MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 6: Calculating the Magnetic Field using Ampere's Law

OBJECTIVES

1. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius *a* and outer radius *b* using Ampere's Law.

2. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: Section 9-3, 8.02 Course Notes.

Problem-Solving Strategy for Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$ around any close loop is proportional to the total steady current passing through any surface that is bounded by the close loop:

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

(1) Draw the Amperian loop.

(2) Find the current enclosed by the Amperian loop.

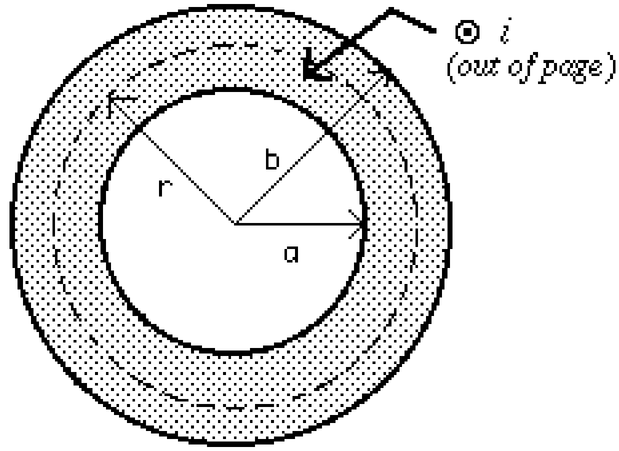
(3) Calculate the line integral $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$ around the closed loop.

(4) Equate $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

Example 1 : Magnetic Field of a Cylindrical Shell

We now apply this strategy to the following problem. Consider the cylindrical conductor with a hollow center and copper walls of thickness $b - a$ as shown . The radii of the inner and outer walls are *a* and *b* respectively, and the current *i* is uniformly spread over the cross section of the copper (shaded region). We want to calculate the magnetic field in the region $a < r < b$.

Question 1 ***(write your answer on the tear-sheet at the end!!!)*:** What is the magnitude of the current per unit area J in the region $a < r < b$? Remember we are assuming that the current I in amps is uniformly spread over the area $a < r < b$, and also remember that current density J is defined as the current per unit area.



Problem Solving Strategy Step (1) <u>Draw Amperian Loop</u>:

Here we take a loop that is a circle of radius r with $a < r < b$ (see figure).

Problem Solving Strategy Step (2) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is that part of the area of the imaginary loop in which the current density J is non-zero, times the current density J.

Question 2 ***(write your answer on the tear-sheet at the end!!!)*:** What is the total current in amps enclosed by your imaginary loop of radius r, when $a < r < b$?

Question 3 ***(write your answer on the tear-sheet at the end!!!)*:** Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

Problem Solving Strategy Step (3): <u>The line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$</u>:

Question 4 ***(write your answer on the tear-sheet at the end!!!)*****:** What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

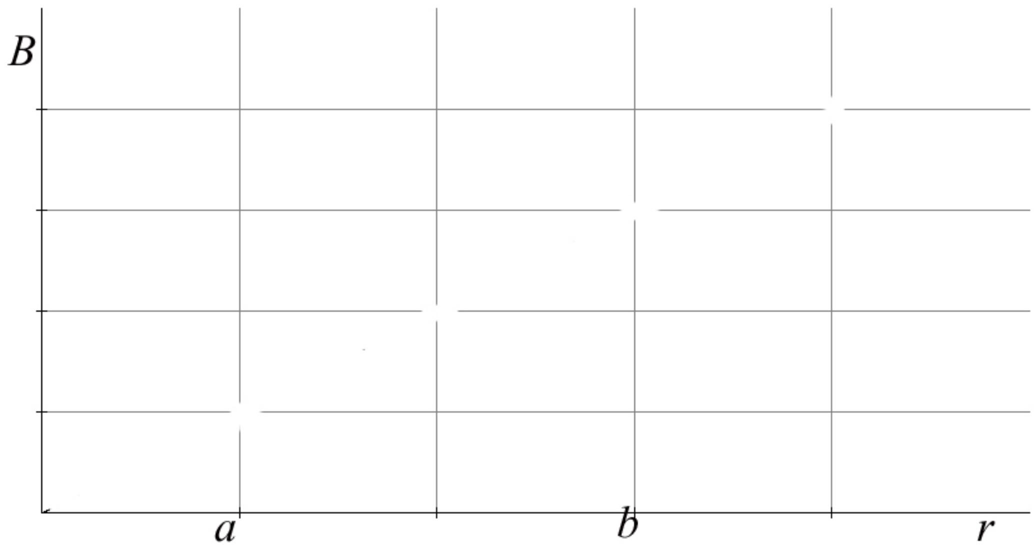
Problem Solving Strategy Step (4): <u>Solve for</u> $\vec{\mathbf{B}}$:

Question 5 (*write your answer on the tear-sheet at the end!!!*): If you equate you answers in Question 4 to your answer in Question 3 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $a < r < b$?

