8.022 (E&M) - Lecture 1

Gabriella Sciolla

Topics:

- How is 8.022 organized?
- Brief math recap
- Introduction to Electrostatics

Welcome to 8.022!

- 8.022: advanced electricity and magnetism for freshmen or electricity and magnetism for advanced freshmen?
  - Advanced!
    - Both integral and differential formulation of E&M
    - Goal: look at Maxwell's equations

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

... and be able to tell what they really mean!

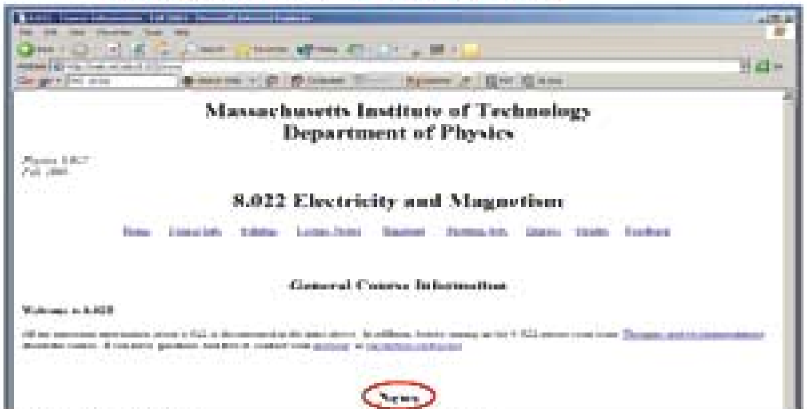
- Familiar with math and very interested in physics
- Fun class but pretty hard: 8.022 or 8.02T?

8.022 web page

Bookmark…
…and watch out for typos!

Everything You Always Wanted to Know About 8.022 But Were Afraid to Ask.

http://web.mit.edu/8.022/www/



Staff and Meetings

Lecturer	Prof. Gabriella Sciolla
Recitations	Prof. Erik Katsavounidis

Lecture	Prof. Sciolla	Tue & Thu	9:30-11:00 AM
Rec. Section #1	Prof. Katsavounidis	Mon & Wed	10-11 AM
Rec. Section #2	Prof. Katsavounidis	Mon & Wed	11-12 AM
Rec. Section #3	Prof. Katsavounidis	Tue & Thu	2-3 PM
Rec. Section #4	Prof. Katsavounidis	Tue & Thu	3-4 PM

Textbook

E. M. Purcell

Electricity and Magnetism

Volume 2 - Second edition

- Advantages:
  - Bible for introductory E&M
    for generations of physicists
- Disadvantage:
  - cgs units!!!

Problem sets

- Posted on the 8.022 web page on Thu night and due on Thu at 4:30 PM of the following week
  - Leave them in the 8.022 lockbox at PEO
- Exceptions:
  - Pset 0 (Math assessment) due on Monday Sep. 13
  - Pset 1 (Electrostatiscs) due on Friday Sep. 17
- How to work on psets?
  - Try to solve them by yourself first
  - Discuss problems with friends and study group
  - Write your own solution

Grades

How do we grade 8.022?

- Homeworks and Recitations (25%)
- Two quizzes (20% each)
- Final (35%)
- Laboratory (2 out of 3 needed to pass)

NB: You *may not* pass the course without completing the laboratories!

More info on exams:

- Two in-class (26-100) quiz during normal class hours:
  - Tuesday October 5 (Quiz #1)
  - Tuesday November 9 (Quiz #2)
- Final exam
  - Tuesday, December 14 (9 AM - 12 Noon), location TBD

All grades are available online through the 8.022 web page

...Last but not least...

Come and talk to us if you have problems or questions

- 8.022 course material
  - I attended class and sections and read the book but I still don't understand concept xyz and I am stuck on the pset!
- Math
  - I can't understand how Taylor expansions work or why I should care about them...
- Curriculum
  - is 8.022 right for me or should I switch to TEAL?
- Physics in general!
  - Questions about matter-antimatter asymmetry of the Universe, elementary constituents of matter (Sciolla) or gravitational waves (Kats) are welcome!

