

Class 25: Outline

Hour 1:

Expt. 10: Part I: Measuring L
LC Circuits

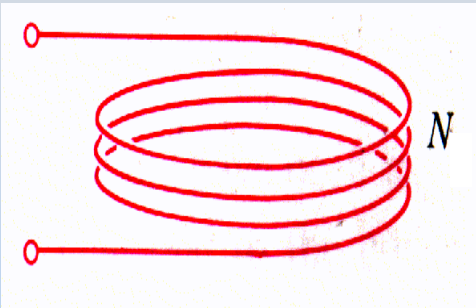
Hour 2:

Expt. 10: Part II: LRC Circuit

Last Time: Self Inductance

Self Inductance

To Calculate: $L = N\Phi/I$

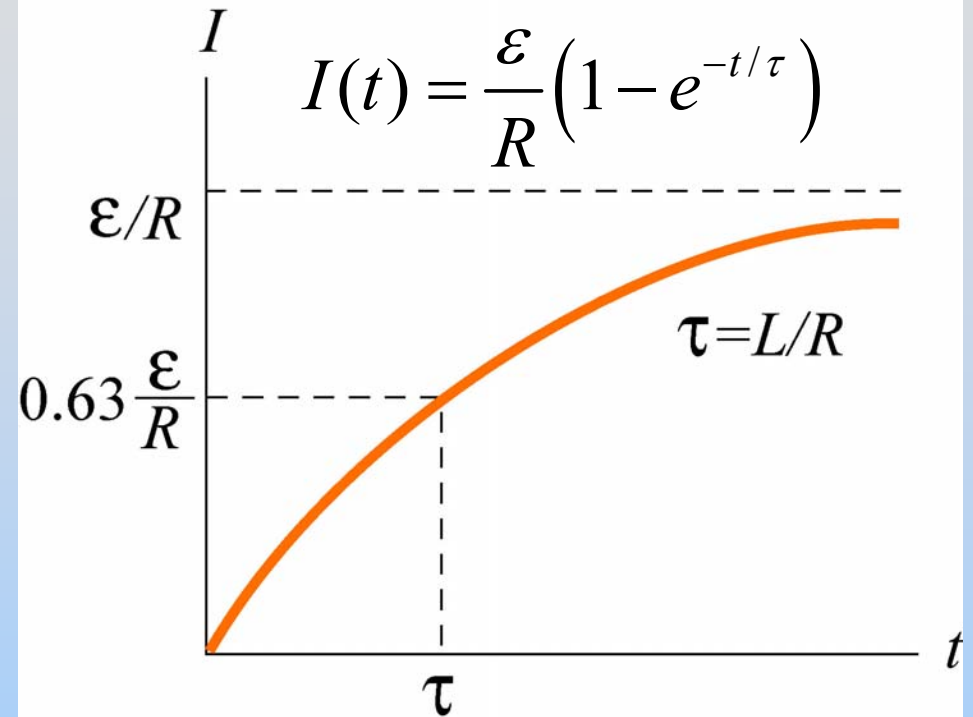
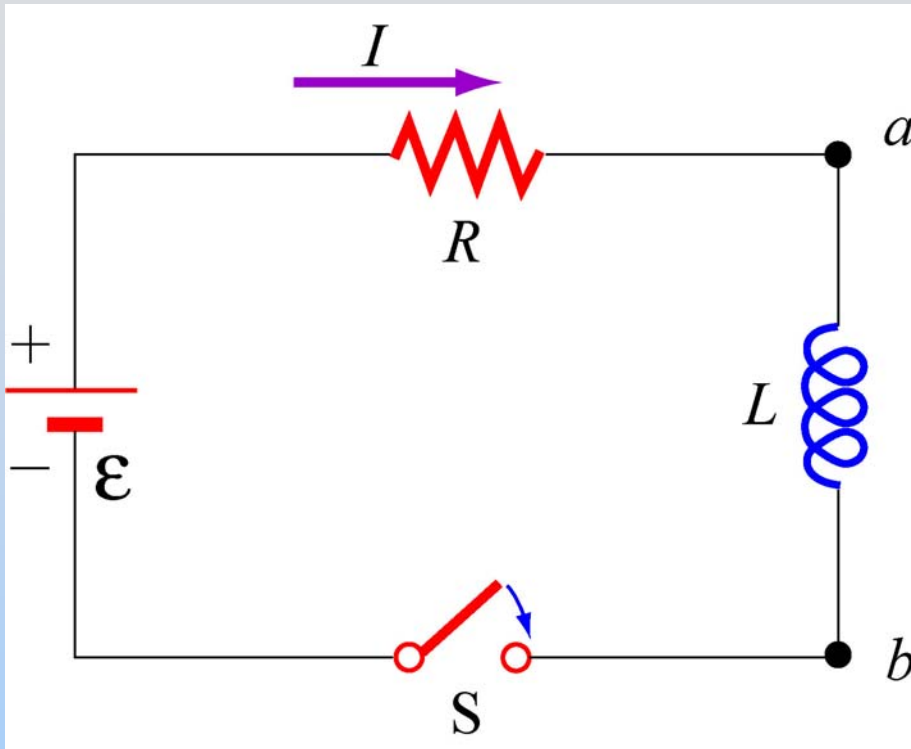


