Massachusetts Institute of technology
Department of Physics
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Final: Formula sheet

Potential: $\phi(a)-\phi(b)=-\int_b^a \vec{E}\cdot d\vec{s}$

Energy of E: The energy of an electrostatic configuration $U=\frac{1}{2}\int_V \rho\phi dV=\frac{1}{8\pi}\int E^2 dV$.

Pressure: A layer of surface charge density σ exerts a pressure $P=2\pi\sigma^2$.

Current density: $\vec{J}=\rho\vec{v}$ **.**

Current: $I=dQ/dt=\int_S \vec{J}\cdot d\vec{a}$ (I is the current through surface S).

Continuity: $\vec{\nabla}\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$

Ohm's law: $\vec{J}=\sigma_c\vec{E}$ (microscopic form); $V=IR$ (macroscopic form)

Kirchhoff's laws: Sum of the EMFs and voltage drops around a closed loop is zero; Current into a junction equals current out.

Capacitance: $Q=CV$. **Energy stored in capacitor:** $U_C=\frac{Q^2}{2C}=\frac{1}{2}CV^2$

Lorentz force: $\vec{F}=q\vec{E}+q\frac{\vec{v}}{c}\times\vec{B}$

Magnetic force on current: $\vec{F}=\frac{I}{c}d\vec{l}\times\vec{B}$**;** or $\vec{F}/L=\frac{\vec{I}}{c}\times\vec{B}$

Vector potential: $\vec{B}=\nabla\times\vec{A}$; $\vec{A}=\frac{I}{c}\int\frac{d\vec{l}}{r}$

Biot-Savart law: $d\vec{B}=Id\vec{l}\times\hat{r}/(cr^2)$

Maxwell's equations in differential form :

$$\vec{\nabla}\cdot\vec{E}=4\pi\rho \quad \text{(Gauss's law)}$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \text{(Faraday's law)}$$

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\vec{J}+\frac{1}{c}\frac{\partial\vec{E}}{\partial t} \quad \text{(Ampere's law)}$$

$$=\frac{4\pi}{c}(\vec{J}+\vec{J}_d) \qquad \vec{J}_d=\frac{1}{4\pi}\frac{\partial\vec{E}}{\partial t}=\text{displacement current density}$$

Maxwell's equations in integral form

$\int_S \vec{E}\cdot d\vec{a} = 4\pi Q$ (Gauss's law. Q is charge enclosed by surface S)

$\int_C \vec{E}\cdot d\vec{s} = -\frac{1}{c}\frac{\partial \phi_B}{\partial t}$ =e.m.f. (Faraday's law. ϕ_B is $\vec{B}$ flux through surface bounded by C.)

$\int_C \vec{B}\cdot d\vec{s} = \frac{4\pi}{c} I + \frac{1}{c}\frac{\partial \phi_E}{\partial t}$ (Ampere's law. I is current enclosed by contour C;

$\phi_E = \vec{E} -$ flux through surface bounded by C)

$$= \frac{4\pi}{c}(I + I_d) \qquad I_d = \frac{1}{4\pi}\frac{\partial \phi_E}{\partial t} = \text{displacement current.}$$

Self inductance: $\varepsilon = -L dI/dt$

Mutual Inductance: $\mathcal{E}_1 = -M_{12} dI_2/dt\,; \mathcal{E}_2 = -M_{21} dI_1/dt\,; M_{12} = M_{21}$

Magnetic energy: $U = \frac{1}{8\pi}\int B^2 dV$

Energy stored in an inductor: $U_L = \frac{1}{2} L I^2$

Impedance: $\tilde{V} = \tilde{I} Z_{tot.} \quad Z_R = R \quad Z_L = i\omega L \quad Z_C = 1/(i\omega C)$

Complex numbers: Some handy things to remember.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\text{if} \quad z = a + ib \quad \text{then z may be rewritten}$$

$$z = |\mathrm{z}| e^{i\theta}$$

$$\text{where} \quad |\mathrm{z}| = \sqrt{a^2 + b^2}$$

$$\tan\theta = b/a$$

Time dilation: Moving clocks run slow: $\Delta t_{stationary} = \gamma \Delta t_{moving}$

Length contraction: Moving rulers are shortened: $L_{stationary} = L_{moving}/\gamma$

Transformation of fields: $||$ denotes parallel to $\vec{v}$, $\perp$ denotes perpendicular to $\vec{v}$

$$\vec{E}'_{||} = \vec{E}_{||} \qquad \vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \frac{\vec{v}}{c}\times\vec{B}_{\perp})$$

$$\vec{B}'_{||} = \vec{B}_{||} \qquad \vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c}\times\vec{E}_{\perp})$$

Plane wave: a plane wave propagating with wave vector $\vec{k}$ is described by

$$\vec{E} = \vec{E}_0 f(\vec{k}\cdot\vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_0 f(\vec{k}\cdot\vec{r} - \omega t)$$

$$\rightarrow k = 2\pi/\lambda; \qquad ck = \omega \qquad |\vec{E}_0| = |\vec{B}_0|$$

$\vec{E}\times\vec{B}$ is parallel to $\vec{\mathrm{k}}$, the propagation direction.

Poynting vector: $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$

Electromagnetic energy flow: the rate at which energy flows through a surface S is given by $P = \int_S \vec{S} \cdot d\vec{a}$.

Useful Math

Cartesian.

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial t}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial t}{\partial z}\hat{\boldsymbol{z}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \vec{\boldsymbol{v}} = (\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z})\hat{\boldsymbol{x}} + (\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x})\hat{\boldsymbol{y}} + (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y})\hat{\boldsymbol{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical.

Gradient: $\nabla t = \frac{\partial t}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\varphi}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl:

$$\nabla \times \vec{\boldsymbol{v}} = \frac{1}{r\sin\theta}[\frac{\partial(\sin\theta v_\phi)}{\partial \theta} - \frac{\partial v_\theta}{\partial \phi}]\hat{\boldsymbol{r}} + \frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial(r v_\phi)}{\partial r}]\hat{\boldsymbol{\theta}} + \frac{1}{r}[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}]\hat{\boldsymbol{\varphi}}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial t}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$

Cylindrical.

Gradient: $\nabla t = \frac{\partial t}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial t}{\partial z}\hat{\boldsymbol{z}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho v_\rho) + \frac{1}{\rho}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \vec{\mathbf{v}} = [\frac{1}{\rho}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}]\hat{\boldsymbol{\rho}} + [\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho}]\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}[\frac{\partial(\rho v_\phi)}{\partial \rho} - \frac{\partial v_\rho}{\partial \theta}]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \frac{\partial t}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

Binomial expansion:

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \cdots (x^2 < 1); (1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \cdots (x^2 < 1)$$

Gradient theorem: $\int_{\vec{a}}^{\vec{b}} \text{grad} f \cdot d\vec{s} = f(\vec{b}) - f(\vec{a})$

Stokes' theorem: $\oint_C \vec{F} \cdot d\vec{l} = \int_S \left(\vec{\nabla} \times \vec{F}\right) \bullet d\vec{a}$

Gauss' theorem: $\oint_S \vec{F} \cdot d\vec{A} = \int_V \left(\nabla \bullet \vec{F}\right) dV$