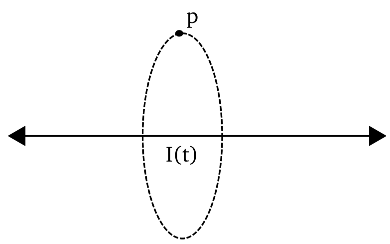
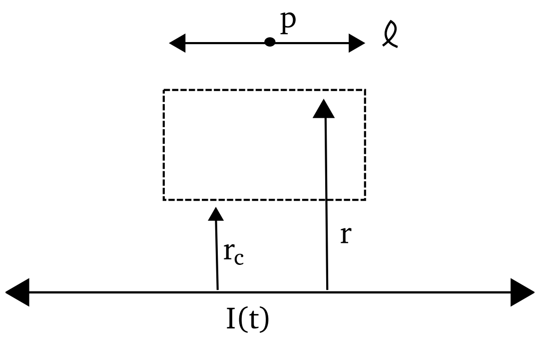
8.022 Lecture Notes Class 35 - 11/20/2006



Hint: Faraday Loop

Find $\vec{E}$ at p

$$\begin{aligned}
\int \vec{B} \cdot d\vec{l} &= \mu_0 I_{enc} \\
B \cdot 2\pi r &= \mu_0 I \\
\vec{B} &= \tfrac{\mu_0 I}{2\pi r}\hat{r} \\
\oint \vec{E} \cdot d\vec{l} &= -\int \tfrac{\partial B}{\partial t} dA \\
&= -\int \tfrac{\partial}{\partial t}(\tfrac{\mu_0 I}{2\pi r})\hat{r} \cdot \hat{n} dA \\
E \cdot 2\pi r &= -\mu_0 \tfrac{\partial}{\partial t}(I(t))\tfrac{1}{2\pi r} \cdot \pi r^2 \\
\vec{E} &= -\tfrac{\mu_0}{4\pi}\tfrac{\partial}{\partial t}(I(t))
\end{aligned}$$

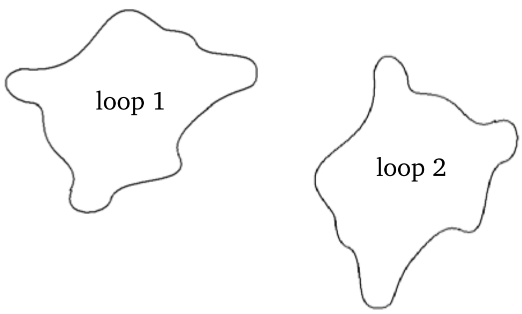


$$B = \frac{\mu_0 I}{2\pi r} \longleftarrow$$ oops doesn't work for rapidly changing I

$$
\begin{aligned}
-\oint \vec{E}\cdot d\vec{l} = &\quad -\tfrac{d}{dt}\int \vec{B}\cdot d\vec{a} \\
= &\quad -\tfrac{d}{dt}\int \tfrac{\mu_0 I}{2\pi r} da \\
= &\ -\tfrac{\mu_0}{2\pi}\cdot\tfrac{dI}{dt}\cdot l\int_{r_0}^{r}\tfrac{1}{r'}dr'
\end{aligned}
$$

$$
\begin{aligned}
-l\cdot E(r) + l\cdot E(r_0) = &\quad -\tfrac{\mu_0}{2\pi}\tfrac{d\hat{I}}{dt}\cdot l(lnr - lnr_0) \\
-E(r) + E(r_0) = &\quad -\tfrac{\mu_0}{2\pi}\cdot\tfrac{dI}{dt}(lnr - lnr_0) \\
\text{Let } r_0 = 1 & \\
-E(r) + E(1) = &\quad -\tfrac{\mu_0}{2\pi}\cdot\tfrac{dI}{dt}lnr \\
E(r) = &\ \tfrac{\mu_0}{2\pi}\cdot\tfrac{dI}{dt}lnr + C \quad \epsilon \to \infty \text{ as } r \to \infty?
\end{aligned}
$$

Inductance



$$\vec{B}_1 = \frac{\mu_0}{2\pi} I_1 \int \frac{d\vec{l_1}\times\hat{r}}{r^2}$$

$$\vec{B}_1 \alpha I_2$$

$$\Phi_2 = \int \vec{B}_1\cdot d\vec{a_2} = M_{21}\cdot I_1$$

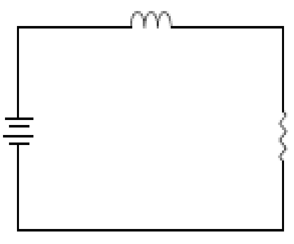
Φ_2 is flux from 1 thru 2

$$
\begin{aligned}
M_{21}I_1 &= \int \vec{B}_1 \cdot d\vec{a}_2 \\
&= \vec{\nabla} \times \vec{A}_1 d\vec{a}_2 \\
&= \int A_1 \cdot d\vec{l_2} \quad (A_1 = \tfrac{\mu_0}{4\pi} \oint \tfrac{dl_1}{r}) \\
&= \tfrac{\mu_0 I_1}{4\pi} \oint\oint (\tfrac{dl_1}{r}))dl_2 = \tfrac{\mu_0}{4\pi} \oint\oint dl_1 dl_2 I_1
\end{aligned}
$$

$$
\begin{aligned}
M_{21} &= \tfrac{\mu_0}{4\pi} \int\oint dl_1 dl_2 \cdot M_{12} = M \\
\varepsilon_2 &= -\tfrac{d\Phi_2}{dt} \\
\varepsilon_2 &= -M \cdot \tfrac{dI_1}{dt}
\end{aligned}
$$

$$\Phi = LI$$

$$\boxed{\varepsilon = -L\frac{dI}{dt}}$$



$$V - L \cdot \frac{dI}{dt} - IR = 0$$

$$I(t) = \frac{V}{R}(1 - e^{-(\frac{R}{L})t}) \qquad \frac{R}{L} = -\tau$$