1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

The Effect: Back EMF: $\mathcal{E} \equiv -L \frac{dI}{dt}$

Inductors hate change, like steady state
They are the opposite of capacitors

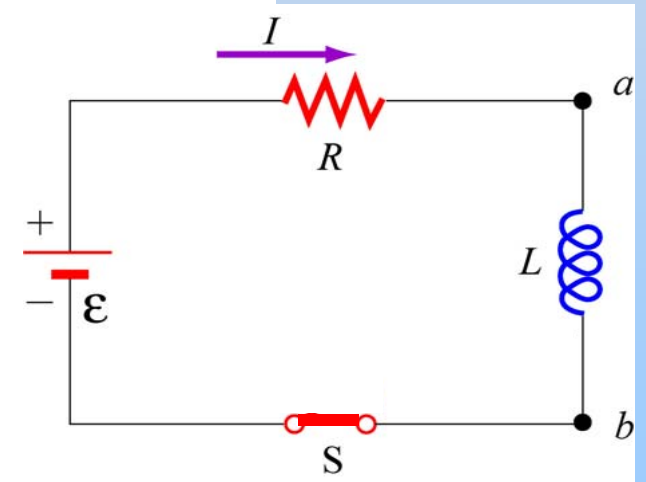
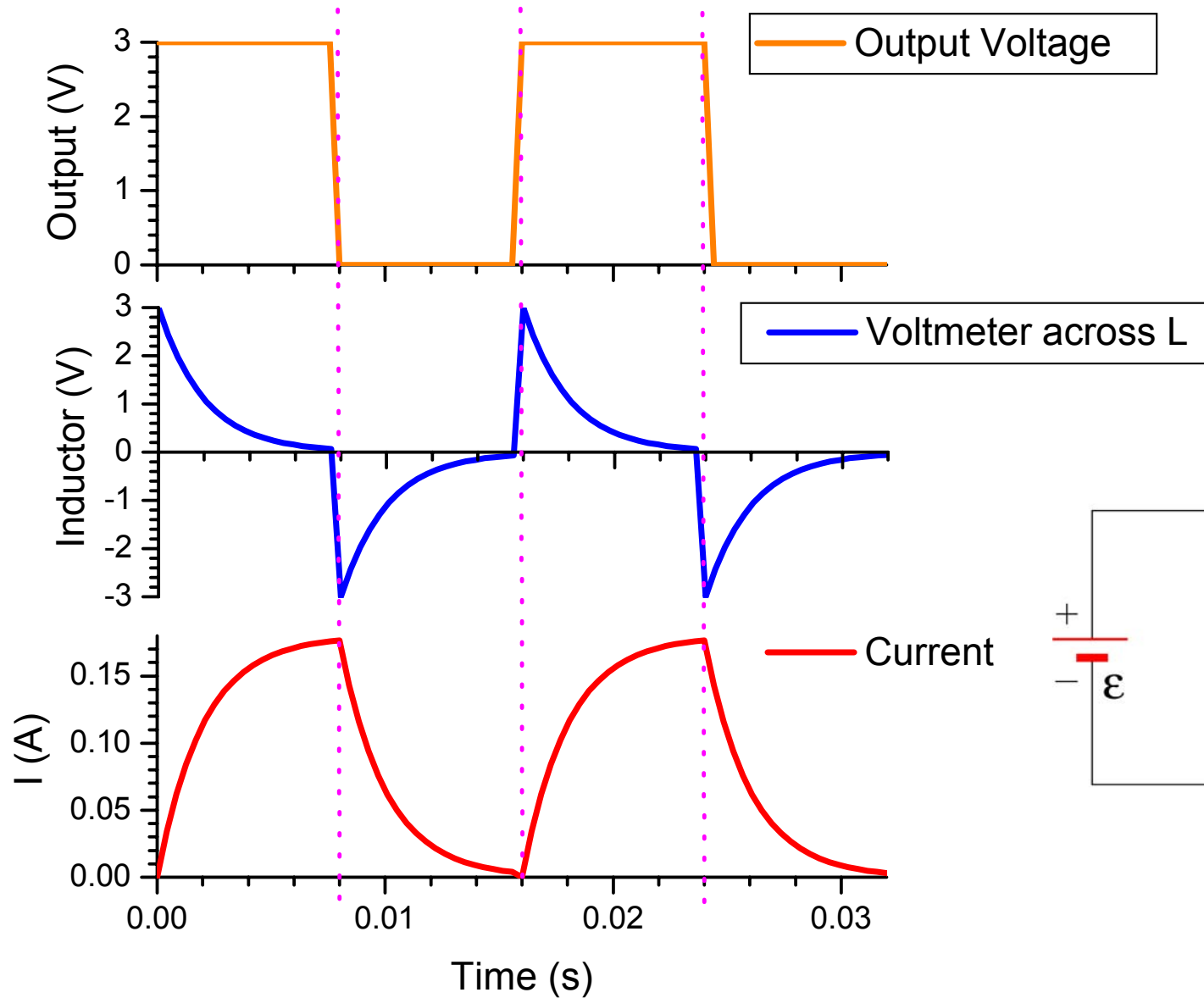
LR Circuit



$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

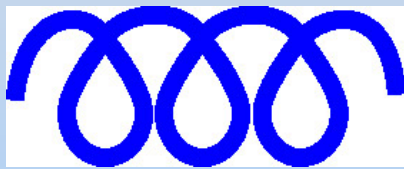
$t=\infty$: Current is steady. Inductor does nothing.

