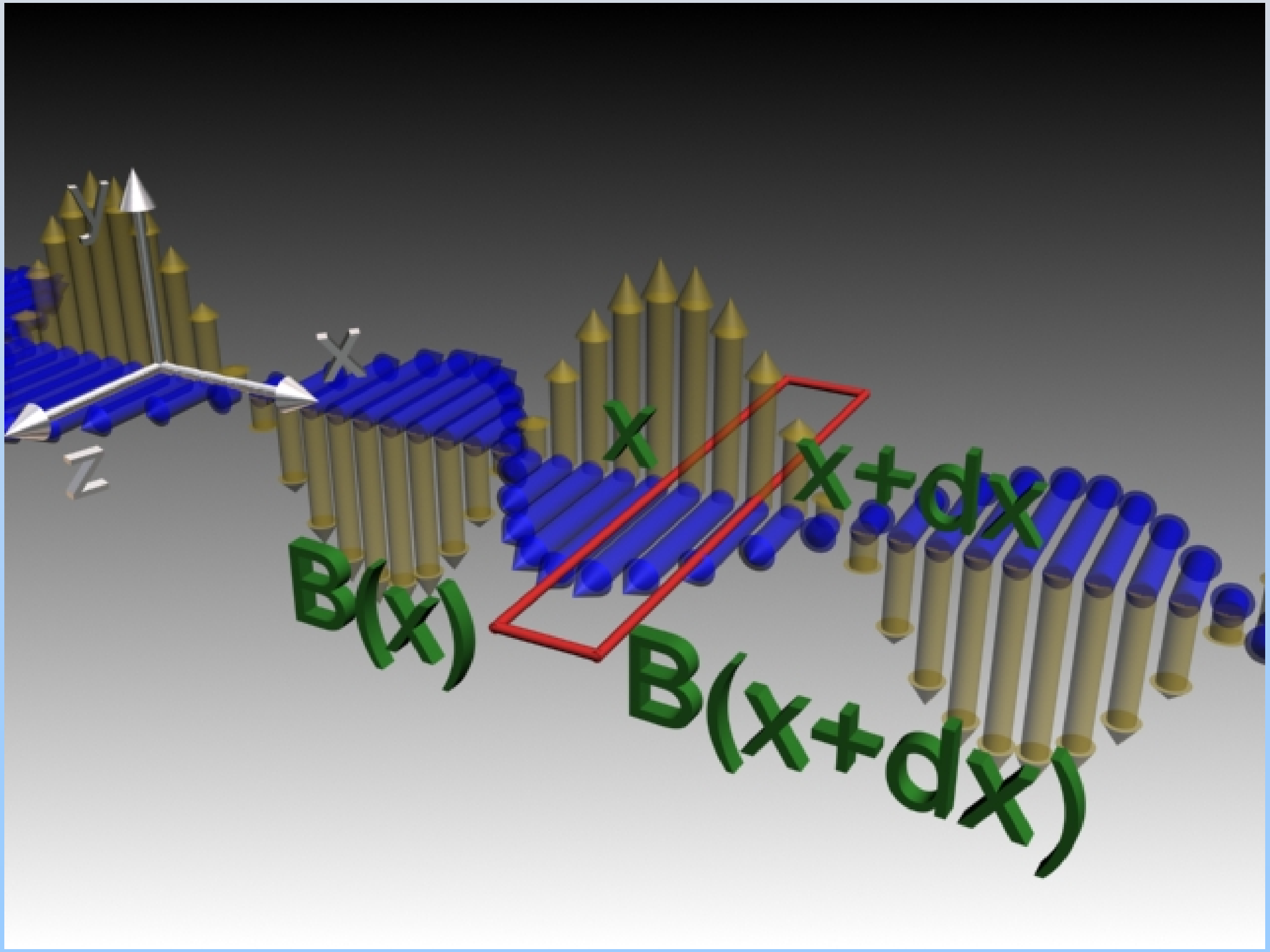
PRS Questions:
Direction of Propagation

How Do Maxwell's Equations Lead to EM Waves?

