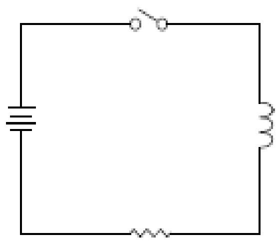
8.022 Lecture Notes Class 36 - 11/21/2006



<u>RLC</u> (Undriven)

V across capacitor

$$\begin{aligned} I &= -\tfrac{dq}{dt} \\ Q &= CV \\ V &= L\tfrac{dI}{dt} + IR \\ I &= -C\tfrac{dV}{dt} \quad (\tfrac{d}{dt}(Q = CV)) \\ V &= L\tfrac{d^2V}{dt^2}C + RC\tfrac{dV}{dt} \end{aligned}$$

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{1}{LC}V = 0$$

Try

$$V = Ae^{-\alpha t}\cos\omega t$$

$$\begin{aligned}\frac{dV}{dt} &= \quad Ae^{-\alpha t}(-\alpha\cos\omega t - \omega\sin\omega t)\\ \frac{d^2V}{dt^2} &= Ae^{-\alpha t}[(\alpha^2-\omega^2)\cos\omega t + 2\alpha\omega\sin\omega t]\end{aligned}$$

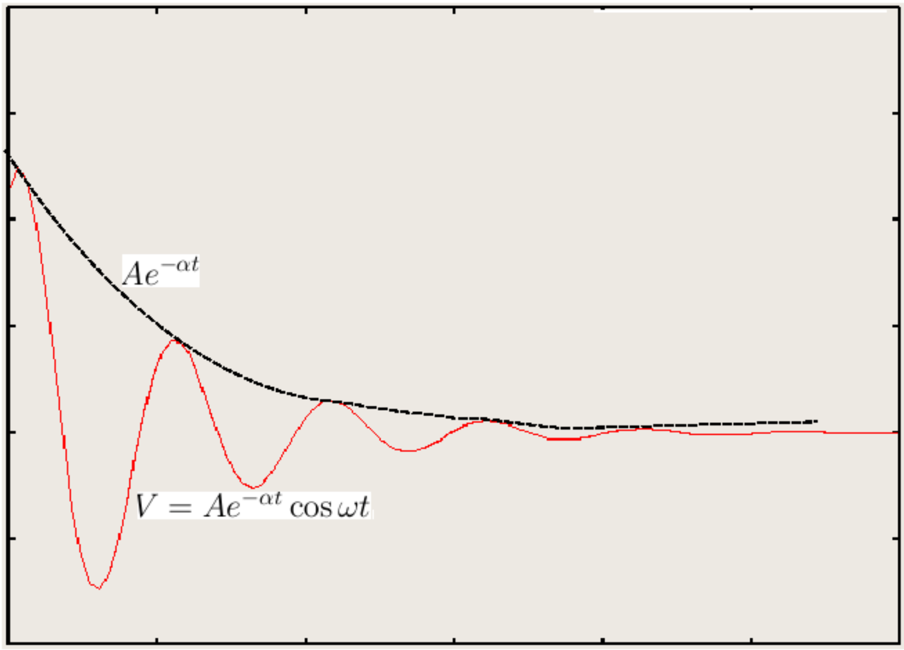
$$(\alpha^2-\omega^2)\cos\omega t + 2\alpha\omega\sin\omega t - \frac{R}{L}(\alpha\cos\omega t + \omega\sin\omega t) + \frac{1}{LC}\cos\omega t = 0$$

$$\begin{cases} 2\alpha\omega - \frac{R\omega}{L} = 0 \Rightarrow \alpha = \frac{R}{2L} \longrightarrow R \\ \alpha^2 - \omega^2 - \frac{\alpha R}{L} + \frac{1}{LC} = 0 \end{cases}$$

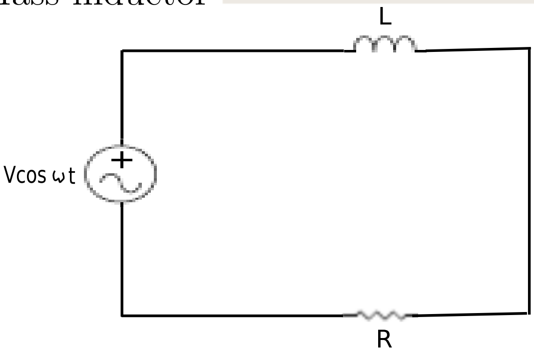
$$\omega^2 = \frac{R^2}{4L^2} - \frac{R^2}{2L^2} + \frac{1}{LC} = \frac{1}{LC} - \frac{R^2}{4L^2}$$

$$\boxed{\begin{gathered}\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\\[1em] R < 2\sqrt{4c}\end{gathered}}$$

$$\longrightarrow \omega = \sqrt{\frac{1}{LC}}$$



Mass-inductor



$$V\cos\omega t = L\frac{dI}{dt} + RI$$

$$I = I\cos(\omega t + \phi)$$