LR Circuit: AC Output Voltage

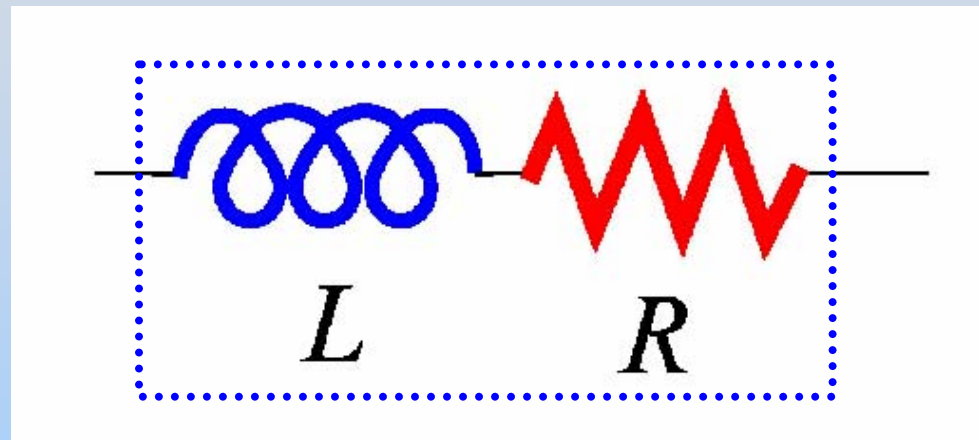


Non-Ideal Inductors

Non-Ideal (Real) Inductor: Not only L but also some R

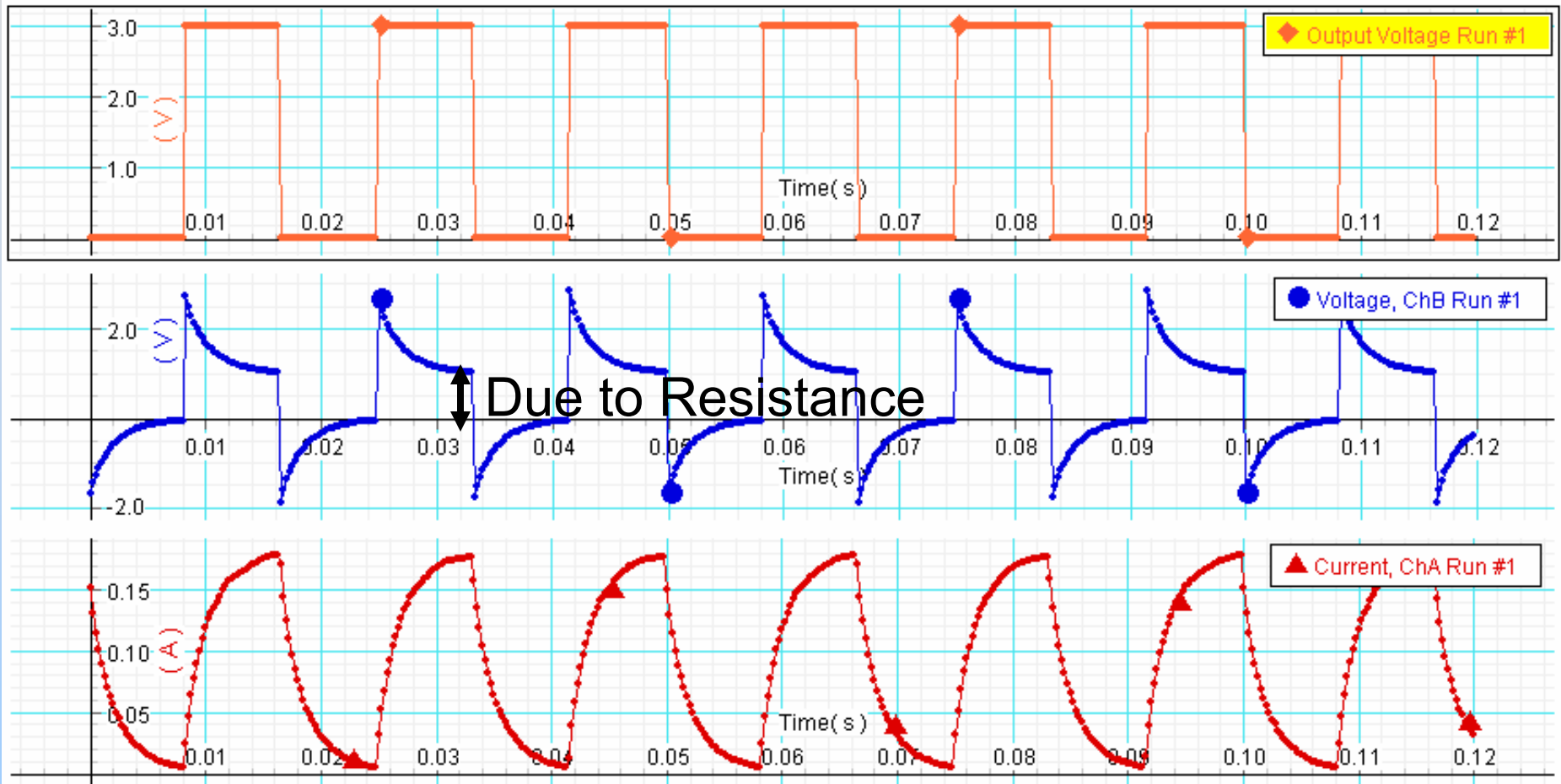


=



In direction of current: $\mathcal{E} = -L \frac{dI}{dt} - IR$

LR Circuit w/ Real Inductor



1. Time constant from I or V
2. Check inductor resistance from V just before switch

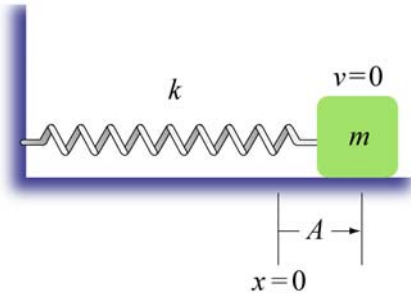
**Experiment 10:
Part I: Measure L, R**

STOP
after you do Part I of Experiment
10 (through page E10-5)

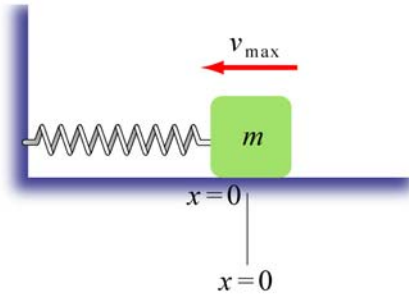
LC Circuits
Mass on a Spring:
Simple Harmonic Motion
(Demonstration)

Mass on a Spring

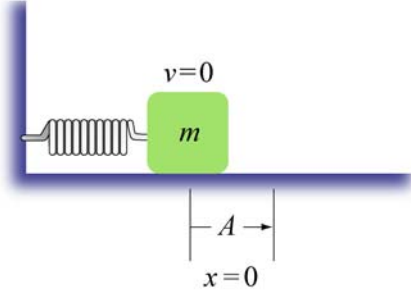