Derive Wave Equation

Wave Equation

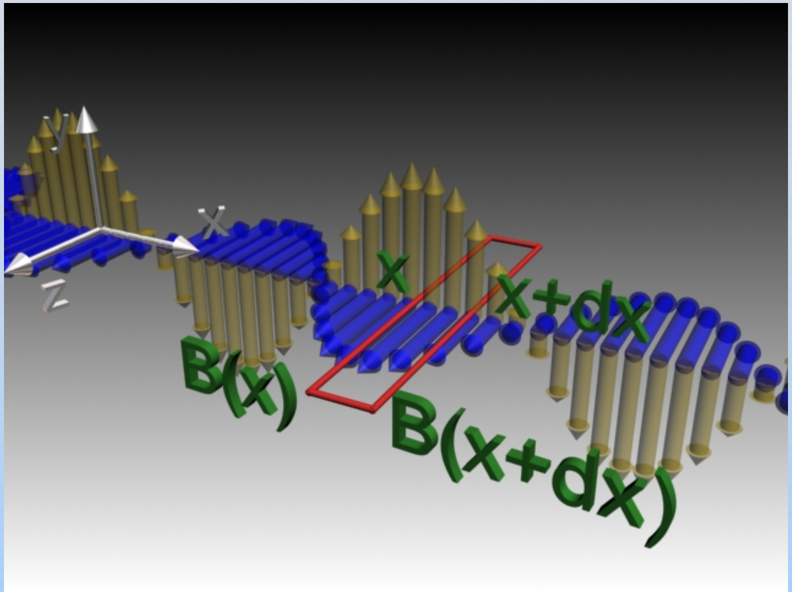
Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$



Wave Equation

Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B_z(x,t)l - B_z(x+dx,t)l$$

$$\mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \mu_0 \varepsilon_0 \left(l\, dx \frac{\partial E_y}{\partial t} \right)$$

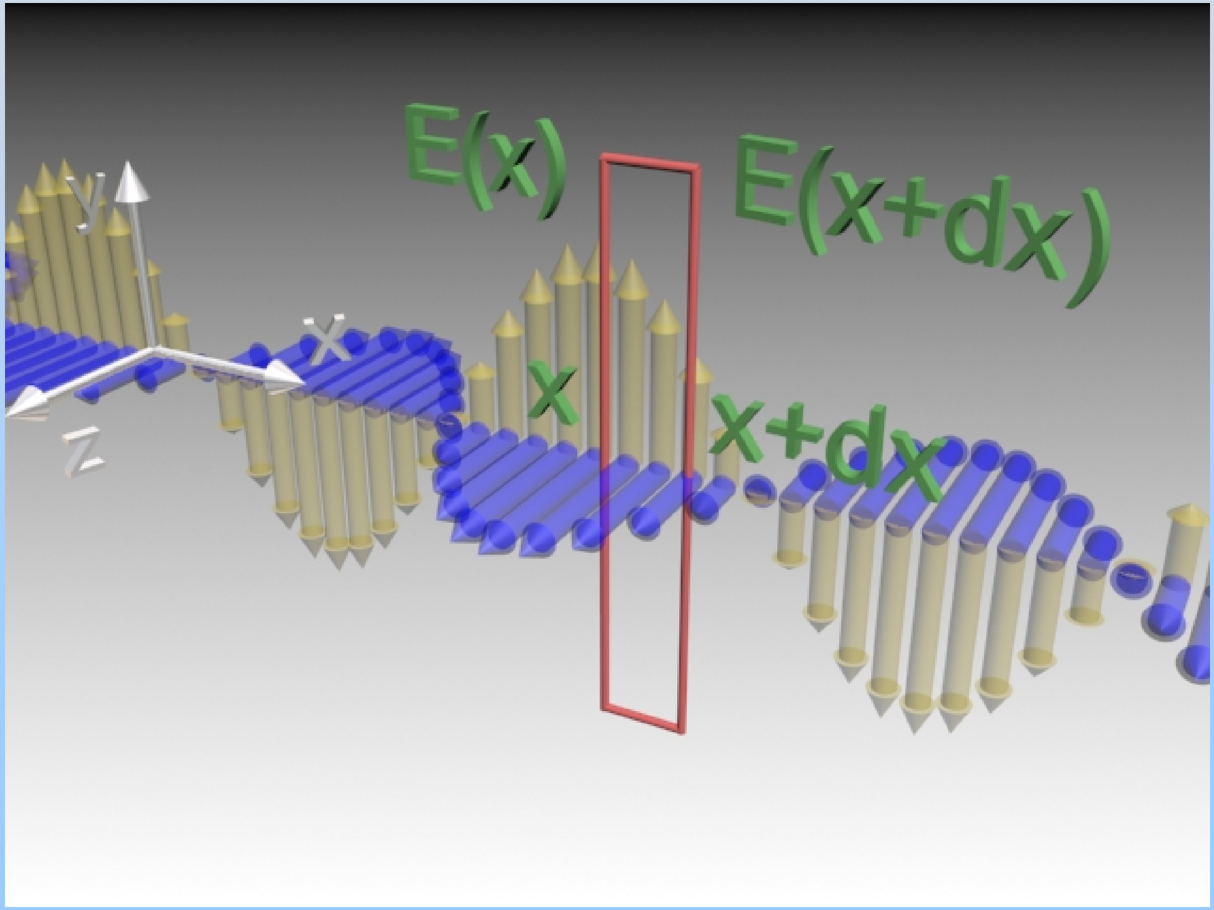
$$-\frac{B_z(x+dx,t) - B_z(x,t)}{dx} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