Question 6 (*write your answer on the tear-sheet at the end!!!*): Repeat the steps above to find the magnetic field in the region $r < a$.

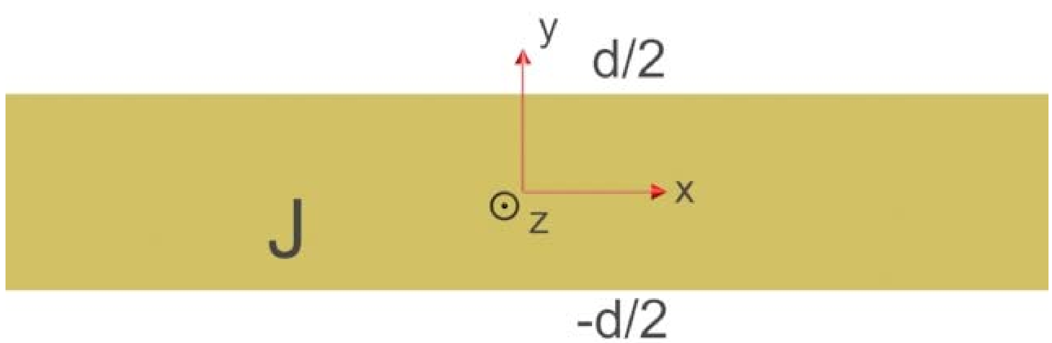
Question 7 (*write your answer on the tear-sheet at the end!!!*): Repeat the steps above to find the magnetic field in the region $r > b$.

Question 8 ***(put your answer on the tear-sheet at the end!!!)*****:** Plot B on the graph below.



Example 2: Magnetic Field of a Slab of Current

We want to find the magnetic field $\vec{\mathbf{B}}$ due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{\mathbf{J}} = J\,\hat{\mathbf{z}}$, where dimensions of J are amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y-direction.



We first want to find the magnetic field in the region $y > d/2$.

Question 9 (***write your answer on the tear-sheet at the end!!!*****):** What is the magnetic field at $y = 0$, where $y = 0$ is the exact center of the loop?

Problem Solving Strategy Step (1) Draw Amperian Loop:

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 9 the magnetic field at $y = 0$. Therefore it makes sense to take what Amperian loop if we want to find the magnetic field for $y > d/2$? .

Question 10 (*write your answer on the tear-sheet at the end!!!*): What Amperian loop do you take to find the magnetic field for $y > d/2$. Draw it on the figure above and also on the tear-sheet at the end, and indicate its dimensions.

Problem Solving Strategy Step (2) Current enclosed by Amperian Loop:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is that part of the area of the imaginary loop in which the current density J is non-zero, times the current density J.

Question 11 *(write your answer on the tear-sheet at the end!!!*): What is the total current in amps enclosed by your Amperian loop from Question 10?

Problem Solving Strategy Step (3): The line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$:

Question 12 (*write your answer on the tear-sheet at the end!!!*): What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Problem Solving Strategy Step (4): <u>Solve for B:</u>

Question 13 (*write your answer on the tear-sheet at the end!!!*): If you equate your answers in Question 12 to your answer in Question 11 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

We now want to find the magnetic field in the region $0 < y < d/2$.

Problem Solving Strategy Step (1) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 9 that the magnetic field at $y = 0$. Therefore it makes sense to take what Amperian loop if we want to find the magnetic field for $0 < y < d/2$? .

Question 14 *(write your answer on the tear-sheet at the end!!!*): What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and on the tear-sheet at the end, and indicate its dimensions.

Problem Solving Strategy Step (2) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is that part of the area of the imaginary loop in which the current density J is non-zero, times the current density J.

Question 15 (*write your answer on the tear-sheet at the end!!!*): What is the total current in amps enclosed by your Amperian loop from Question 14?