(1)



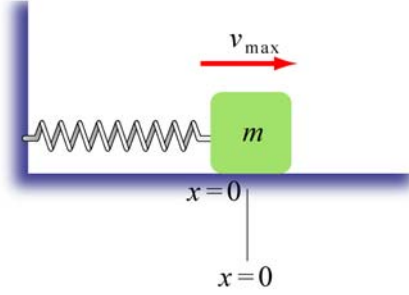
(2)



(3)



(4)



What is Motion?

$$F = -kx = ma = m \frac{d^2 x}{dt^2}$$

$$m \frac{d^2 x}{dt^2} + kx = 0$$

Simple Harmonic Motion

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

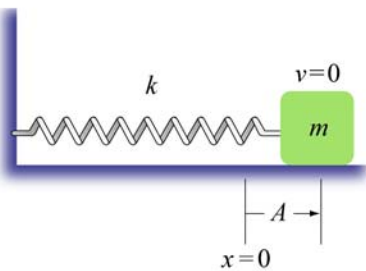
x_0 : Amplitude of Motion

ϕ : Phase (time offset)

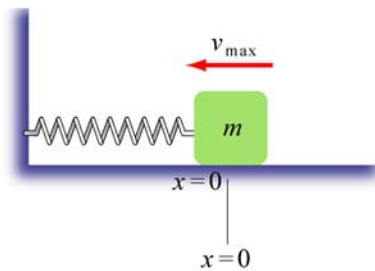
$$\omega_0 = \sqrt{\frac{k}{m}} = \text{Angular frequency}$$

Mass on a Spring: Energy

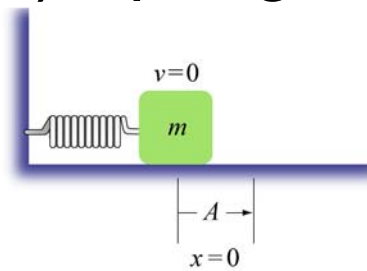
(1) Spring



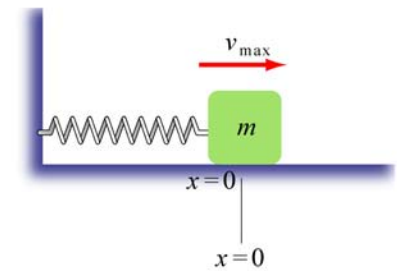
(2) Mass



(3) Spring



(4) Mass



$$x(t) = x_0 \cos(\omega_0 t + \phi) \quad x'(t) = -\omega_0 x_0 \sin(\omega_0 t + \phi)$$