Your best friend in 8.022: math

- Math is an essential ingredient in 8.022
  - Basic knowledge of multivariable calculus is essential
  - You must be enrolled in 18.02 or 18.022 (or even more advanced)
- To be proficient in 8.022, you don't need an A+ in 18.022
  - Basic concepts are used!
- Assumption: you are familiar with these concepts already but are a bit rusty…

Let's review some basic concepts right now!

NB: excellent reference: D. Griffiths, Introduction to electrodynamics, Chapter 1.

Derivative

- Given a function f(x), what is it's derivative?

$$df = \frac{\partial f}{\partial x} dx$$

  - The derivative $\frac{\partial f}{\partial x}$ tells us how fast f varies when x varies.

→ The derivative is the proportionality factor between a change in x and a change in f.

- What if f=f(x,y,z)?

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

Gradient

Let's define the infinitesimal displacement $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \bullet (dx, dy, dz) = \nabla f \bullet d\vec{l}$$

Definition of Gradient:

$$\text{grad}\, f \equiv \nabla f \equiv \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \equiv \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

Conclusions:

- ∇f measures how fast f(x,y,z) varies when x, y and z vary
- Logical extension of the concept of derivative!
- f is a scalar function but ∇f is a vector!

The "del" operator

Definition:

$$\vec{\nabla} \equiv \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Properties:

- It looks like a vector
- It works like a vector
- But it's not a real vector because it's meaningless by itself. It's an operator.

How it works:

It can act on both scalar and vector functions:

- Acting on a scalar function: gradient $\vec{\nabla} f$ (vector)
- Acting on a vector function with dot product: divergence $\vec{\nabla} \bullet \vec{f}$ (scalar)
- Acting on a scalar function with cross product: curl $\vec{\nabla} \times \vec{f}$ (vector)

Divergence

Given a vector function $\vec{v}(x, y, z)$

$$\vec{v}(x, y, z) \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z} \equiv (v_x, v_y, v_z)$$

we define its divergence as:

$$\text{div}\,\vec{v} \equiv \vec{\nabla} \bullet \vec{v} \equiv \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Observations:

- The divergence is a scalar
- Geometrical interpretation: it measures how much the function $\vec{v}(x, y, z)$ "spreads around a point".

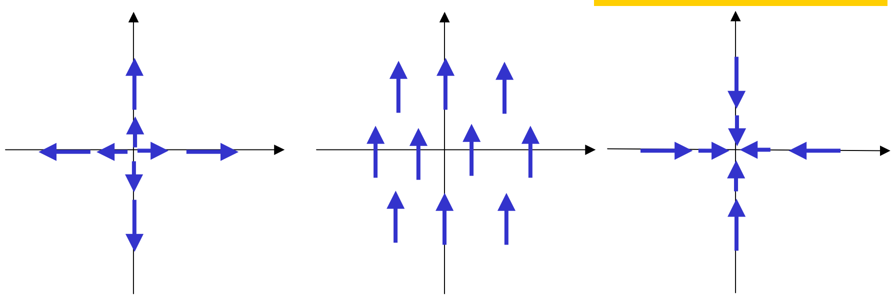
Divergence: interpretation

Calculate the divergence for the following functions:

$\vec{v}(x,y,z) = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{v}(x,y,z) = \hat{z}$

$\vec{v}(x,y,z) = -x\hat{x} - y\hat{y} - z\hat{z}$



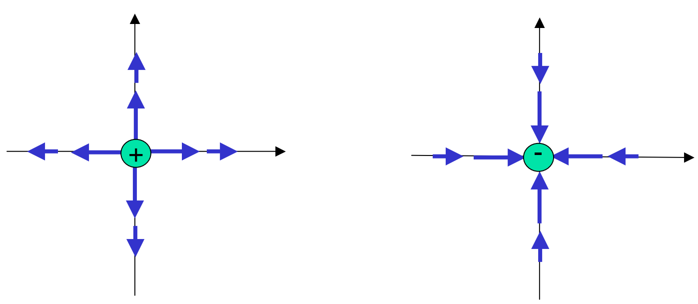
div v=3>0 (faucet)

div v=0

div v = -3 (sink)

Does this remind you of anything?