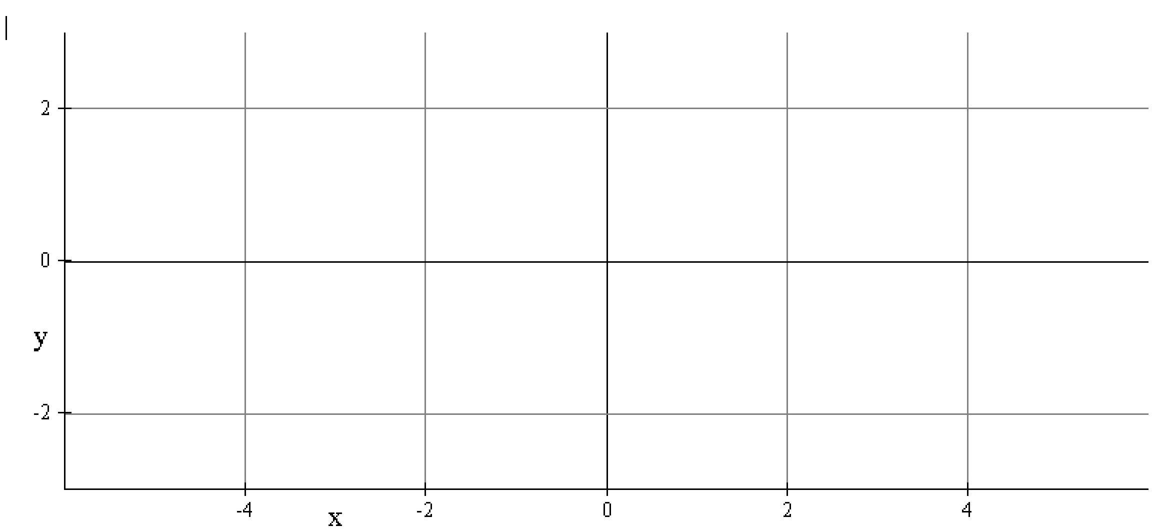
Problem Solving Strategy Step (3) **<u>The line integral</u>** $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ **:**

Question 16 (*write your answer on the tear-sheet at the end!!!*): What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Problem Solving Strategy Step (4) **<u>Solve for B:</u>**

Question 17 (*write your answer on the tear-sheet at the end!!!*): If you equate you answers in Question 16 to your answer in Question 15 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

Question 18 *(put your answer on the tear-sheet at the end!!!)*: Plot B_x on the graph below.



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Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a Committee on Discipline offense.

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Group ____________________________________ **(e.g. 6A Please Fill Out)**

Names ____________________________________

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Question 1: What is the magnitude of the current per unit area J in the region $a < r < b$?

Question 2: What is the total current in amps enclosed by your imaginary loop of radius r, when $a < r < b$?

Question 3: Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

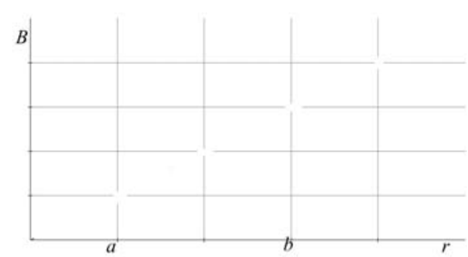
Question 4: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 5: What do you get for the magnetic field in the region $a < r < b$?

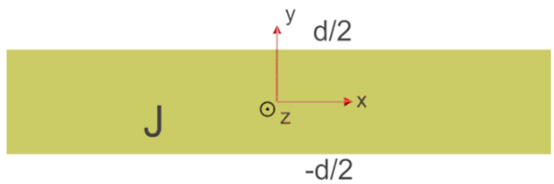
Question 6: What is the magnetic field in the region $r < a$.

Question 7: What is the magnetic field in the region $r > b$.

Question 8: Plot B on the graph.



Example 2: Magnetic Field of a Slab of Current



Question 9: What is the magnetic field at $y = 0$?

Question 10: What Amperian loop do you take to find the magnetic field for $y > d/2$. Draw it on the figure above, and indicate its dimensions.

Question 11: What is the total current in amps enclosed by your Amperian loop from Question 10?

Question 12: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 13: If you equate your answers in Question 12 to your answer in Question 11 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

Question 14: What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and indicate its dimensions.

Question 15: What is the total current in amps enclosed by your Amperian loop from Question 14?

Question 16: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 17: If you equate you answers in Question 16 to your answer in Question 15 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

Question 18: Plot B_x on the graph.