Energy has 2 parts: (Mass) Kinetic and (Spring) Potential

$$K = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 = \frac{1}{2} k x_0^2 \sin^2(\omega_0 t + \phi)$$

$$U_s = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^2 \cos^2(\omega_0 t + \phi)$$

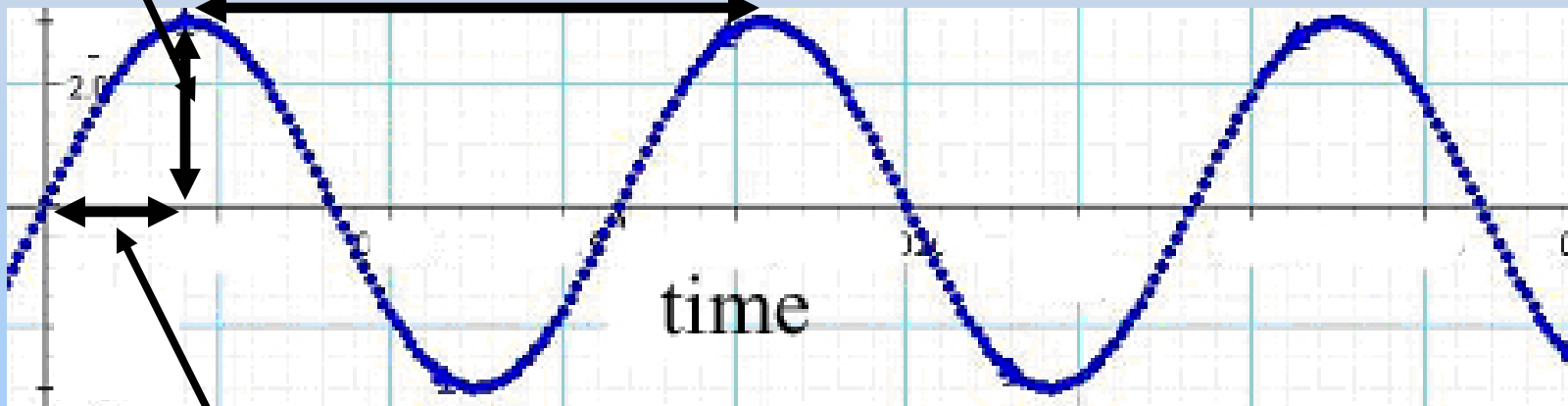
Energy
sloshes back
and forth

Simple Harmonic Motion

$$\text{Period } (T) = \frac{1}{\text{frequency } (f)}$$

$$= \frac{2\pi}{\text{angular frequency } (\omega)}$$

Amplitude (x_0)



$$x(t) = x_0 \cos(\omega_0 t - \phi)$$

Phase Shift (ϕ) = $\frac{\pi}{2}$

Electronic Analog: LC Circuits

Analog: LC Circuit

Mass doesn't like to accelerate

Kinetic energy associated with motion

$$F = ma = m \frac{dv}{dt} = m \frac{d^2 x}{dt^2}; \quad E = \frac{1}{2} mv^2$$

Inductor doesn't like to have current change

Energy associated with current

$$\mathcal{E} = -L \frac{dI}{dt} = -L \frac{d^2 q}{dt^2}; \quad E = \frac{1}{2} LI^2$$

Analog: LC Circuit

Spring doesn't like to be compressed/extended

Potential energy associated with compression

$$F = -kx; \quad E = \frac{1}{2} kx^2$$

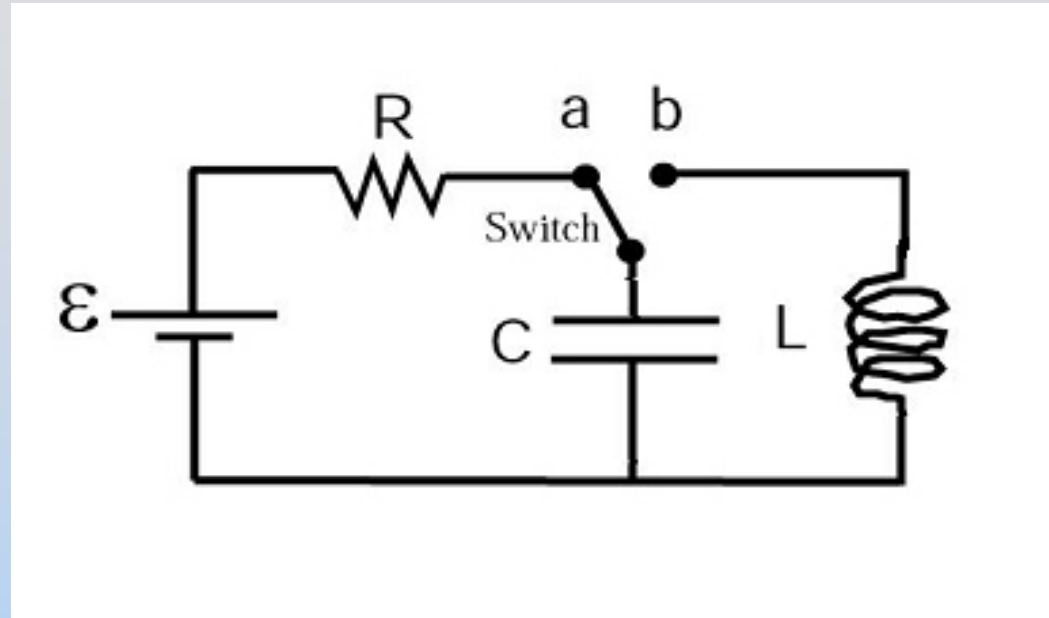
Capacitor doesn't like to be charged (+ or -)