Electric field around a charge has divergence .ne. 0 !



div E>0 for + charge: faucet

div E <0 for – charge: sink

Curl

Given a vector function $\vec{v}(x, y, z)$

$$\vec{v}(x, y, z) \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z} \equiv (v_x, v_y, v_z)$$

we define its curl as:

$$\vec{\nabla} \times \vec{v} \equiv \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

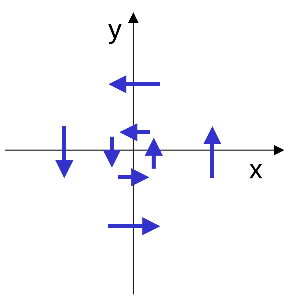
Observations:

- The curl is a vector
- Geometrical interpretation: it measures how much the function $\vec{v}(x, y, z)$ "curls around a point".

Curl: interpretation

Calculate the curl for the following function:

$$\vec{v}(x, y, z) = -y\hat{x} + x\hat{y}$$

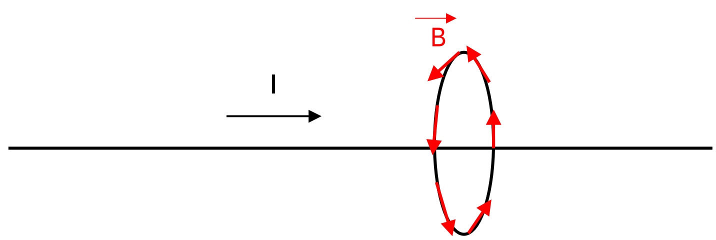


$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix} = 2\hat{k}$$

This is a vortex: non zero curl!

Does this sound familiar?

Magnetic filed around a wire :



$$\vec{\nabla} \times \vec{B} \neq 0$$

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An now, our feature presentation:
Electricity and Magnetism

The electromagnetic force:
Ancient history...

- 500 B.C. – Ancient Greece
  - Amber (ελεχτρον="electron") attracts light objects
  - Iron rich rocks from μαγνεσια (Magnesia) attract iron
- 1730 - C. F. du Fay: Two flavors of charges
  - Positive and negative
- 1766-1786 – Priestley/Cavendish/Coulomb
  - EM interactions follow an inverse square law:
  - Actual precision better than $2/10^9$!

$$F_{em} \propto \frac{q_1 q_2}{r^2}$$

- 1800 – Volta
  - Invention of the electric battery

N.B.: Till now Electricity and Magnetism are disconnected!

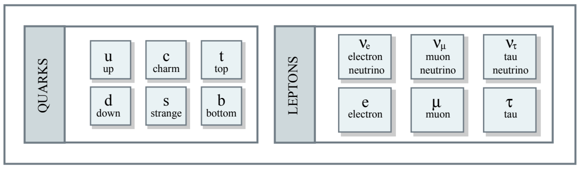
The electromagnetic force:
...History... (cont.)

- 1820 – Oersted and Ampere
  - Established first connection between electricity and magnetism
- 1831 – Faraday
  - Discovery of magnetic induction
- 1873 – Maxwell: Maxwell's equations
  - The birth of modern Electro-Magnetism
- 1887 – Hertz
  - Established connection between EM and radiation
- 1905 – Einstein
  - Special relativity makes connection between Electricity and Magnetism as natural as it can be!

The electromagnetic force:

Modern Physics!

- The Standard Model of Particle Physics
  - Elementary constituents: 6 quarks and 6 leptons



  - Four elementary forces mediated by 5 bosons:

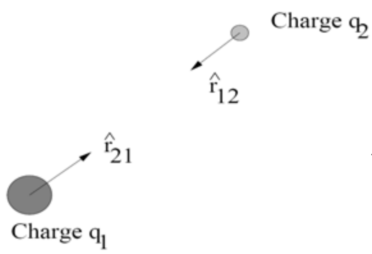
Interaction	Mediator	Relative Strength	Range (cm)
Strong	Gluon	10^{37}	10^{-13}
Electromagnetic	Photon	10^{35}	Infinite
Weak	$W^{+/-}$, Z^0	10^{24}	10^{-15}
Gravity	Graviton?	1	Infinite

The electric charge

D1, D2, D4

- The EM force acts on charges
  - 2 flavors: positive and negative
    - Positive: obtained rubbing glass with silk
    - Negative: obtained rubbing resin with fur
- Electric charge is quantized (Millikan)
  - Multiples of the **e** = elementary charge
    - **e** = 1.602 10^{-19} C (SI), 4.803 10^{-10} esu (cgs)
    - $Q_{electron}$ = -**e**; Q_{proton} = +**e**
- Electric charge is conserved
  - In any isolated system, the total charge cannot change
    - If the total charge of a system changes, then it means the system is not isolated and charges came in or escaped.