So in the limit that *dx* is very small:

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Wave Equation

Now go to Faraday's Law $\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

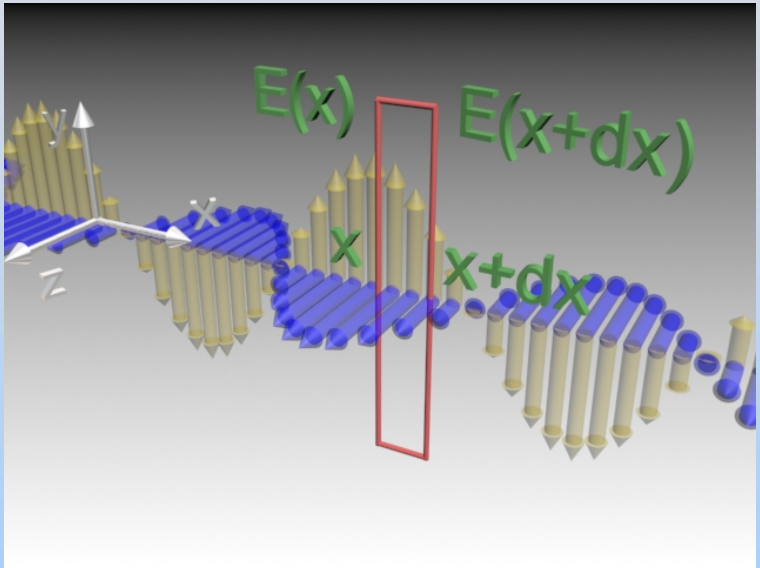


Wave Equation

Faraday's Law:

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = E_y(x+dx,t)l - E_y(x,t)l$$

$$-\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -l\,dx\frac{\partial B_z}{\partial t}$$

$$\frac{E_y(x+dx,t) - E_y(x,t)}{dx} = -\frac{\partial B_z}{\partial t}$$

So in the limit that *dx* is very small:

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

1D Wave Equation for E

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = \underline{\frac{\partial^2 E_y}{\partial x^2}} = \frac{\partial}{\partial x}\left(-\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \underline{\mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

1D Wave Equation for E

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

This is an equation for a wave. Let: $E_y = f\left(x - vt\right)$

$$\left.\begin{aligned}\frac{\partial^2 E_y}{\partial x^2} &= f''\left(x - vt\right)\\ \frac{\partial^2 E_y}{\partial t^2} &= v^2 f''\left(x - vt\right)\end{aligned}\right\} \quad v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

1D Wave Equation for B

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial t}\right) = \underline{\frac{\partial^2 B_z}{\partial t^2}} = \frac{\partial}{\partial t}\left(-\frac{\partial E_y}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial t}\right) = \underline{\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 B_z}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}}$$