Energy associated with stored charge

$$\varepsilon = \frac{1}{C} q; \quad E = \frac{1}{2} \frac{1}{C} q^2$$

$$F \rightarrow \varepsilon; \quad x \rightarrow q; \quad v \rightarrow I; \quad m \rightarrow L; \quad k \rightarrow C^{-1}$$

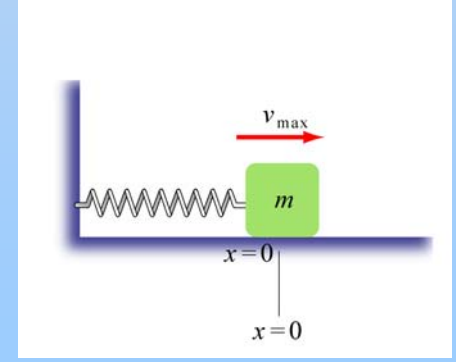
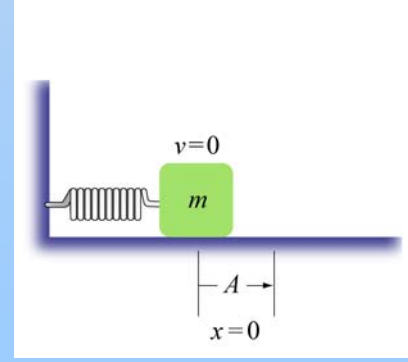
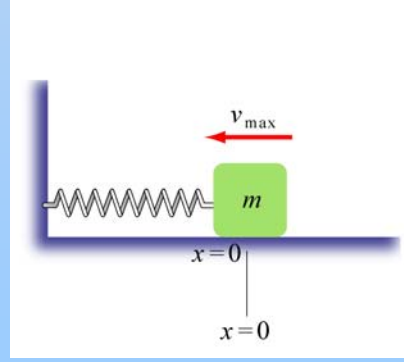
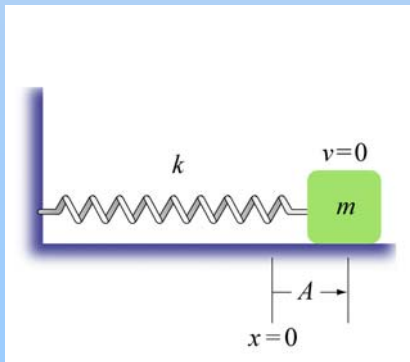
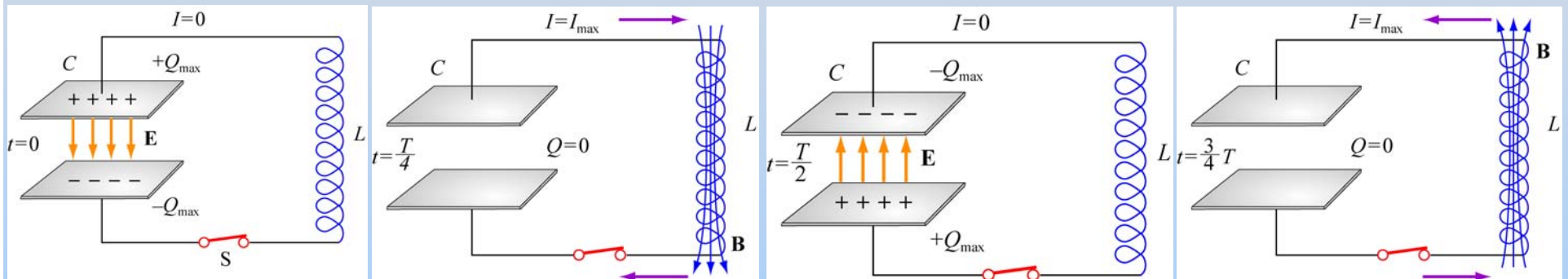
LC Circuit



1. Set up the circuit above with capacitor, inductor, resistor, and battery.
2. Let the capacitor become fully charged.
3. Throw the switch from a to b
4. What happens?