Coulomb's law



$$\vec{F}_2 = k \frac{q_1 q_2}{|r_{21}|^2} \hat{r}_{21}$$

- Where:
  - $\vec{F}_2$ is the force that the charge q_2 feels due to q_1
  - $\hat{r}_{21}$ is the unit vector going from q_1 to q_2
- Consequences:
  - Newton's third law: $\vec{F}_2 = -\vec{F}_1$
  - Like signs repel, opposite signs attract

Units: cgs vs SI

- Units in cgs and SI (Sisteme Internationale)

	cgs	SI
Length	cm	m
Mass	g	Kg
Time	s	s
Charge	electrostatic units (e.s.u.)	Coulomb (C)
Current	e.s.u./s	Ampere (A)

- In cgs the esu is defined so that k=1 in Coulomb's law →

$$1 \text{ dyne} = \frac{(1\text{esu})^2}{(1\text{cm})^2} \longrightarrow 1 \text{ esu} = \text{cm}\sqrt{\text{dyne}}$$

- In SI, the Ampere is a fundamental constant
  - $k=1/(4\pi\varepsilon_0)=8.99\ 10^9$ N C^{-2} m^2
  - $\varepsilon_0=8.8\text{x}10^{-12}$ C^2 N^{-1} m^{-2} is the permittivity of free space

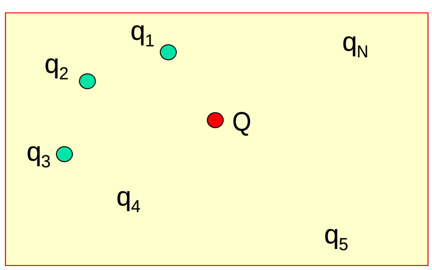
Practical info: cgs - SI conversion table

	SI Units		CGS units
Energy	1 Joule	=	10^7 erg
Force	1 Newton	=	10^5 dyne
Charge	1 Coulomb	=	"3" $\times 10^9$ esu
Current	1 Ampere	=	"3" $\times 10^9$ esu/sec
Potential	"3" $\times 10^2$ Volts	=	1 statvolt
Electric field	"3" $\times 10^4$ Volts/m	=	1 statvolt/cm
Magnetic field	1 Tesla	=	10^4 gauss
Capacitance	1 Farad	=	"9" $\times 10^{11}$ cm
Resistance	"9" $\times 10^{11}$ Ohm	=	1 sec/cm
Inductance	"9" $\times 10^{11}$ Henry	=	1 sec^2/cm

"3"=2.9979... =c

- FAQ: why do we use cgs?
  - Honest answer: because Purcell does...

The superposition principle: discrete charges

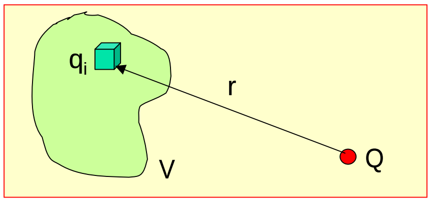


The force on the charge Q due to all the other charges is equal to the vector sum of the forces created by the individual charges:

$$\vec{F}_Q = \frac{q_1 Q}{|r_1|^2}\hat{r}_1 + \frac{q_2 Q}{|r_2|^2}\hat{r}_2 + \ldots + \frac{q_N Q}{|r_N|^2}\hat{r}_N = \sum_{i=1}^{i=N} \frac{q_i Q}{|r_i|^2}\hat{r}_i$$

The superposition principle: continuous distribution of charges

What happens when the distribution of charges is continuous?
Take the limit for $q_i \rightarrow dq$ and $\Sigma \rightarrow$ integral:



$$\vec{F}_Q = \sum_{i=1}^{i=N} \frac{q_i\ Q}{|r_i|^2}\hat{r}_i \rightarrow \int_V \frac{dq\ Q}{|r|^2}\hat{r} = \int_V \frac{\rho\ dV\ Q}{|r|^2}\hat{r}$$

where ρ = charge per unit volume: "volume charge density"

The superposition principle: continuous distribution of charges (cont.)

