8.022 Lecture Notes Class 16 - 10/05/2006

q at $\vec{x} = 0$

$$V(r) = \begin{cases} \frac{q_0}{4\pi\epsilon_0 r} & r < r_0 \\ \frac{q_0[r^2+(r-r_0)^2]}{4\pi\epsilon_0 r^3} & r > r_0 \end{cases}$$

Find $\vec{E}$ everywhere and find σ on surface.

$$V = -\int_\theta^{r_0} \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\nabla \cdot V$$

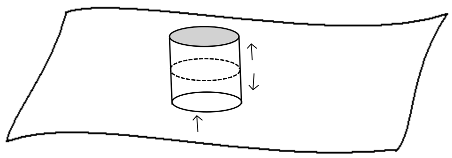
For $r < r_0$

$$\begin{aligned} \vec{E} &= \frac{d}{dr}\left(\frac{q_0}{4\pi\epsilon_0 r}\hat{r}\right) \\ \vec{E} &= \frac{q_0}{4\pi\epsilon_0 r^2}\hat{r} \end{aligned}$$

For $r > r_0$

$$\vec{E} = -\nabla \cdot V$$

$$\begin{aligned} \vec{E} &= -\frac{d}{dr}\left(\frac{q_0(r_0^2+(r-r_0)^2)}{4\pi\epsilon_0 r^3}\right) \\ &= -\frac{q_0}{4\pi\epsilon_0}\left(\frac{r_0^2}{r^3} + \frac{1}{r} - \frac{2r_0}{r^2} + \frac{r_0^2}{r^3}\right) \\ &= -\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r^2} + 4r_0 r^{-3} - 6r_0^2 r^{-4}\right) \\ &= -\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r^2} + \frac{4r_0}{r^3} - \frac{6r_0^2}{r^4}\right) \\ \vec{E} &= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{4r_0}{r^3} + \frac{6r_0^2}{r^4}\right) \end{aligned}$$



Gauss:

$$\frac{Q_{enc}}{\epsilon_0} = \int E \cdot dA$$

$$\begin{aligned} A \cdot (E_a - E_b) &= \frac{(\sigma \cdot A)}{\epsilon_0} \\ E_a - E_b &= \frac{\sigma}{\epsilon_0} \\ \sigma &= \epsilon_0 (E_a - E_b) \end{aligned}$$