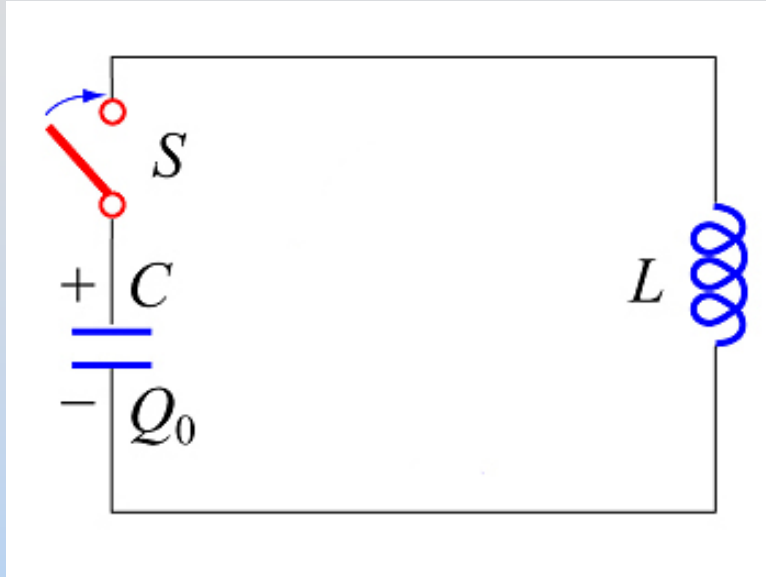
LC Circuit

It undergoes simple harmonic motion, just like a mass on a spring, with trade-off between charge on capacitor (Spring) and current in inductor (Mass)



PRS Questions: LC Circuit

LC Circuit



$$\frac{Q}{C} - L \frac{dI}{dt} = 0 \quad ; \quad I = -\frac{dQ}{dt}$$

$$\frac{d^2 Q}{dt^2} + \frac{1}{LC} Q = 0$$

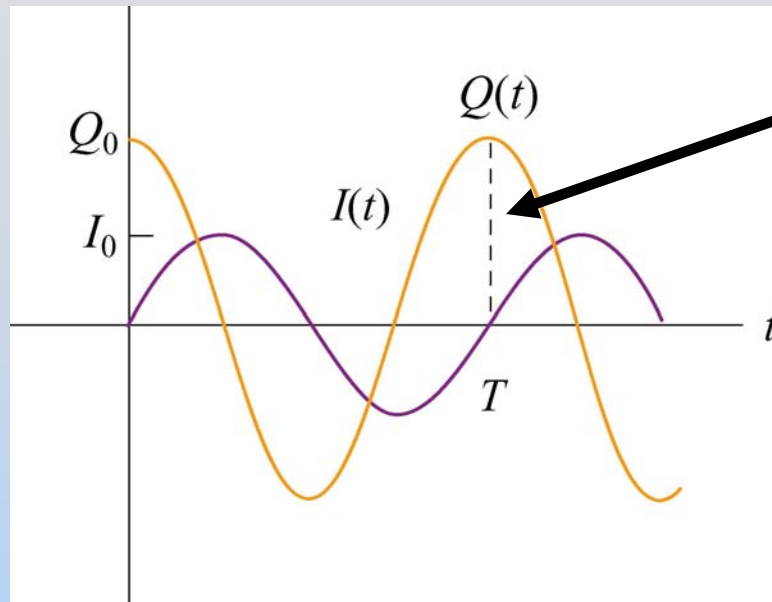
Simple Harmonic Motion

$$Q(t) = Q_0 \cos(\omega_0 t + \phi) \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

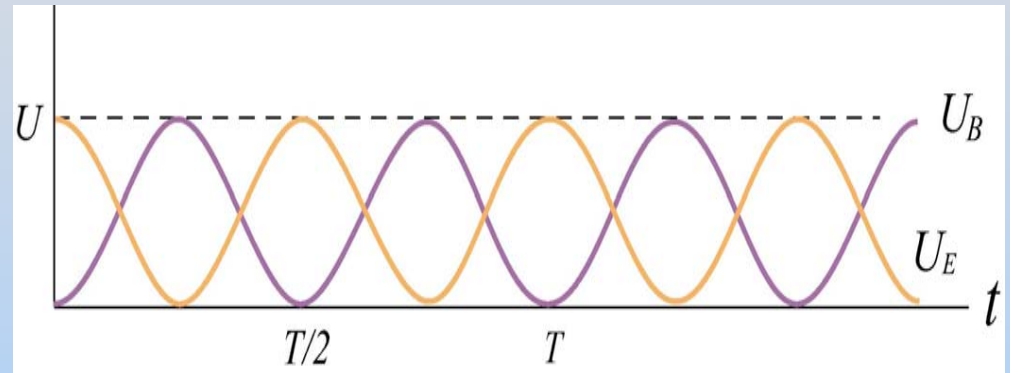
Q_0 : Amplitude of Charge Oscillation

ϕ : Phase (time offset)