- Charges are distributed inside a volume V:

$$\vec{F}_Q = \int_V \frac{\rho\ dV\ Q}{|r|^2}\hat{r}$$

- Charges are distributed on a surface A:

$$\vec{F}_Q = \int_A \frac{\sigma\ da\ Q}{|r|^2}\hat{r}$$

- Charges are distributed on a line L:

$$\vec{F}_Q = \int_L \frac{\lambda\ dl\ Q}{|r|^2}\hat{r}$$

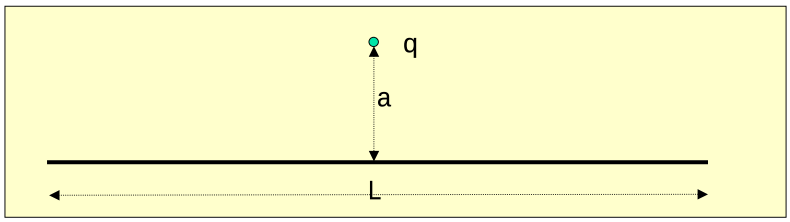
Where:

- ρ = charge per unit volume: "volume charge density"
- σ = charge per unit area: "surface charge density"
- λ = charge per unit length: "line charge density"

Application: charged rod

P: A rod of length L has a charge Q uniformly spread over it. A test charge q is positioned at a distance a from the rod's midpoint.

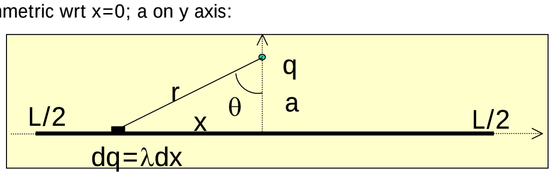
Q: What is the force F that the rod exerts on the charge q?



Answer: $\vec{F} = \dfrac{Qq}{a\sqrt{a^2 + \left(\frac{L}{2}\right)^2}}\hat{y}$

Solution: charged rod

- Look at the symmetry of the problem and choose appropriate coordinate system: rod on x axis, symmetric wrt x=0; a on y axis:



- Symmetry of the problem: F // y axis; define λ=Q/L linear charge density
- Trigonometric relations: x/a=tgθ; a=r cosθ → dx=dθ/cos2θ; r=a/cosθ
- Consider the infinitesimal charge dF_y produced by the element dx:

$$dF_y = dF\cos\theta = \frac{\lambda dx}{r^2}q\cos\theta = \lambda q\frac{\frac{a d\theta}{\cos^2\theta}}{\frac{a^2}{\cos^2\theta}}\cos\theta = \frac{\lambda q}{a}\cos\theta d\theta$$

- Now integrate between –L/2 and L/2: $\vec{F} = \hat{y}\int_{-L/2}^{L/2}\frac{\lambda q}{a}\cos\theta d\theta = \dfrac{Qq}{a\sqrt{a^2+\left(\frac{L}{2}\right)^2}}\hat{y}$

Infinite rod? Taylor expansion!

Q: What if the rod length is infinite?

P: What does "infinite" mean? For all practical purposes, infinite means >> than the other distances in the problem: L>>a:

Let's look at the solution: $\vec{F} = \frac{Qq}{a\sqrt{a^2 + \left(\frac{L}{2}\right)^2}}\hat{y}$

Taylor expand using $(2a/L)^2$ as expansion coefficient remembering that

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \ldots \text{ for } x^2 < 1$$

and

$$(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \ldots \text{ for } x^2 < 1$$

→

$$F = \frac{\frac{\lambda L q}{a}}{\frac{L}{2}\left(1 + \frac{2a}{L}\right)^{\frac{1}{2}}} = \frac{\lambda q}{2a}\left(1 + \left(\frac{2a}{L}\right)^2\right)^{-\frac{1}{2}} = \frac{\lambda q}{2a}\left(1 - \frac{1}{2}\left(\frac{2a}{L}\right)^2 + \ldots\right) \sim \frac{\lambda q}{2a}$$

Rusty about Taylor expansions?

Here are some useful reminders...

Exponential function and natural logarithm:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n \quad \text{for } |x| < 1$$

Geometric series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

Trigonometric functions:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \quad \text{for all } x$$