Electromagnetic Radiation

Both E & B travel like waves:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

But there are strict relations between them:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Here, E_y and B_z are "the same," traveling along x axis

Amplitudes of E & B

$$\text{Let } E_y = E_0 f\left(x - vt\right);\ B_z = B_0 f\left(x - vt\right)$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \Rightarrow -vB_0 f'\left(x - vt\right) = -E_0 f'\left(x - vt\right)$$

$$\boxed{\Rightarrow vB_0 = E_0}$$

E_y and B_z are "the same," just different amplitudes

Group Problem: EM Standing Waves

Consider EM Wave approaching a perfect conductor:

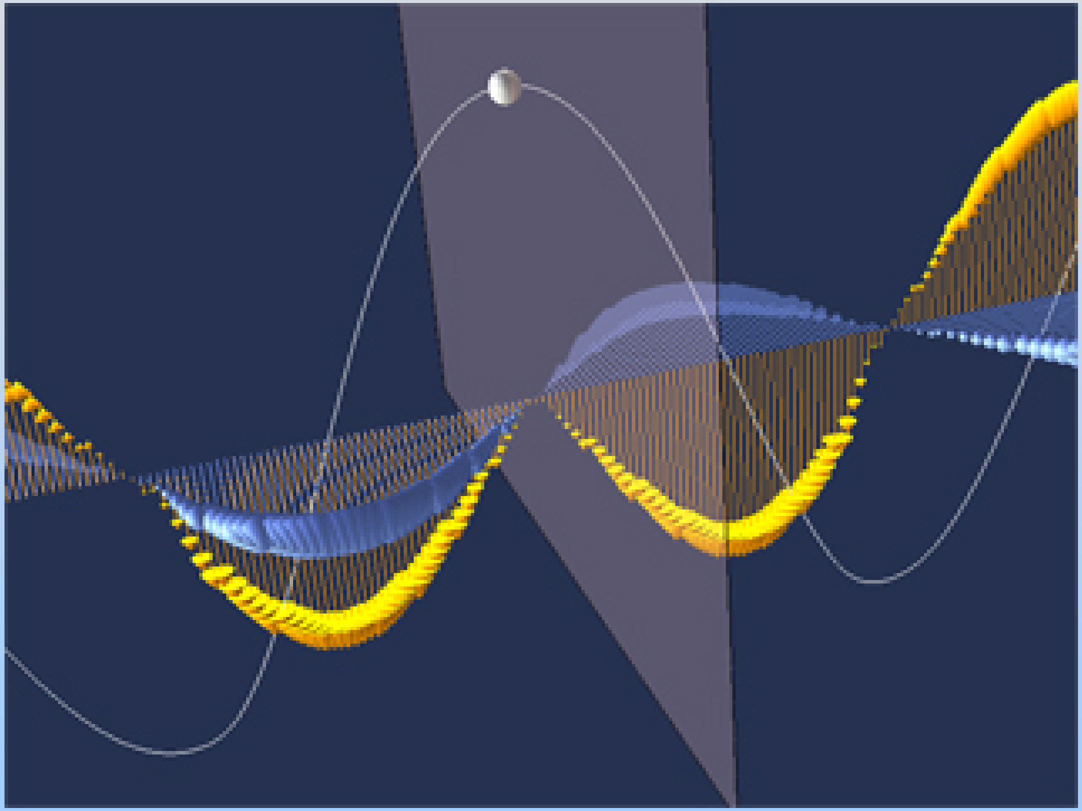
$$\vec{\mathbf{E}}_{\text{incident}} = \hat{x} E_0 \cos(kz - \omega t)$$

If the conductor fills the XY plane at Z=0 then the wave will reflect and add to the incident wave

1. What must the total E field ($E_{inc}+E_{ref}$) at Z=0 be?
2. What is $E_{reflected}$ for this to be the case?
3. What are the accompanying B fields? (B_{inc} & B_{ref})
4. What are E_{total} and B_{total}? What is B(Z=0)?
5. What current must exist at Z=0 to reflect the wave? Give magnitude and direction.

Recall: $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

Next Time: How Do We Generate Plane Waves?



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/09-planewaveapp/09-planewaveapp320.html