LC Oscillations: Energy



Notice relative phases



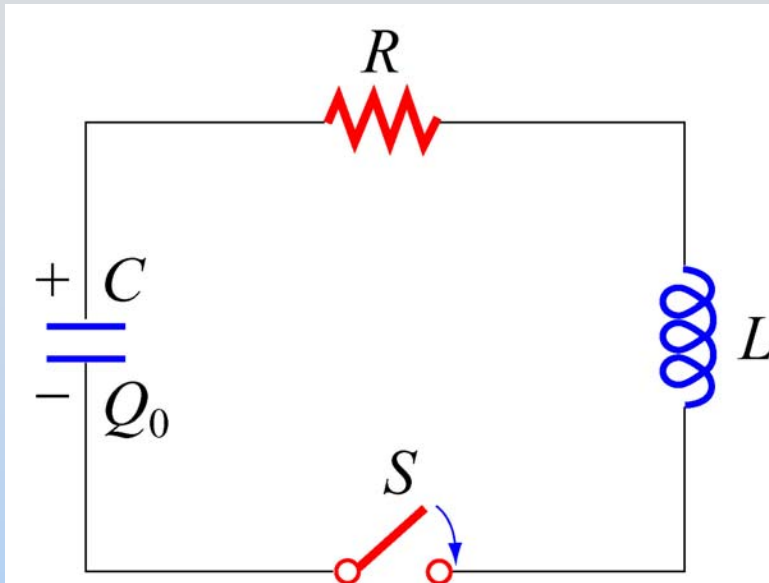
$$U_E = \frac{Q^2}{2C} = \left(\frac{Q_0^2}{2C} \right) \cos^2 \omega_0 t \quad U_B = \frac{1}{2} LI^2 = \frac{1}{2} LI_0^2 \sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C} \right) \sin^2 \omega_0 t$$

$$U = U_E + U_B = \frac{Q^2}{2C} + \frac{1}{2} LI^2 = \frac{Q_0^2}{2C}$$

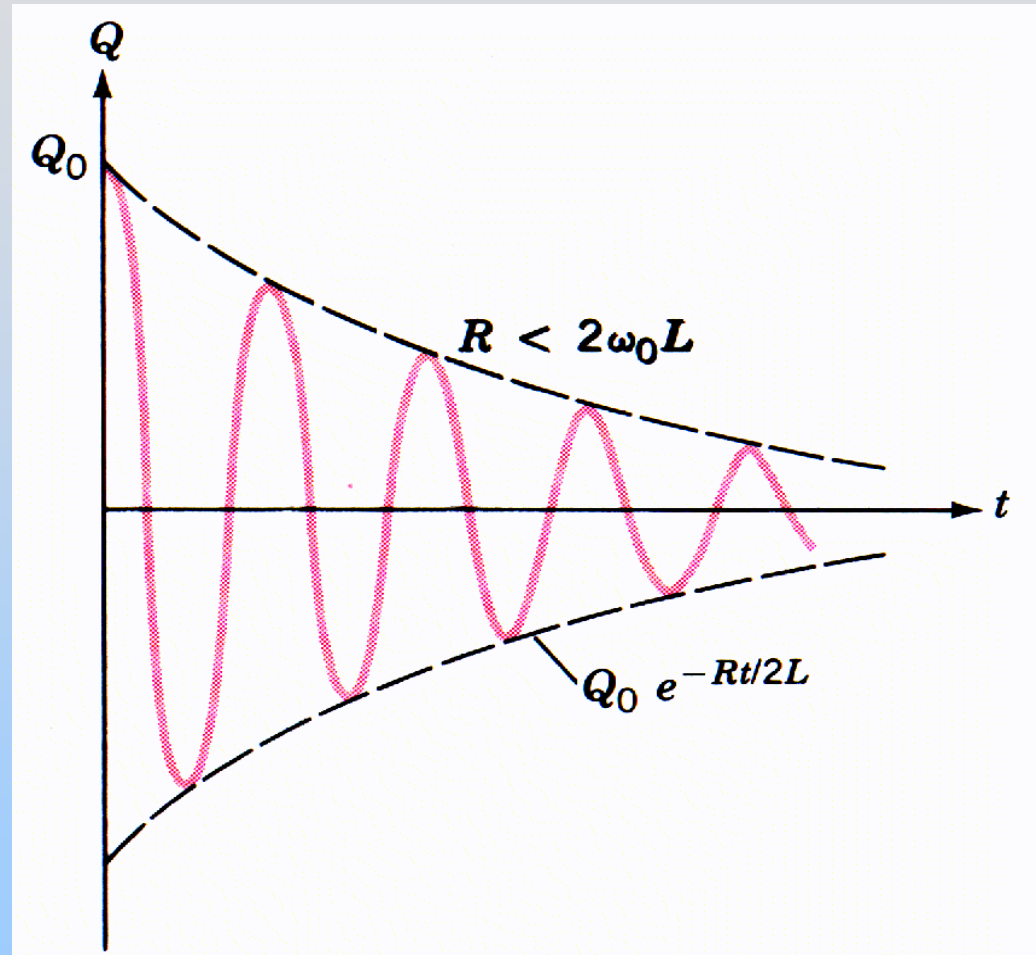
Total energy is conserved !!

Adding Damping: RLC Circuits

Damped LC Oscillations



Resistor dissipates energy and system rings down over time



Also, frequency decreases: $\omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$

Experiment 10: Part II: RLC Circuit

Use Units

PRS Questions: 2 Lab Questions